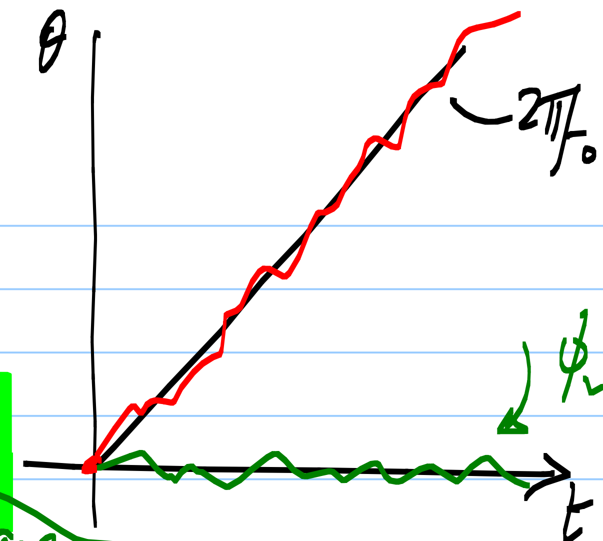
Lecture 23

How much the phase error grows over time T

Noiseless osc — $\theta = 2\pi f_0 t$

Noisey osc — $\theta = 2\pi f_0 t +$

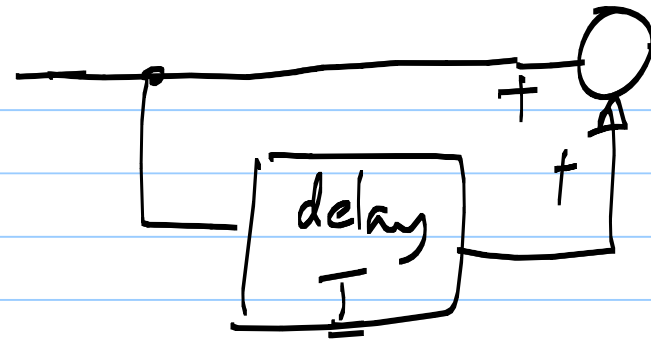
Phase noise



$= \phi(t+T) - \phi(t)$ — phase change over an interval T

Variance given the $S_\phi(f)$

$\phi(t)$



$\phi(t) - \phi(t-T)$

$\phi(f) \; \underbrace{\qquad\qquad}_{1-\exp(-j2\pi f T)}$

$$\boxed{\Delta\phi_T(f) = \phi(f)\underbrace{\left(1-\exp(-j2\pi f T)\right)}_{|\cdot|^2 = 4\sin^2(\pi f T)}}$$

$$S_{\Delta\phi_T}(f) = S_\phi(f)\cdot \underbrace{\underline{4\sin^2(\pi f T)}}_{\cdot\exp(-j\pi f T)\left[\exp(j\pi f T) - \exp(-j\pi f T)\right]}$$

$$\text{Variance} = \int_0^\infty \text{Spectral density} \cdot df \qquad \frac{\sin^2(\pi f T)}{(\pi f T)^2}$$

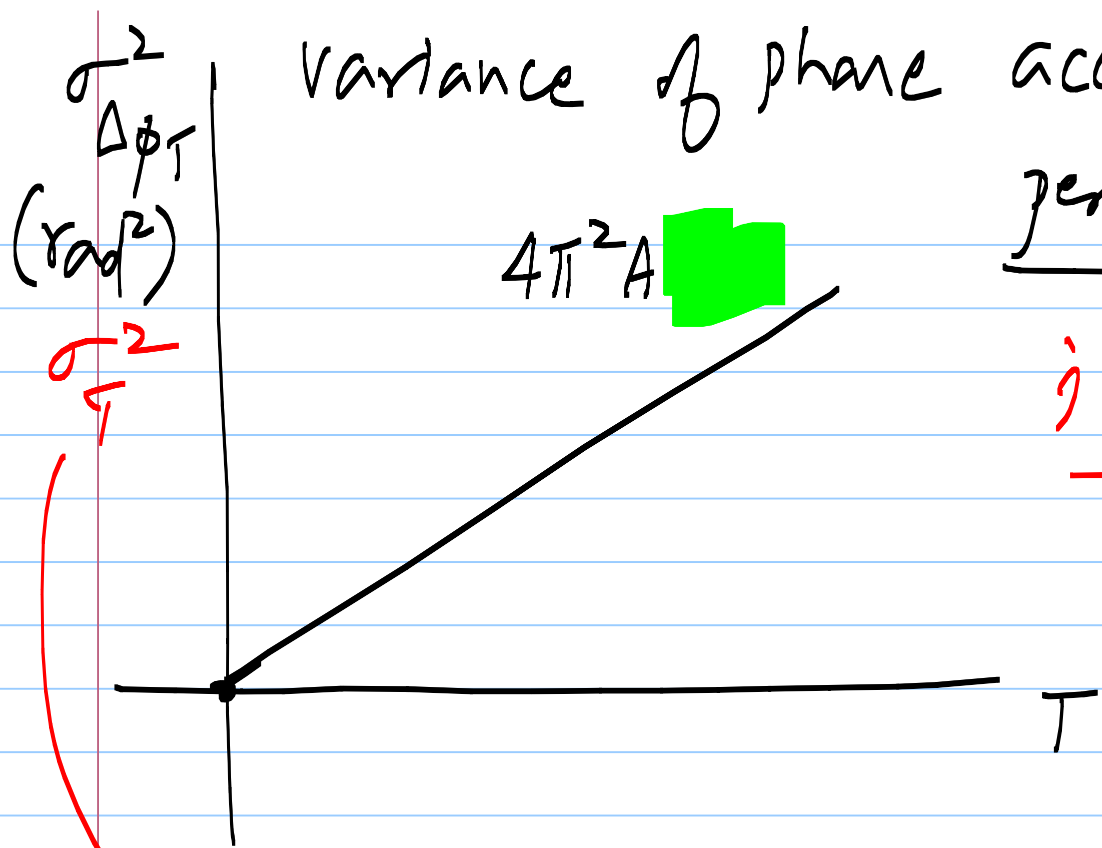
$$\sigma^2_{\Delta\phi_T} = \int_0^\infty \underbrace{S_\phi(f)}_{2\mathcal{L}(f)} \cdot 4\sin^2(\pi f T) \cdot df \qquad \text{sinc}^2()$$

$$= \int_0^\infty \frac{A}{f^2} \cdot 8 \cdot \sin^2(\pi f T) \cdot df$$

Thermal noise in a VCO

$$= \underline{4\pi^2 \cdot A \cdot T}$$

variance of phase accumulated over a period T

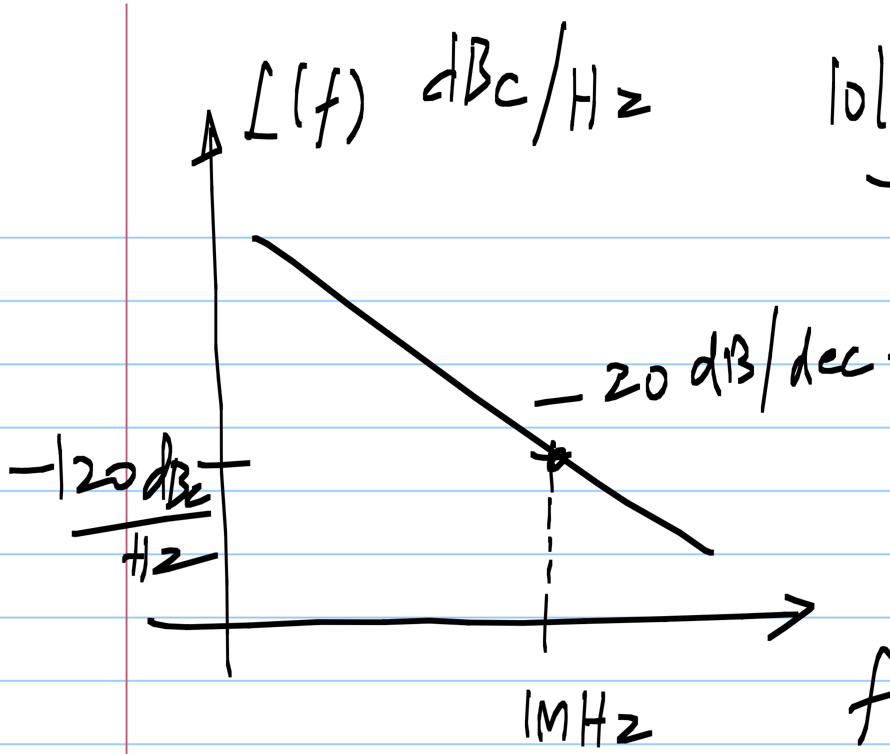


jitter: T $\qquad \dfrac{\Delta\phi_T}{2\pi} \cdot T_0$ (oscillation period)

period jitter

$$4\pi^2 \cdot AT \cdot \left(\frac{T_0}{2\pi}\right)^2 = \underline{A \cdot T_0^{\,2} \cdot T}$$

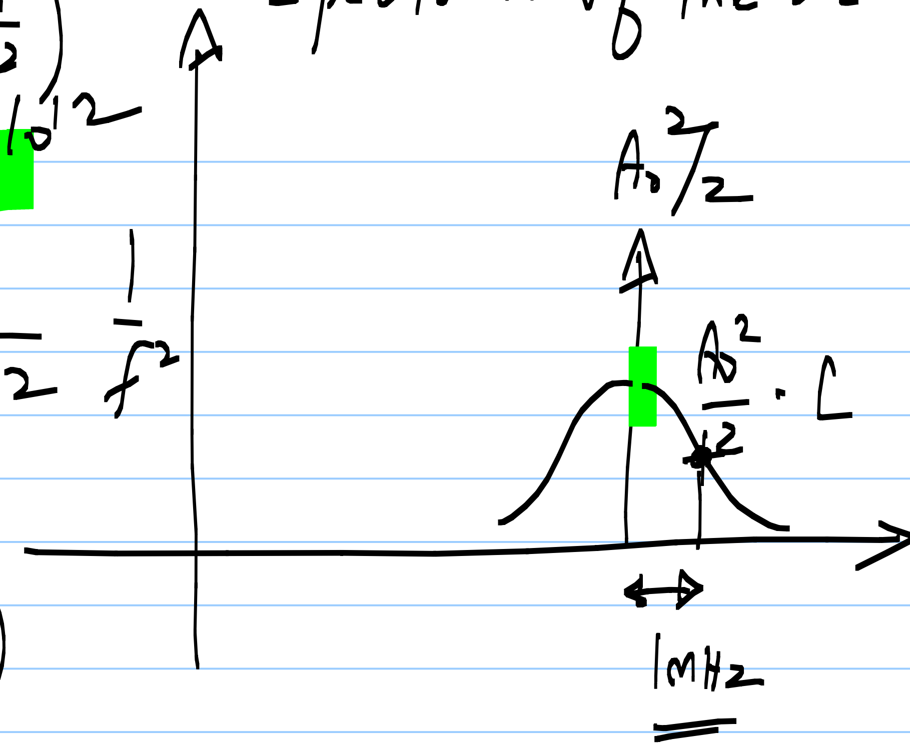
$$= AT_0^{\,3}$$



$10\log\left(\frac{A}{f^2}\right)$ 10^{12}

$\frac{A}{f^2}$ $\frac{1}{f^2}$

spectrum of the osc.



-120 dBc/Hz phase noise @ 1MHz offset

$A = 1 \cdot Hz$

2×10^{-10} s

5GHz osc.

variance of period jitter $= A T_0^{3} = 8 \cdot 10^{-30}$

rms period jitter $= \sqrt{8} \cdot fs \quad 2.8\, fs$

$-120\, dBc/Hz$ $\qquad \dfrac{x}{\sqrt{1000}} \qquad \boxed{90\, fs}$

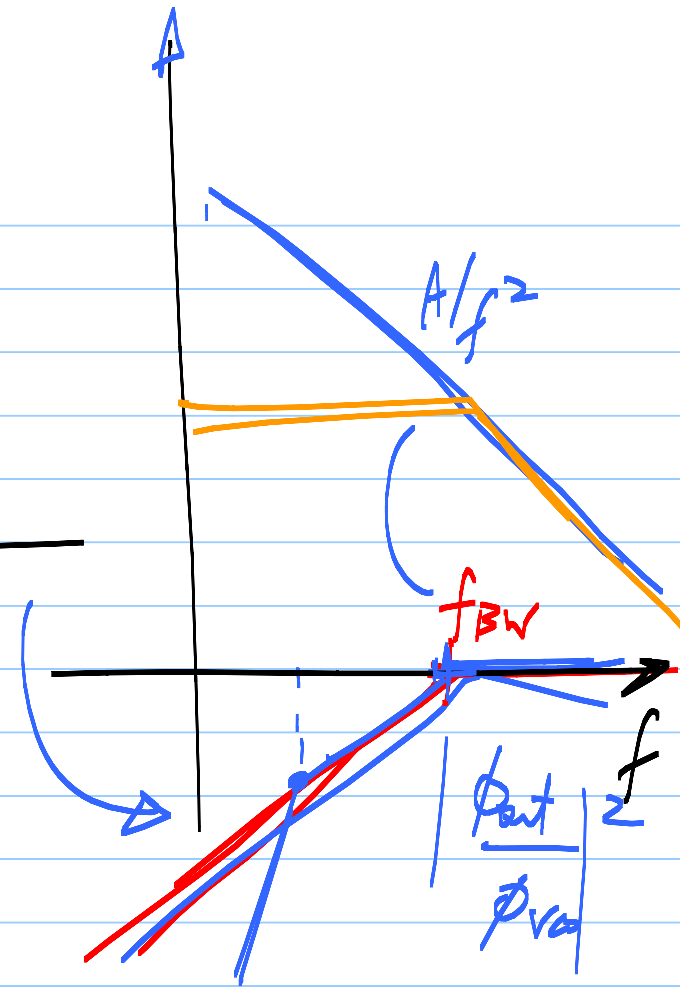
$\downarrow$

$-80\, dBc/Hz$ $\qquad 280\, fs$

VCO in a closed loop:

CDR loop with a linear PD:

$$\frac{\phi_{out}}{\phi_{vco}} = \frac{s^2(\ )}{s^2(\ ) + s(\ ) + 1}$$

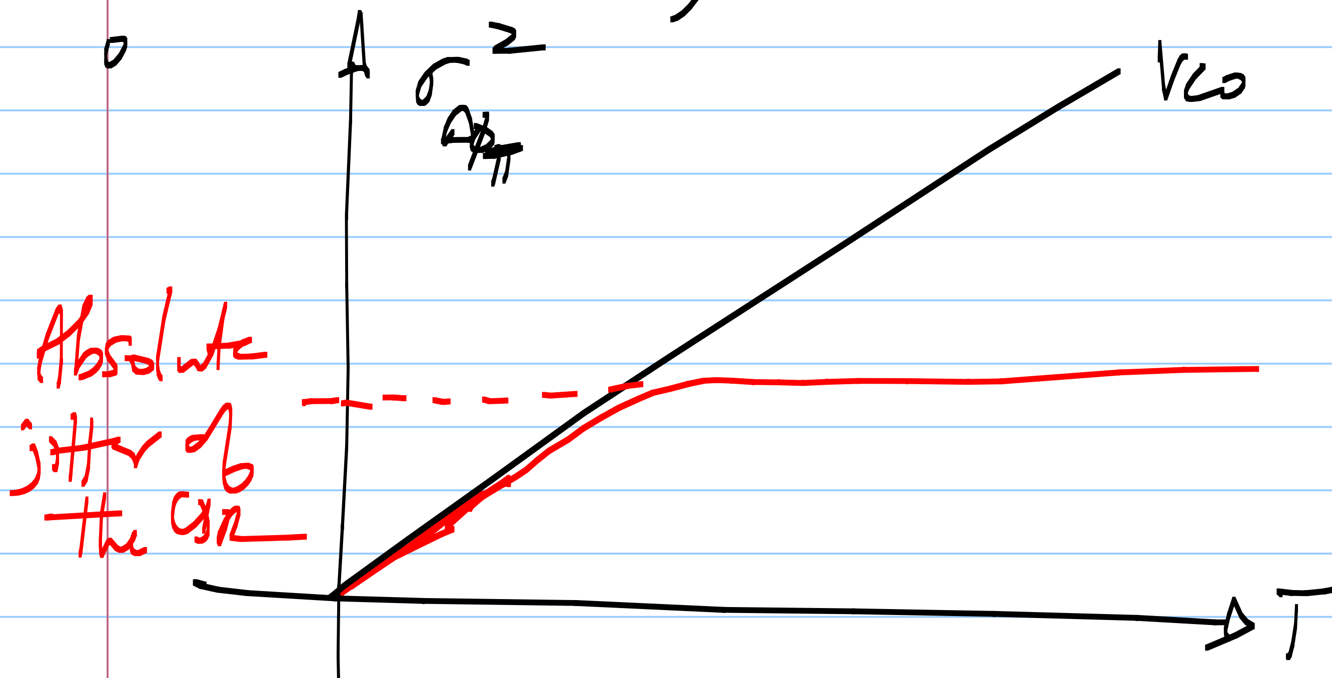


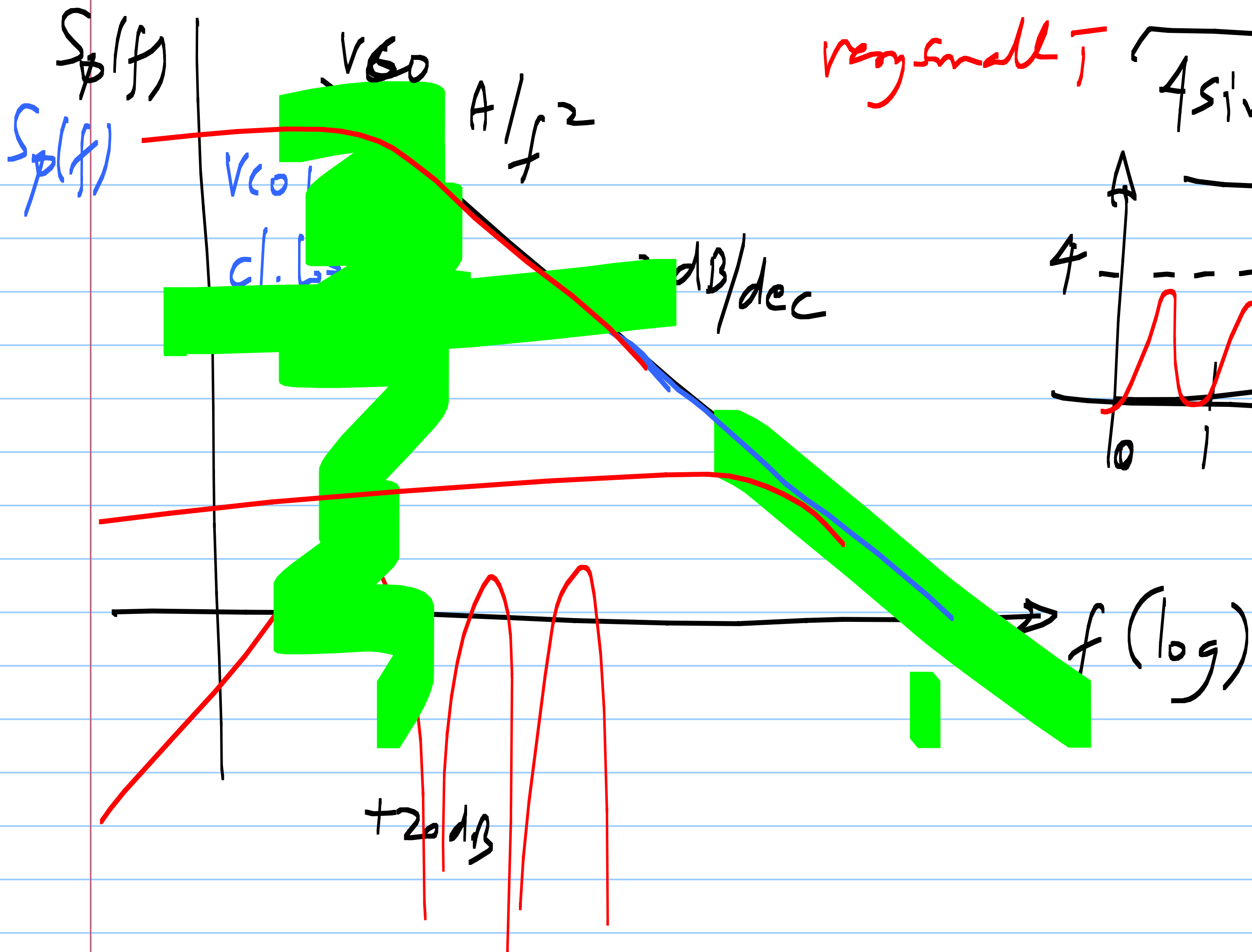
$s = j2\pi f$

$$\frac{\phi_{out}}{\phi_{vco}} = \frac{j\,f/f_{BW}}{1 + j(f/f_{BW})}$$

$$\sigma^2_{\Delta\phi_T} = \int_0^\infty \frac{A}{f^2} \cdot \frac{\left(f/f_{BW}\right)^2}{1+\left(\frac{f}{f_{BW}}\right)^2} \cdot 4\sin^2(\pi f T)\, df$$

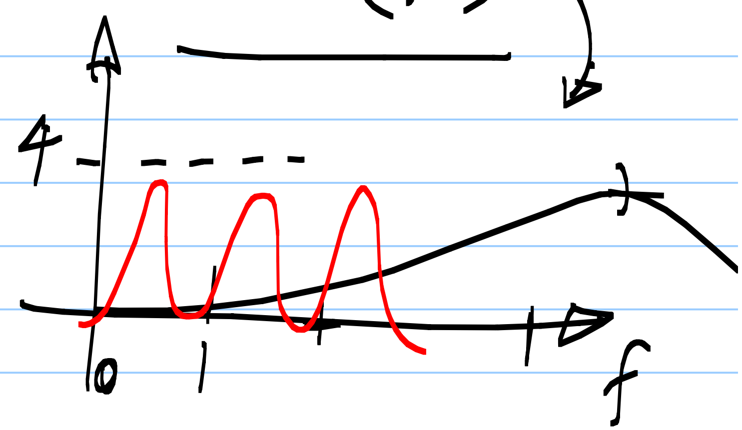
$$\left|1 - \exp(-j2\pi f T)\right|$$





very small T

$$4\sin^2(\pi f T)$$



$$4\pi^2 f^2 T^2$$