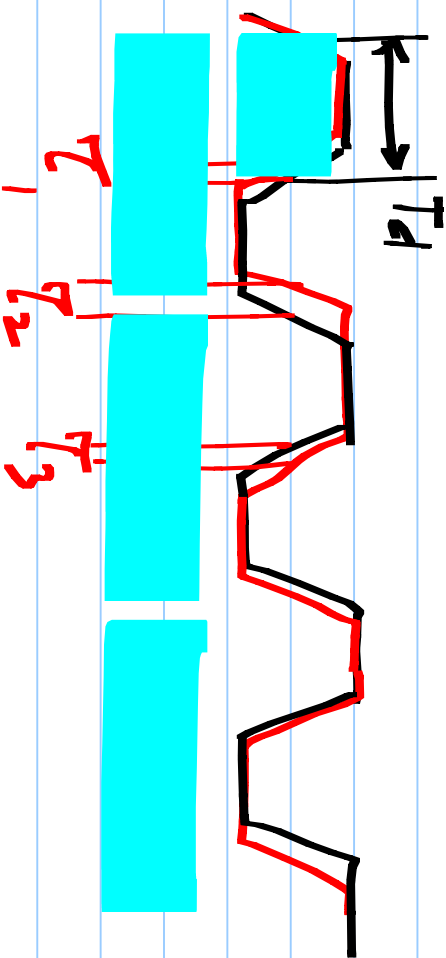


EEG 322

VCO phase noise

1/9/2/20/8



noise less

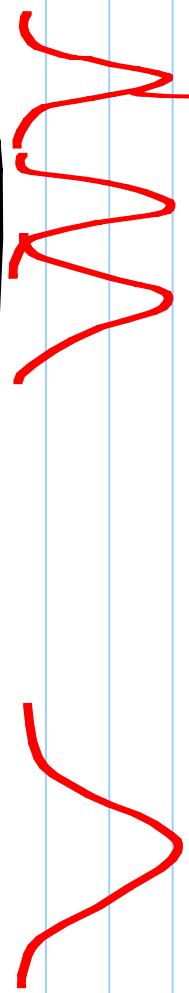
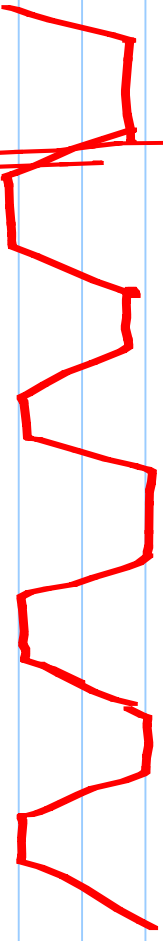
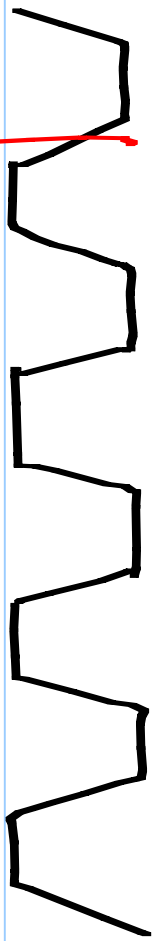
$T_k \cdot \frac{2\pi}{T}$  : phase noise  
in radians

Nominal half period :  $T_d$  (delay of the chain)

Actual  $k^{\text{th}}$  half period :  $T_d + (-1)^{k+1} \cdot \frac{v_n(nT_d)}{S}$

Nominal  $k^{\text{th}}$  edge :  $k \cdot T_d$   
slope

Actual  $k^{\text{th}}$  edge :  $kT_d + \dots$



Absolute jitter:

edge timing compared to an ideal periodic

clock (first edge

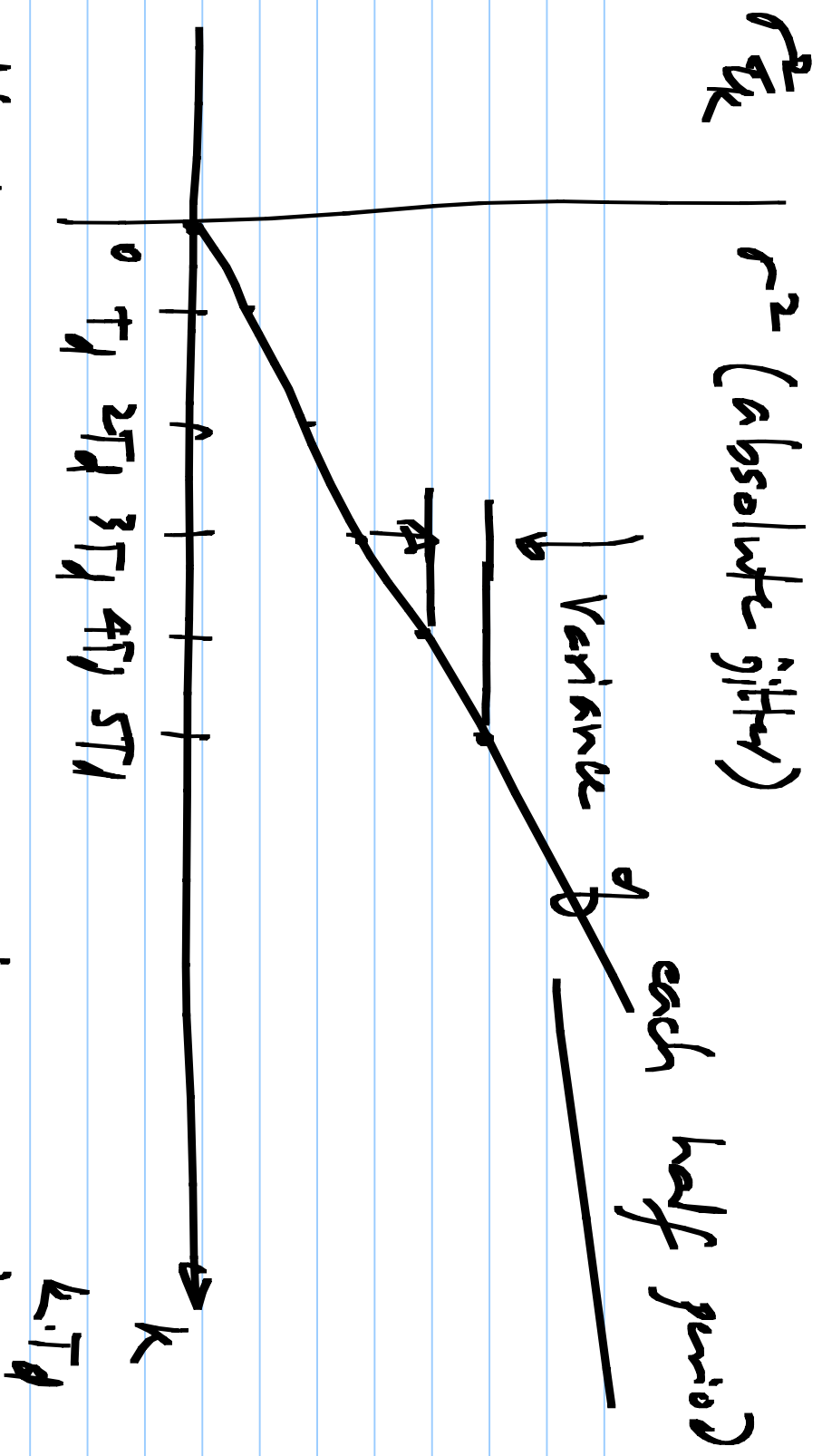
synchronized)

$$y = [x_1 + x_2 + x_3 \dots + x_N]$$

$x_k$ : i.i.d st.d. dev.  $\sigma$

st.d dev.  $\sigma$

$$y = \sqrt{N} \cdot \sigma$$

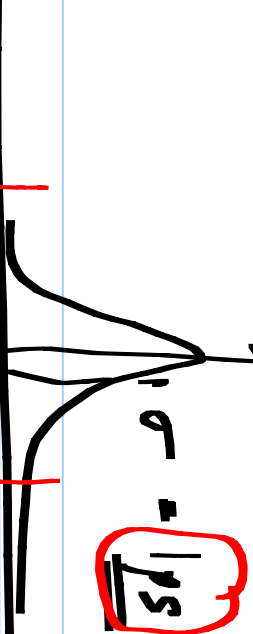


Absolute jitter variance increases linearly with time

(Half) period jitter: constant variance (white)

1st edge .

$$1 \text{ GHz osc.}$$

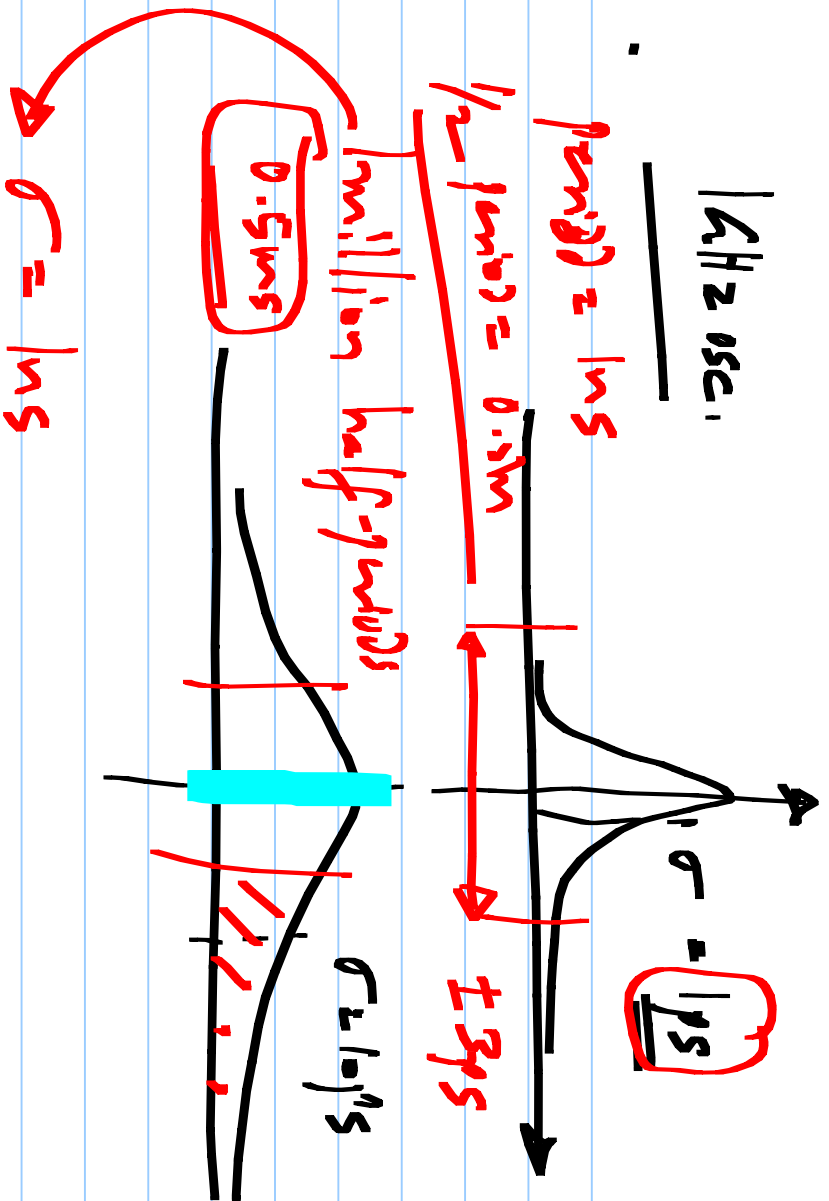


100th edge

$$1/2 \text{ period} = 0.5 \text{ ns}$$

million half-periods

0.5 ns



"Longitudinal" problem

$$\Gamma_k - \Gamma_{k-1} = (-1)^{k+1} \cdot \frac{v_n(kT_0)}{s} = \frac{v_n'(kT_0)}{s}$$

$\Gamma_k$  is white,  $v_n(kT_0)$  is also white]

---

$$\Gamma(z) (1 - z^{-1}) = \frac{v_n'(z)}{s}$$

$$\frac{v_n'}{s} \xrightarrow{\text{white}} \boxed{\frac{1}{1-z^{-1}}} \xrightarrow{\text{white}} \Gamma(z) = \frac{1}{1-z^{-1}} \frac{v_n'(z)}{s}$$

$$C(z) = \frac{1}{1-z^{-1}} \frac{V_n'(z)}{s} \quad z = \exp(j2\pi f/2f_0)$$

(f<sub>0</sub>: oscillation freq.)

$$S_z(f) = S_V(f) \cdot \frac{1}{s^2} \cdot \left| \frac{1}{1 - \exp(-j\frac{\pi f}{f_0})} \right|^2$$

constant

with freq.

$$\left| j2 \cdot \sin\left(\frac{\pi f}{2f_0}\right) \cdot \exp\left(-j\frac{\pi f}{2f_0}\right) \right|^2$$

$$= S_V(f) \cdot \frac{1}{s^2} \cdot \frac{1}{4 \sin^2 \frac{\pi f}{2f_0}}$$

Sampled noise process, white  
(const PSD from 0 to  $f_0$ )

$$S_T(f) = S_V(f) \cdot \frac{1}{s^2} \cdot \frac{1}{4 \sin^2 \frac{\pi f}{2f_0}}$$

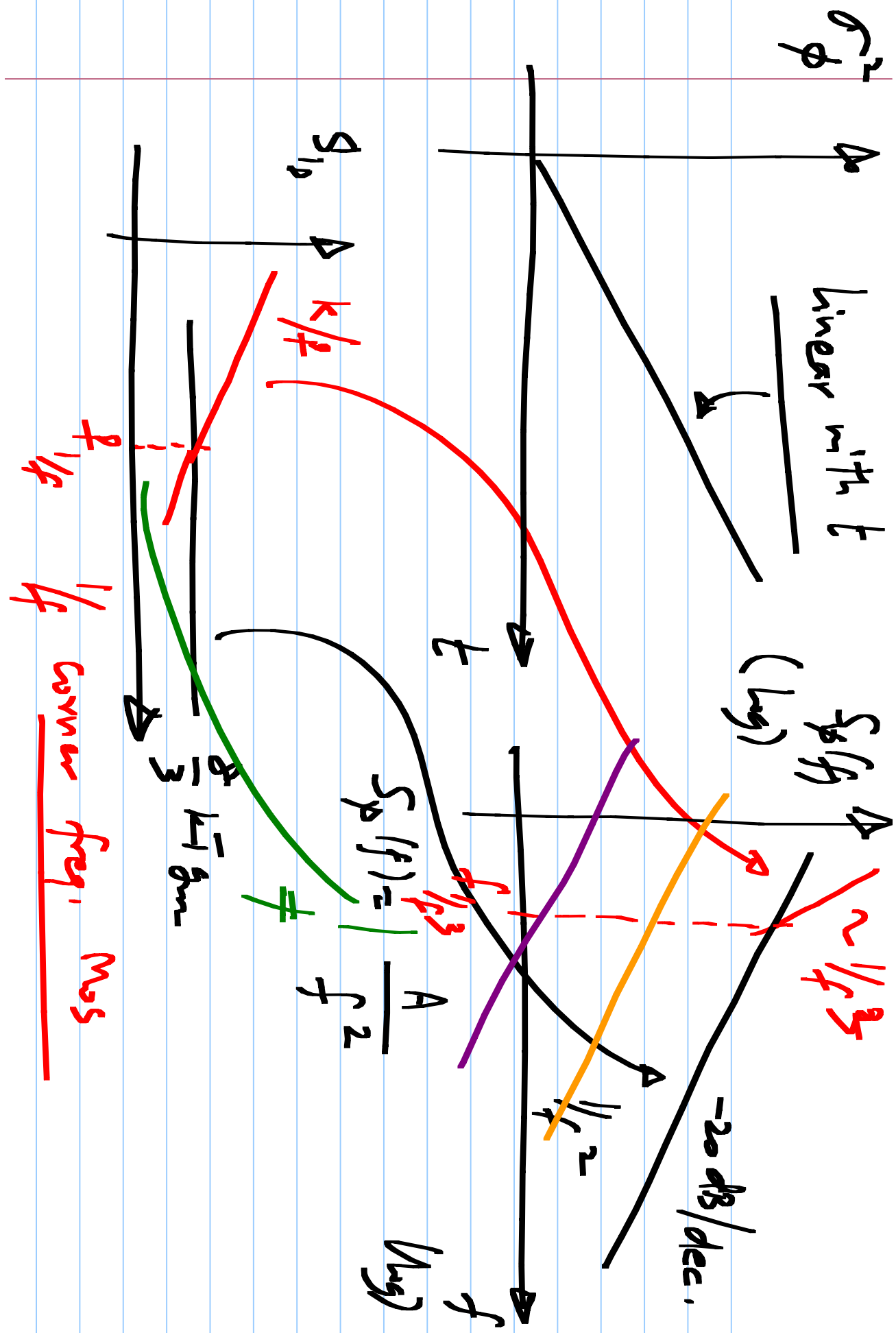
$$\approx S_V(f) \cdot \frac{1}{s^2} \cdot \frac{(2f_0)^2}{4 \cdot (\pi f)^2}$$

$f \ll f_0$

$$= S_V(f) \cdot \frac{1}{s^2} \cdot \frac{1}{\pi^2} \cdot \left(\frac{f_0}{f}\right)^2$$

$\phi = 2\pi f_0 \cdot T_k$

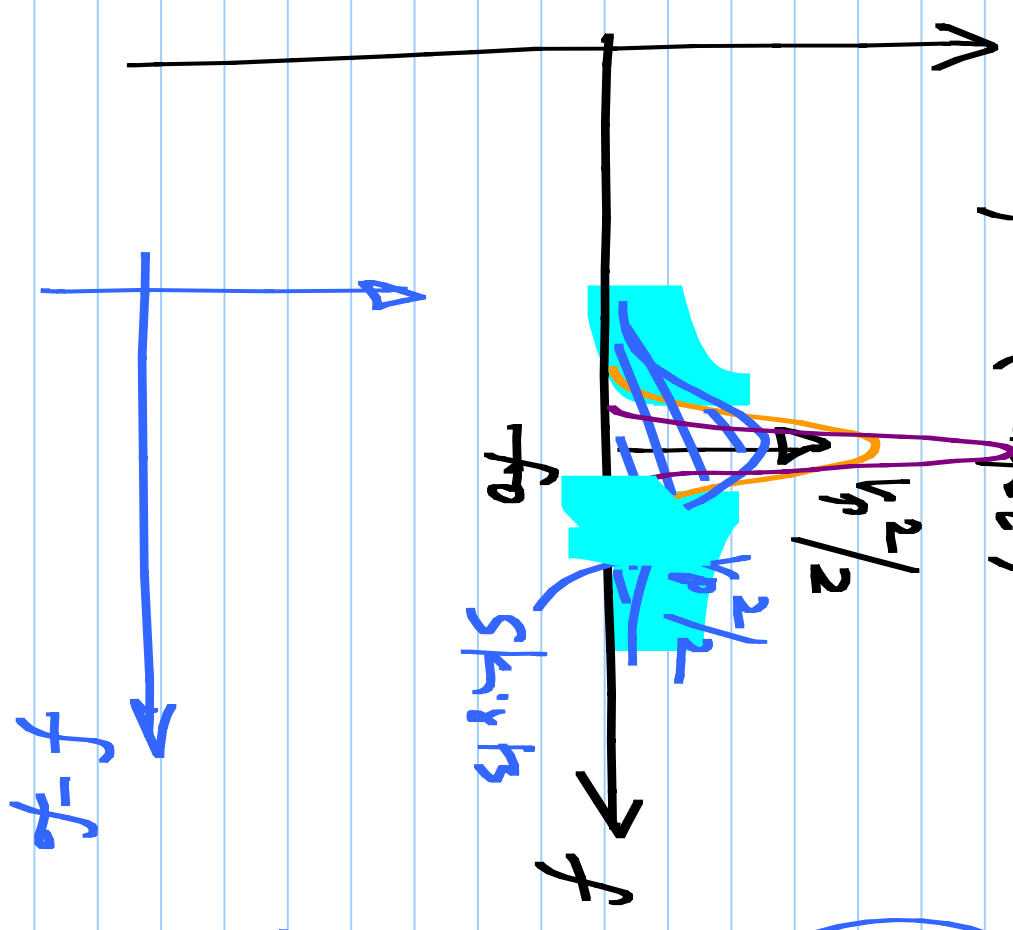
$$S_p(f) = \frac{S_V(f) \cdot \frac{1}{s^2} \cdot 4 \frac{f_0^4}{f^4}}$$





ideal oscillator:

$$V_p \cos(2\pi f_c t)$$



$$V_p \cos(2\pi f_c t + \phi(t))$$

phase noise

$$V_p \cos(2\pi f_c t) \cdot \cos(\phi(t))$$

$$-V_p \sin(2\pi f_c t) \cdot \sin(\phi(t))$$

$$\approx V_p \cos(2\pi f_c t)$$

$$-V_p \cdot \phi(t) \cdot \sin(2\pi f_c t)$$

$$|f - f_0|$$

$$V_p \cos(2\pi f_0 t)$$

$$V_p \cdot \text{sgn}(\cos 2\pi f_0 t) + \phi(t)$$

$$V_p \cos(2\pi f_0 t + \phi(t))$$

$$V_p \cdot \sum_{n: \text{odd}} \frac{4}{n\pi} \cos(2\pi n f_0 t)$$

$$\sum_{n\pi} \frac{4}{n\pi} \cos(2\pi n f_0 t + n\phi(t))$$