Lecture 38

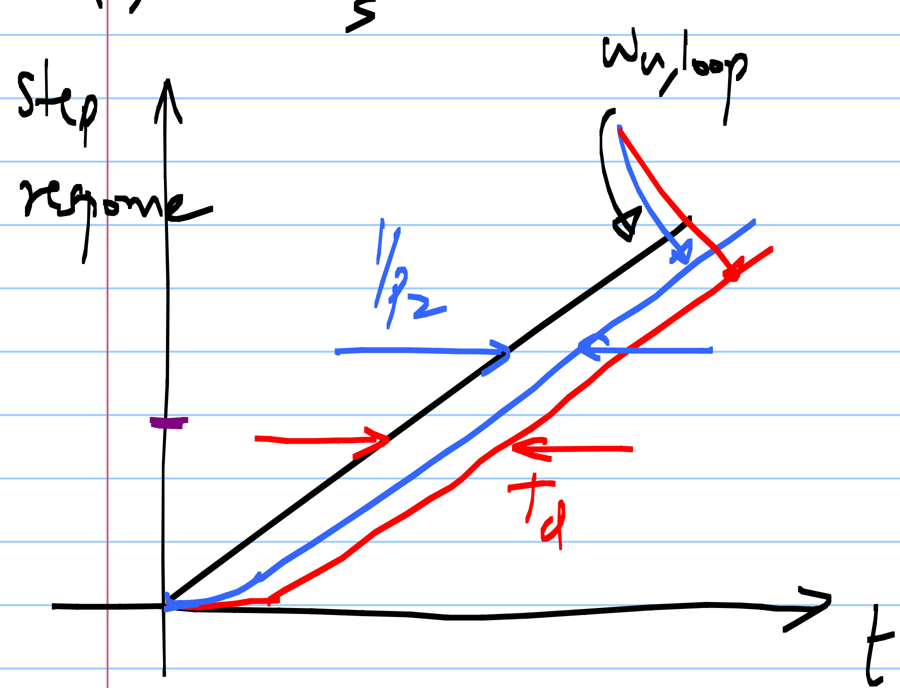
5/4/2016

$\dfrac{1}{\omega_{u,loop}}$

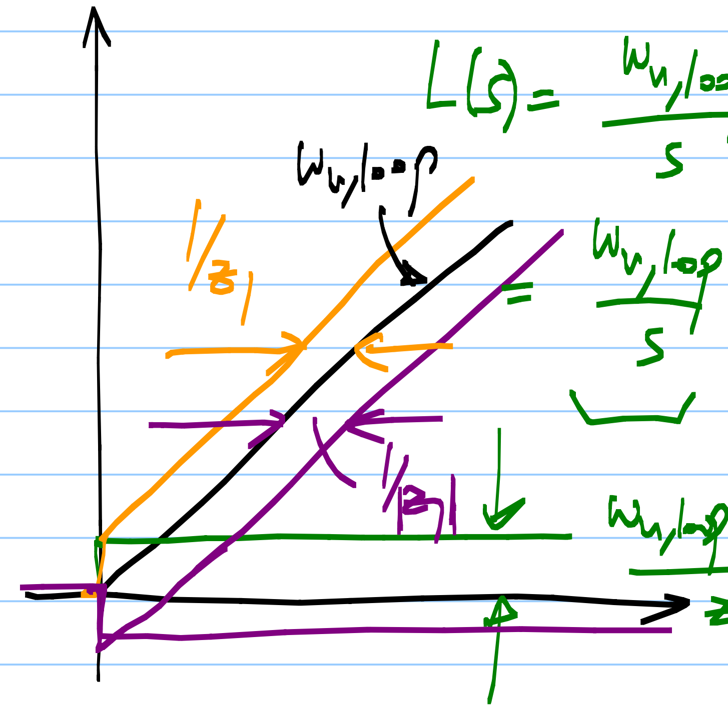
$$L(s) = \frac{\omega_{u,loop}}{s}$$

$$L(s) = \frac{\omega_{u,loop}}{s} \exp(-sT_d)$$

$$L(s) = \frac{\omega_{u,loop}}{s} \cdot \frac{1}{1 + s/p_2}$$



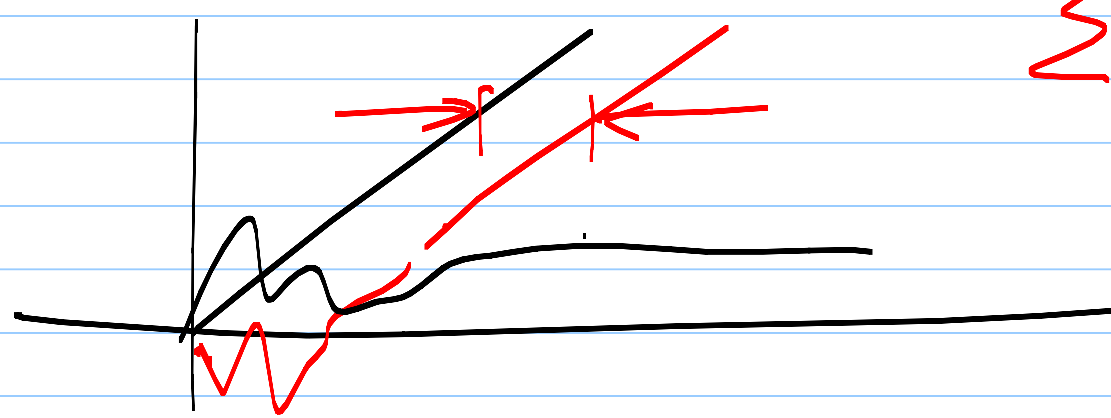
$$L(s) = \frac{\omega_{u,loop}}{s}\left(1 + s/z_1\right)$$



$$= \frac{\omega_{u,loop}}{s} + \frac{\omega_{u,loop}}{z_1}$$

$$\left(\frac{\omega_{n,loop}}{s}\right)\left(\frac{\cancel{\sum b_n s^n}\ 1}{\sum a_n s^n}\right)$$

$$\omega_{n,loop}\left[\frac{1}{s} - \underbrace{\frac{(\ )}{\sum a_n s^n}}\right]$$



$$\sum \frac{1}{p_k}$$

Excess delay T_d can cause instability

Degrade stability

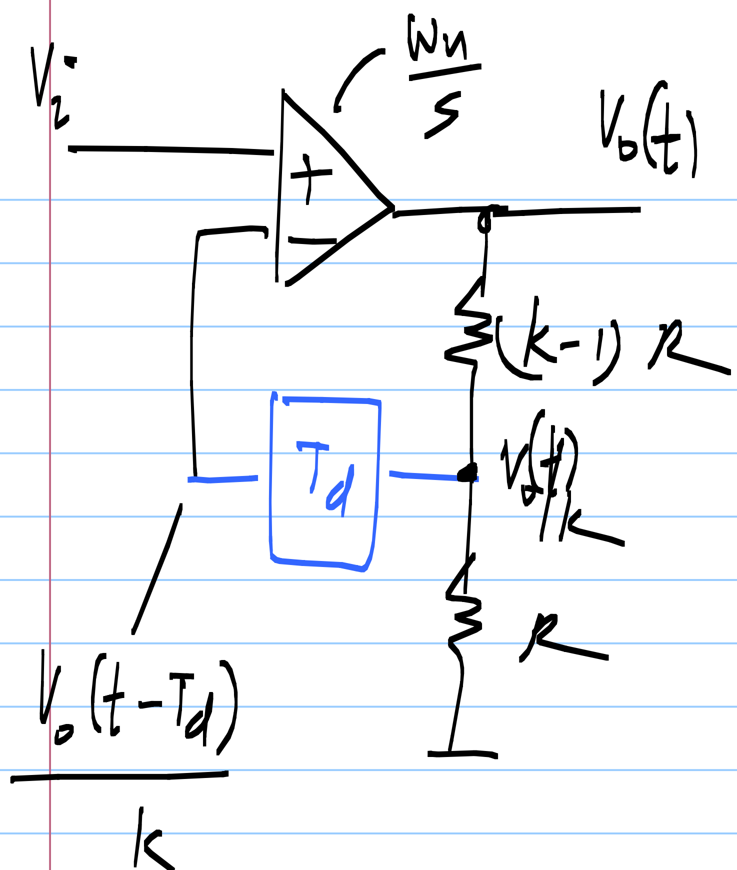
$T_d \ll \frac{1}{\omega_{u,loop}}$, no instability

(LHP) p_2 in the loop gain $\equiv$ Delay of $\frac{1}{p_2}$

Enhances stability

(LHP) z_1 $\{z_1 > 0\}$ in the loop gain $\equiv$ Advance of $\frac{1}{z_1}$

(RHP) z_1 $\{z_1 < 0\}$ in the loop gain $\equiv$ Delay of $\frac{1}{z_1}$



$$\left(\frac{dV_o}{dt} + \frac{\omega_u}{k} \cdot V_o = \omega_u \cdot V_i \right)$$

$$\frac{k}{\omega_u} \cdot \frac{dV_o(t)}{dt} + V_o(t) = k \cdot V_i(t)$$

Delay differential equation

$$\underbrace{\frac{k}{\omega_u} \cdot \frac{dV_o(t)}{dt} + V_o\left(t - T_d\right) = k \cdot V_i(t)}$$

Assume $V_i = 0$ find T_d, find ω

Solution: $V_p \cos(\omega t)$ $\left.\frac{}{V_p \exp(j\omega t)}\right\}$

$$\frac{k}{\omega_n} \cdot \frac{dV_o(t)}{dt} + V_o(t - T_d) = \cancel{k \cdot V_i(t)}\ 0 \qquad V_o(t) = V_p \cos \omega t$$

$$-\frac{k}{\omega_n} \cdot \omega \cancel{V_p} \sin \omega t + \cancel{V_p} \cos(\omega(t - T_d)) = 0$$

$$\cos \omega t \cdot \underbrace{\cos \omega T_d}^{\pi/2} + \sin \omega t \sin \omega T_d = \frac{k}{\omega_n} \cdot \omega \cdot \sin \omega t$$

$$\omega T_d = \pi/2 \qquad \sin \omega t = \frac{k}{\omega_n} \cdot \omega \sin \omega t$$

$$\omega = \frac{\omega_n}{k} \ ; \qquad T_d = \frac{\pi}{2} \cdot \frac{k}{\omega_n} \ .$$

instability

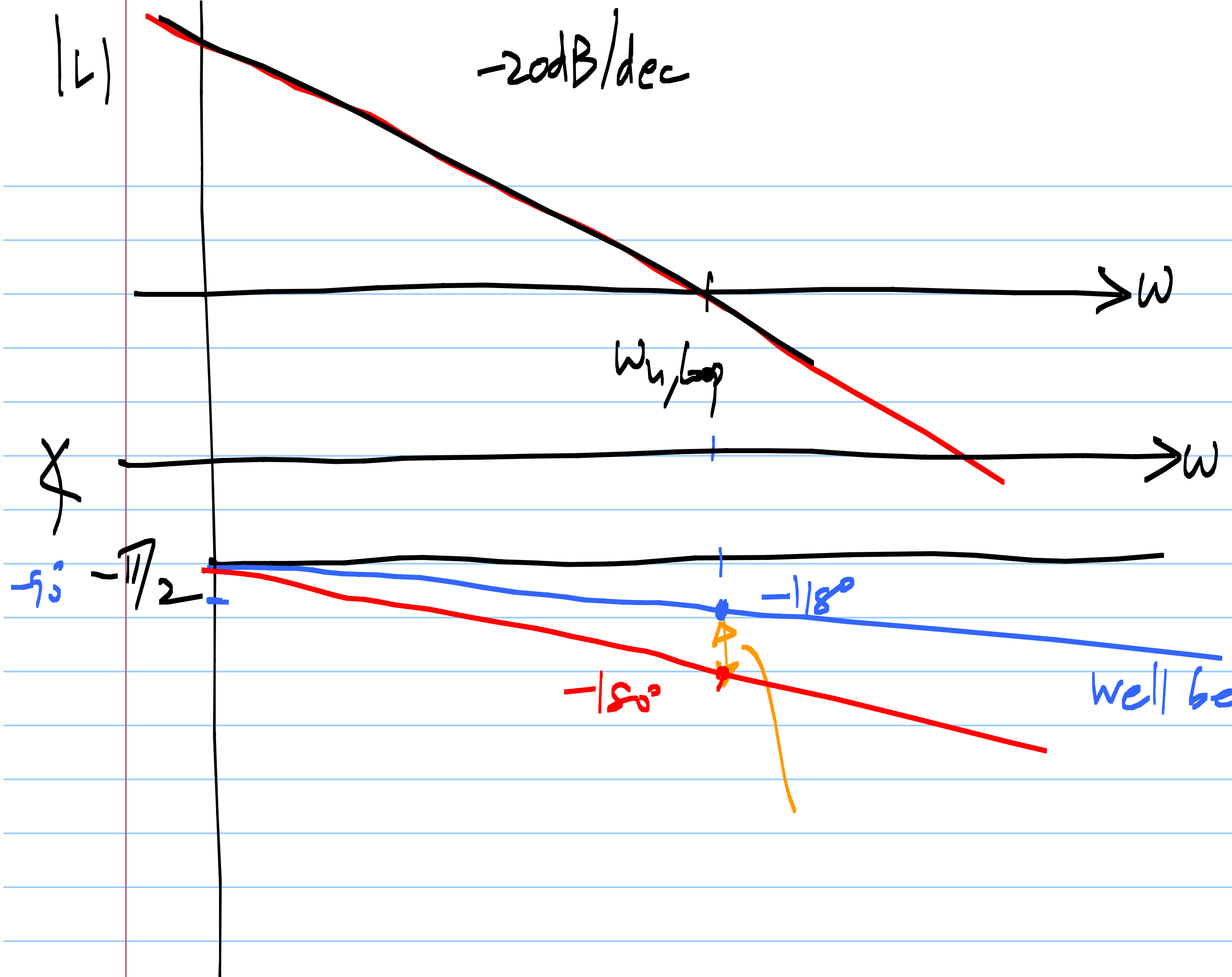
$T_d = 0$ stable $0 \leq T_d < \pi/2 \cdot k/\omega_n$

$T_d = \frac{\pi}{2} \cdot \frac{k}{\omega_n}$. Unstable – constant amplitude sinusoid

$T_d > \frac{\pi}{2} \frac{k}{\omega_n}$: Solution blows up

Well behaved: $0 \leq T_d < 0.5 \frac{k}{\omega_n}$

$0.5 \frac{1}{\omega_{n,loop}}$



$$L(s) = \frac{\omega_{n,loop}}{s}$$

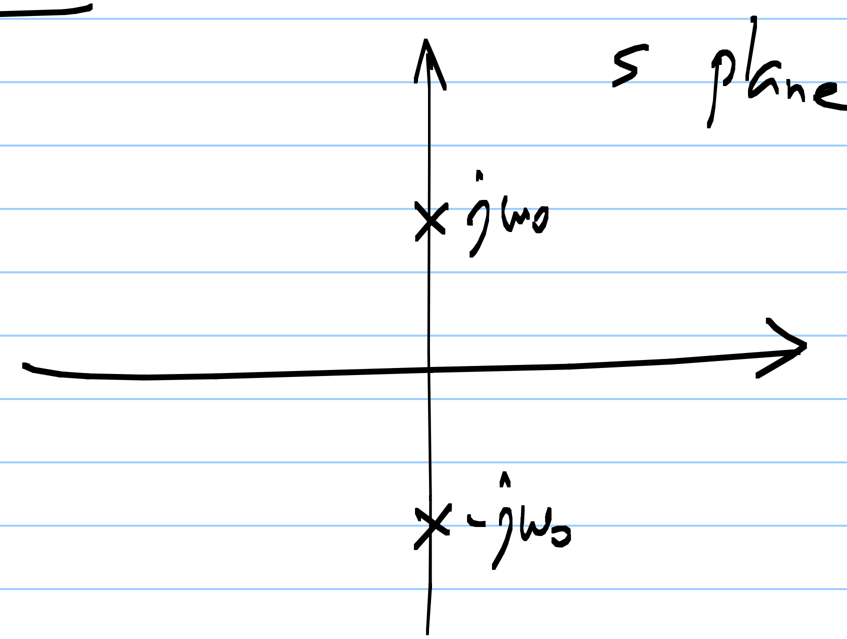
$$L(s) = \underbrace{\underbrace{\frac{\omega_{n,loop}}{s}}_{-\pi/2} \exp(-sT_d)}_{-\pi/2 - \omega T_d}$$

$$T_d \le 0.5 \frac{1}{\omega_{n,loop}}$$

$$T_d = \frac{\pi}{2} \cdot \frac{1}{\omega_{n,loop}}$$

$$H(s) = \left(\quad \right) \cdot \frac{L}{1+L}$$

$$L(j\omega_0) = -1$$



$L(\underbrace{j\omega_0}_{\omega_{u,loop}}) = -1 \Rightarrow |L| = 1$ and $\angle L = -\pi$ at some frequency ω_0 implies instability

---

If $|L(j\omega_{u,loop})| = 1$ and $\angle L(j\omega_0) > -\pi$

then the system is stable

If $|L(j\omega_{u,loop})| = 1$ and $\angle L(j\omega_{u,loop}) < -\pi$

then the system is ~~st~~ unstable
