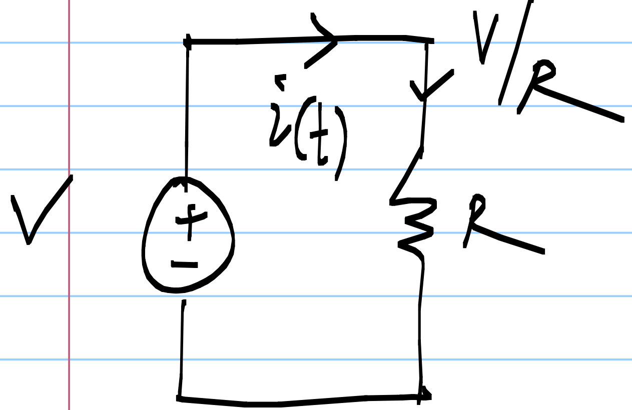
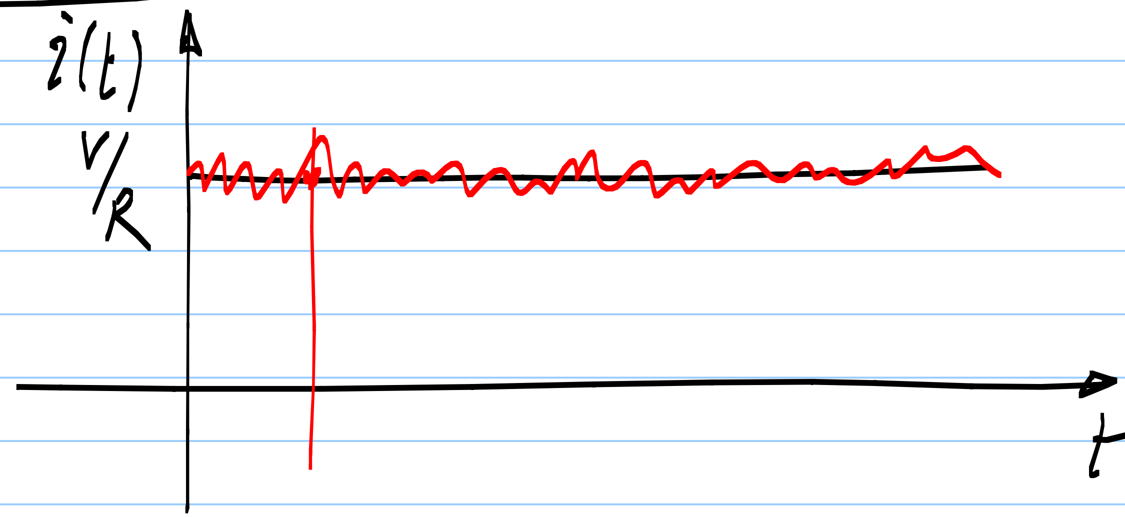
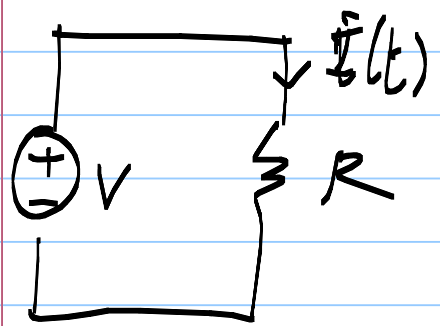
Noise : Random noise

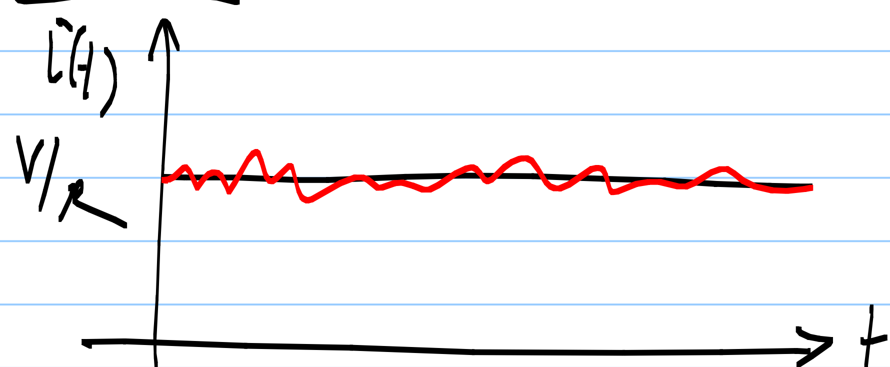




Lecture 10 9/2/2016

Noise in resistors





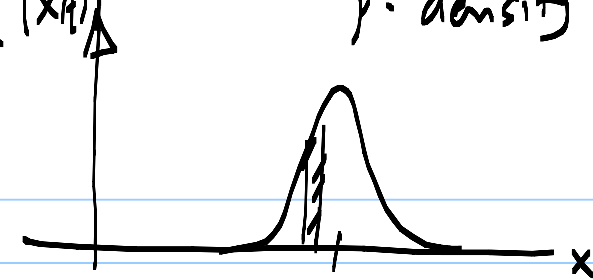
Mean = 0

Random process $x(t)$

Mean : $E(x(t))$

$$= \int_{-\infty}^{\infty} x \cdot p_x(x) \cdot d_x$$

$p_x(x(t))$ p. density fn.



Probability that the r.v has a value between x & $x+\Delta x$

$p_x(x) \cdot \Delta x$

Variance : $E\left(\left(x - E(x)\right)^2\right)$

"spread"

mean = 0 $\longrightarrow$ $E(x^2) = E(x^2(t))$

Autocorrelation: $E(x(t)\, x(t+\tau))$

Auto correlation: $E\left(x(t)\, x(t+\tau)\right)$ $\int_{-\infty}^{\infty} x(t)\, x(\underbrace{t-\tau}_{t})\, dt$

$t+\tau$

$x(t)$ $R_x(t) = \int_{-\infty}^{\infty} x(t)\, x(t+\tau)\, dt$

$\updownarrow \mathcal{F}$

$X(f)$

$x * y \; (\tau)$

$(t-\tau)$

$\int x(t)\, y(\tau - t)\, dt$

$\underline{x(t) * x(-t)}$ $y(t) = x(-t)$

$= \int_{-\infty}^{\infty} x(t) \exp(-j 2\pi f t)\, dt$

$X(f) \cdot X(-f)$

$X(f) \cdot X^*(f) = \underline{\underline{|X(f)|^2}}$

$$\mathcal{F}\left(x(-t)\right) = \int_{-\infty}^{\infty} x(t) \exp\left(-j 2\pi f t\right) \, dt$$

$$= \int_{-\infty}^{\infty} \underbrace{x(\tau) \exp\left(-j 2\pi (-f) \tau\right)} \, d\tau$$

$$= \underline{X(-f)}$$

$R_x(\tau) = E\left(x(t)\,x(t+\tau)\right)$

$E\left(x(t)\,x(t-\tau)\right)$

Correlation of noise between two time instants separated by τ

$R_x(-\tau) = R_x(\tau)$

Power Spectral Density

$\mathcal{F}$

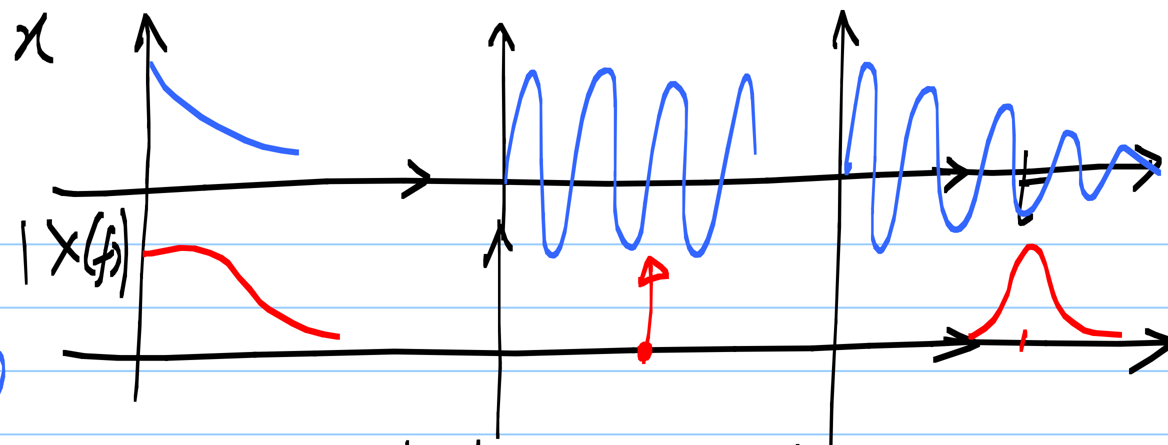
$$S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) \exp(-j2\pi f\tau)\, d\tau$$

Deterministic signal :

$$x(t) \xleftrightarrow{\mathcal{F}} X(f)$$

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t+\tau)\, dt \xrightarrow{\mathcal{F}} |X(f)|^2$$

$|X(f)|$, $|X(f)|^2 \longrightarrow$ Energy distribution of $x(t)$ over frequency



Random signal $x(t)$

$$R_x(\tau) = E(x(t)x(t+\tau)) \xrightarrow{\mathcal{F}} \underline{S_x(f)}$$ (Power) spectral density

$S_x(f)$: Energy distribution of $x(t)$ over frequency

variance of $x(t)$: $R_x(0) = E(x^2(t))$

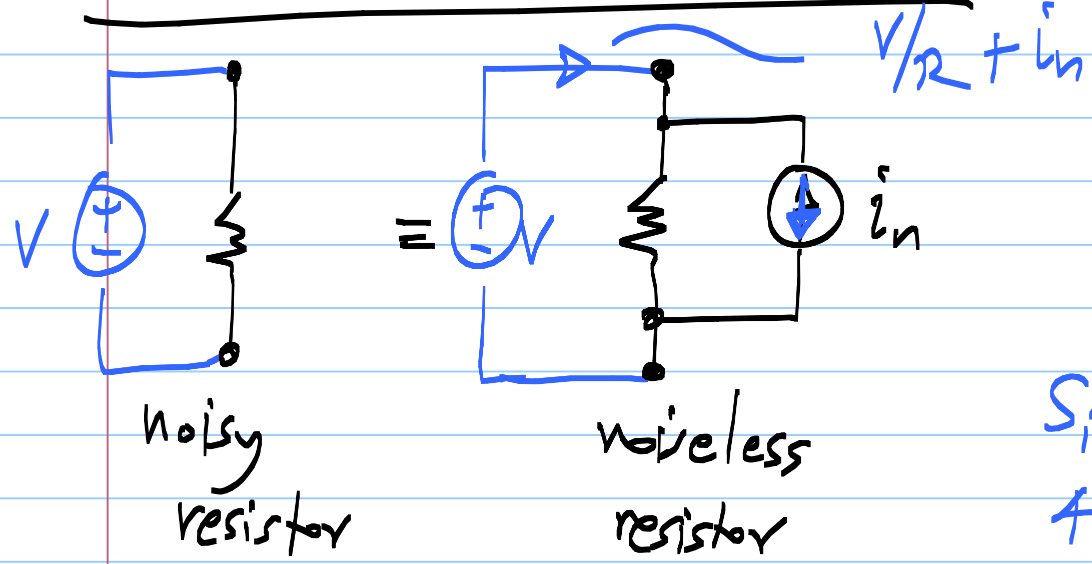
Variance of $x(t) = R_x(0) = \int_0^\infty \underbrace{S_x(f)}_{\text{defined for } f \geq 0} \, df$

$x(t)$: real ;

<u>Variance or Spectral density of noise</u>

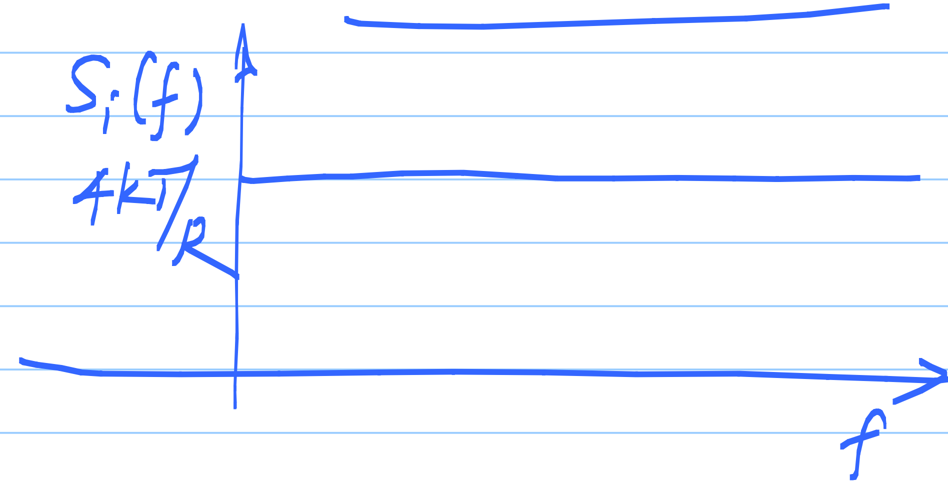
Lossless passive components L, C : noiseless

Noise in a resistor

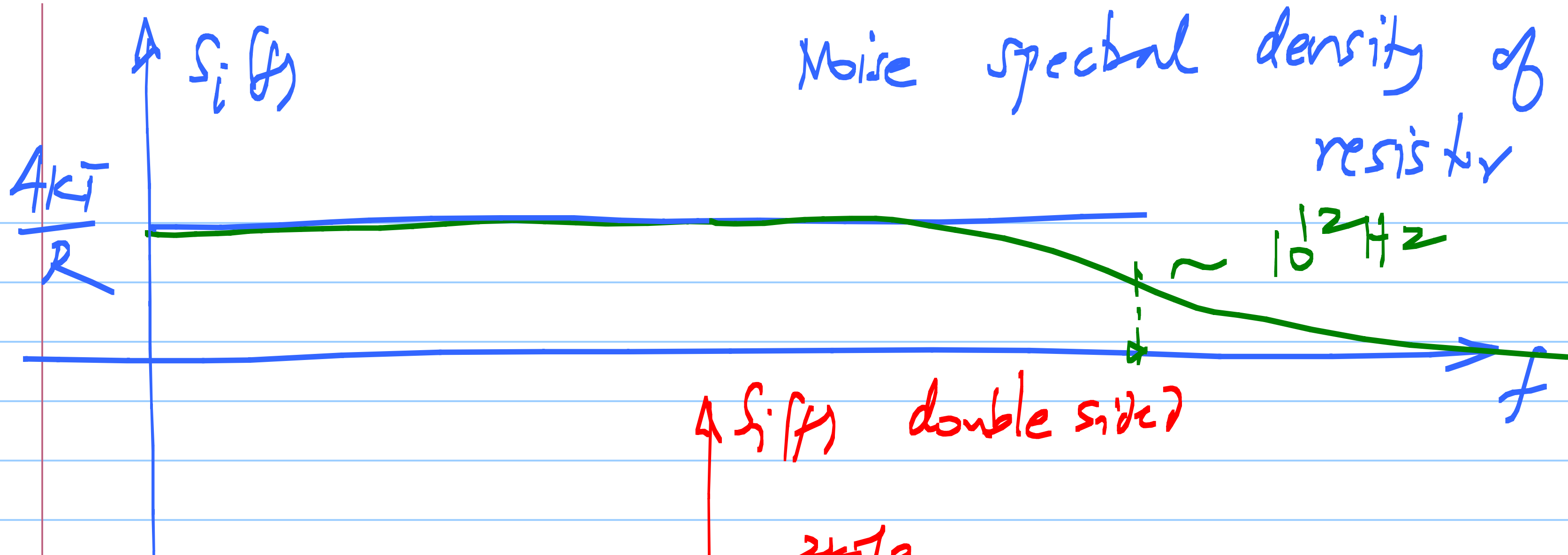


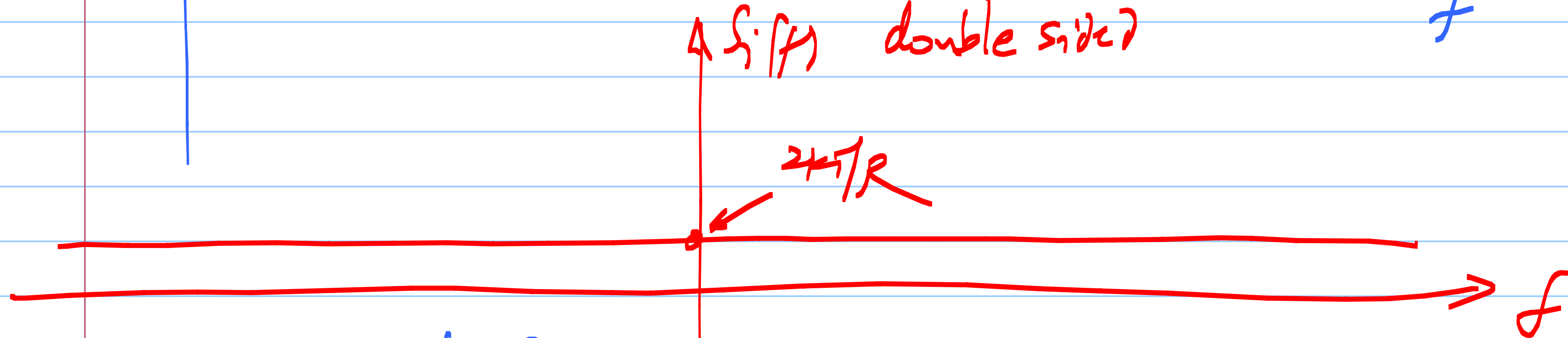
i_n : noise current of the resistor

zero mean



Noise spectral density of a resistor





Noise uncorrelated for any time shift.