# EE539: Analog Integrated Circuit Design;

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## 1 DIFFERENTIAL AMPLIFIER

Simple differential amplifier circuit is shown in the below figure In this circuit $V_{bias}$, $V_{dd}$, and $I_o$ such that

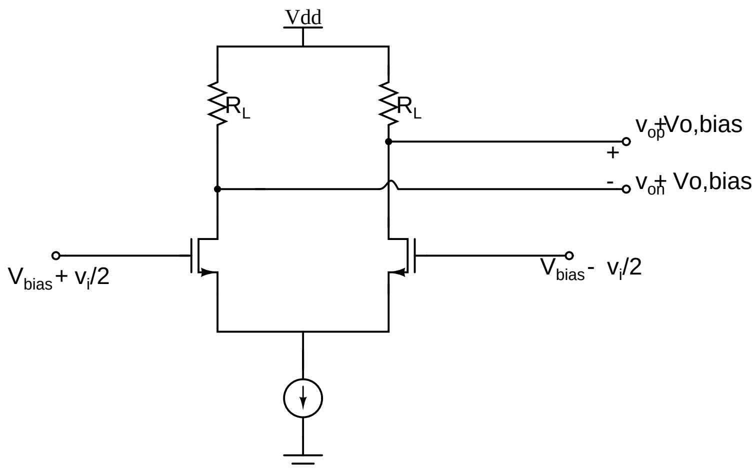


Figure 1: DIFFERENTIAL PAIR

to make MOS transistors in saturation.

## 2 SMALL SIGNAL DC GAIN

Small signal current flowing in the two transistors are equivalent but in opposite directions,so

$$\frac{v_{op}}{R_L} = -\frac{v_{on}}{R_L}$$

$$\Rightarrow v_{op} = -v_{on}$$

The value of $v_x$ can be calculated by applying KCL at node $v_x$

$$-g_m(\frac{v_i}{2} - v_x) + (v_x - v_{on})g_{ds} + (v_o - v_{op})g_{ds} - g_m(\frac{v_i}{2} - v_x) = 0$$

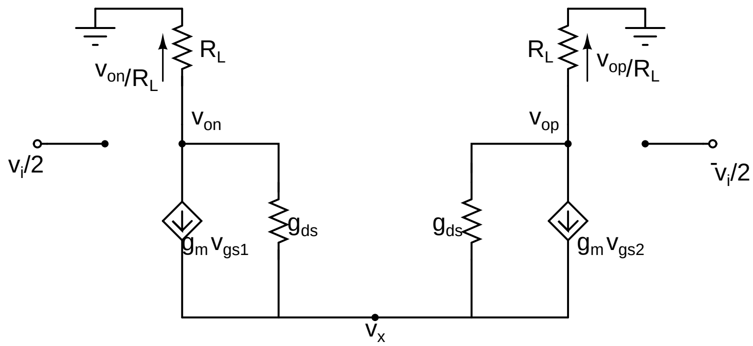


Figure 2: SMALL SIGNAL EQUIVALENT CIRCUIT FOR THE DIFFERENTIAL PAIR

$$\Rightarrow 2(g_m + g_{ds})v_x = 0$$

$$\Rightarrow v_x = 0$$

That implies $v_x$ acts as virtual ground.
This is true iff the circuit is fully symmetrical and with fully symmetrical drives.
Apply KCL at $v_{op}$, then

$$v_{op} = \frac{g_m}{g_{ds} + \frac{1}{R_L}} = -v_{on}$$

The differential output voltage is given by,

$$v_o = v_{op} - v_{on} = \frac{g_m}{g_{ds} + \frac{1}{R_L}} v_i$$

Dc gain,

$$\frac{v_o}{v_i} = \frac{g_m}{g_{ds} + \frac{1}{R_L}}$$

This DC gain value is equivalent to that of the CS Amplifier.But DC gain of the CS Amplifier is negative .Here for the differential pair the DC gain value we got is positive.This is because we have changed the reference.
The small signal equivalent circuit for the CS Amplifier amplifier is shown in the below figure
If $v_x = 0$,then we can analyze only the half circuit,the other is negative of this.

# 3 SMALL SIGNAL EQUIVALENT CIRCUIT AT HIGH FREQUENCIES

If we consider $C_{db}$, then

$$\frac{v_o(s)}{v_i(s)} = \frac{g_m}{g_{ds} + sC_{db}\frac{1}{R_L}}$$

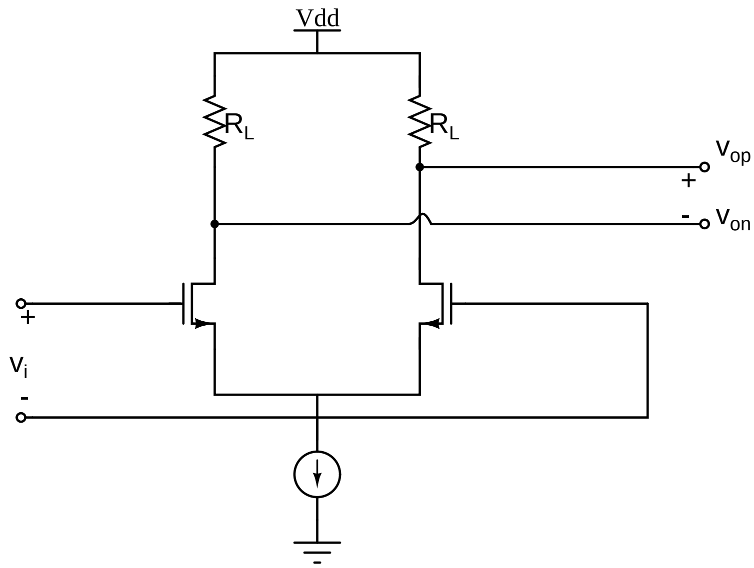


Figure 3: DIFFERENTIAL PAIR

If we have connected a capacitor $C_L$ between two output nodes,then
We can devide $C_L$ in to $2C_L$ and $2C_L$.Due to the circuit is symmetrical the mid point between $2C_L$ and $2C_L$ is grounded.
Now,

$$\frac{v_o(s)}{v_i(s)} = \frac{g_m}{g_{ds} + sC_{db} + \frac{1}{R_L} + s(2C_L)}$$

If we cosider $C_{gs}$,then there is no effect in the Gain equation.
If we cosider $C_{gd}$ only,

$$\frac{v_o(s)}{v_i(s)} = \frac{g_m(1 - \frac{sC_{gd}}{g_m})}{g_{ds} + sC_{db} + \frac{1}{R_L} + s(2C_L) + sC_{gd}}$$

# 4 LARGE SIGNAL ANALYSIS OF DIFFERENTIAL PAIR

If $v_i \neq 0$, then

$$V_{op} = V_{dd} - \frac{I_o R_L}{2} + \frac{v_o}{2}$$

,

$$V_{on} = V_{dd} - \frac{I_o R_L}{2} - \frac{v_o}{2}$$

.

We can also write these equation as,

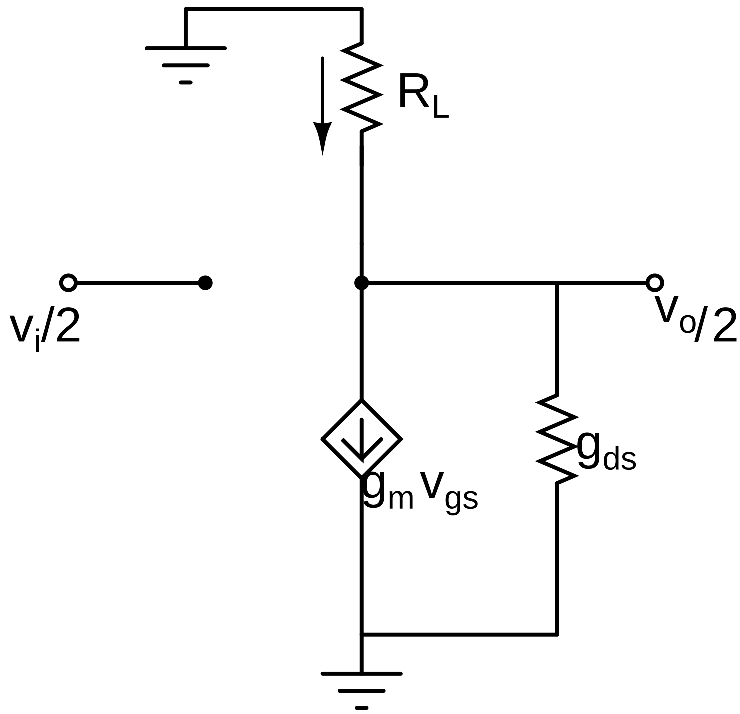


Figure 4: SMALL SIGNAL EQUIVALENT CIRCUIT FOR THE CS Amplifier

$$V_{op} = V_{dd} - \frac{I_o R_L}{2} + v_{op}$$

,

$$V_{on} = V_{dd} - \frac{I_o R_L}{2} - v_{on}$$

.

The sum of the currents flowing through these transistors is $I_o$

$$\Rightarrow \frac{V_{dd} - (V_{dd} - \frac{I_o R_L}{2} + v_{on})}{R_L} + \frac{V_{dd} - (V_{dd} - \frac{I_o R_L}{2} + v_{op})}{R_L} = I_o$$

$$\Rightarrow v_{op} + v_{on} = 0$$

To calculate $v_x$ individually,
Assuming MOSFET as a square law device,

$$\frac{\mu C_{ox} W}{2L}(V_{bias} + \frac{v_i}{2} - v_x - V_T)^2 + \frac{\mu C_{ox} W}{2L}(V_{bias} - \frac{v_i}{2} - v_x - V_T)^2 = I_o$$

$$\Rightarrow v_x = V_{bias} - V_T - \sqrt{\frac{I_o}{\mu C_{ox}\frac{W}{L}} - (\frac{v_i}{2})^2}$$

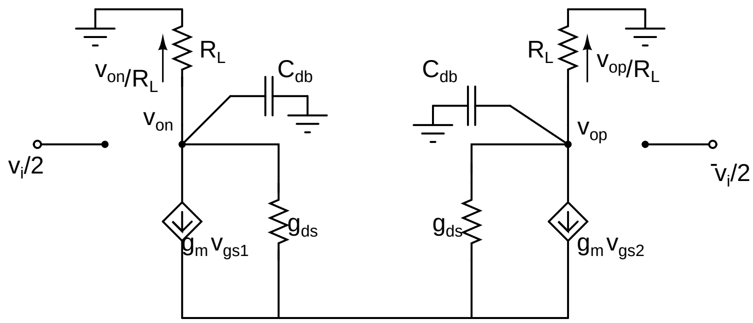


Figure 5: SMALL SIGNAL EQUIVALENT CIRCUIT FOR THE DIFFERENTIAL PAIR

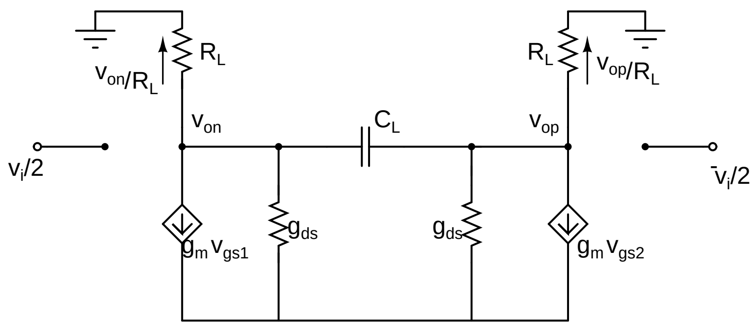


Figure 6: SMALL SIGNAL EQUIVALENT CIRCUIT FOR THE DIFFERENTIAL PAIR

Variation in $v_x$ wrt $v_i$ is shown in the below figure,
This plot is even function in $v_i$.In small signal analysis we got $v_x = 0$.In the above figure slope of the curve at $v_i = 0$is zero.
$\frac{dv_x}{dv_i} = 0$, for large signal analysis.
That implies the first coefficient in the Taylor's sereis is zero.
To find the output voltage $V_o = V_{op} - V_{on}$ in the large signal analysis,

$$V_o = V_{op} - V_{on} = (i_1 - i_2)R_L$$

$$\Rightarrow V_o = \frac{\mu C_{ox} W}{2L}(V_{bias} + \frac{v_i}{2} - v_x - V_T)^2 - \frac{\mu C_{ox} W}{2L}(V_{bias} - \frac{v_i}{2} - v_x - V_T)^2$$

$$\Rightarrow V_o = R_L \frac{\mu C_{ox} W}{L}(V_{bias} - v_x - V_T)v_i$$

$$\Rightarrow V_o = R_L \frac{\mu C_{ox} W}{L}\sqrt{\frac{I_o}{\mu C_{ox}\frac{W}{L}} - (\frac{v_i}{2})^2} v_i$$

The plot for $V_{op} - V_{on}$ VS $v_i$ is shown in the below figure.
This plot is odd function in $v_i$.
Slope reduces as we go away from origin.

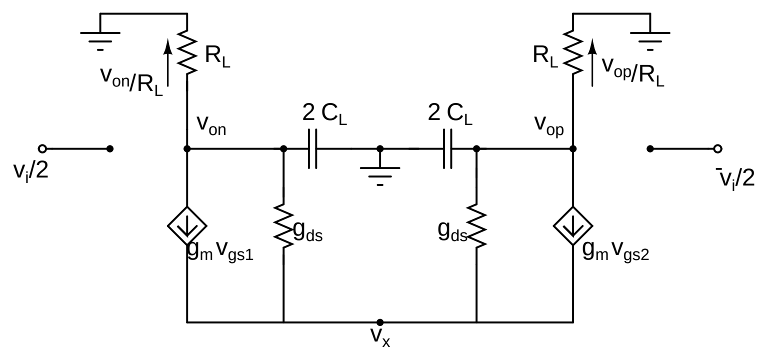


Figure 7: SMALL SIGNAL EQUIVALENT CIRCUIT FOR THE DIFFERENTIAL PAIR

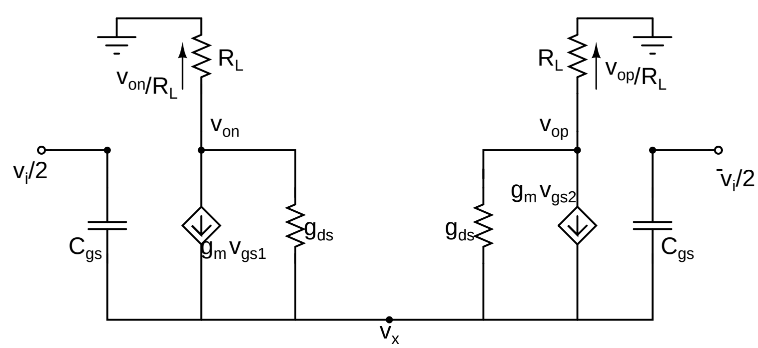


Figure 8: SMALL SIGNAL EQUIVALENT CIRCUIT FOR THE DIFFERENTIAL PAIR

Arguement under the root becomes negative at

$$v_i^1 = 2\sqrt{\frac{I_o}{\mu C_{ox}\frac{W}{L}}}$$

If $v_i$ increases $I_1$ increases and and $I_2$ decreases until carries $I_1$ all the current.

$$\Rightarrow I_1 = I_o$$

$$v_i = \sqrt{\frac{2I_o}{\mu C_{ox}\frac{W}{L}}}$$

This implies it never reaches $v_i^1$, this condition .So after this $v_i$, i saturates.

Small signal gain :

$$\frac{dv_o}{dv_i}|_{v_i=0} = R_L\sqrt{2\mu C_{ox}\frac{W}{L}\frac{I_o}{2}} = g_m R_L$$

For an odd function even ordr derivatives are zero,so for this also. After $\sqrt{\frac{2I_o}{\mu C_{ox}\frac{W}{L}}}$, $V_x$ increaes linearly,because all current flows in one transistor,is shown in the below figure.

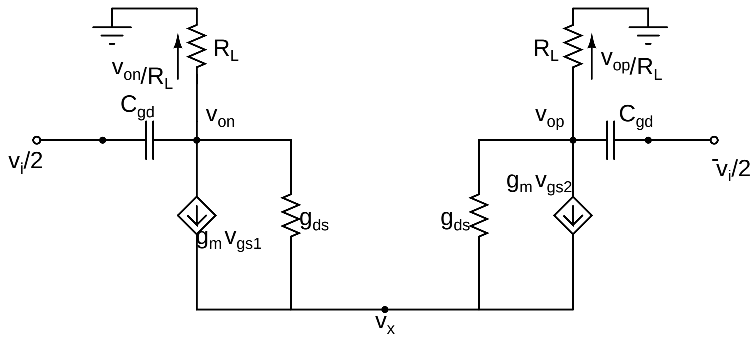


Figure 9: SMALL SIGNAL EQUIVALENT CIRCUIT FOR THE DIFFERENTIAL PAIR

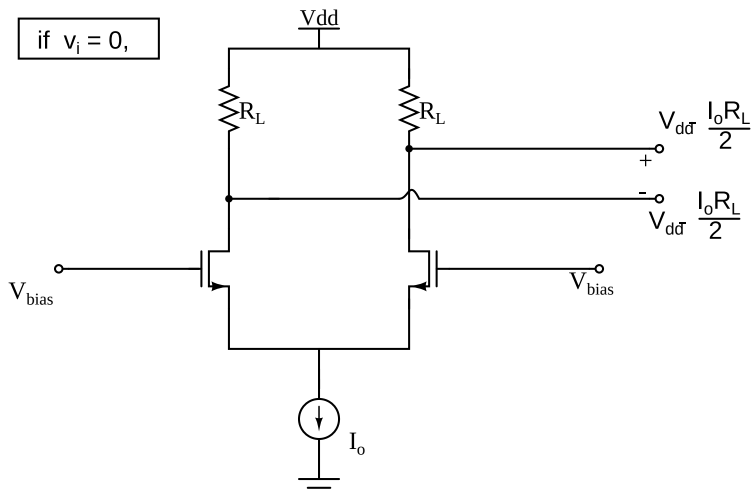


Figure 10: DIFFERENTIAL PAIR

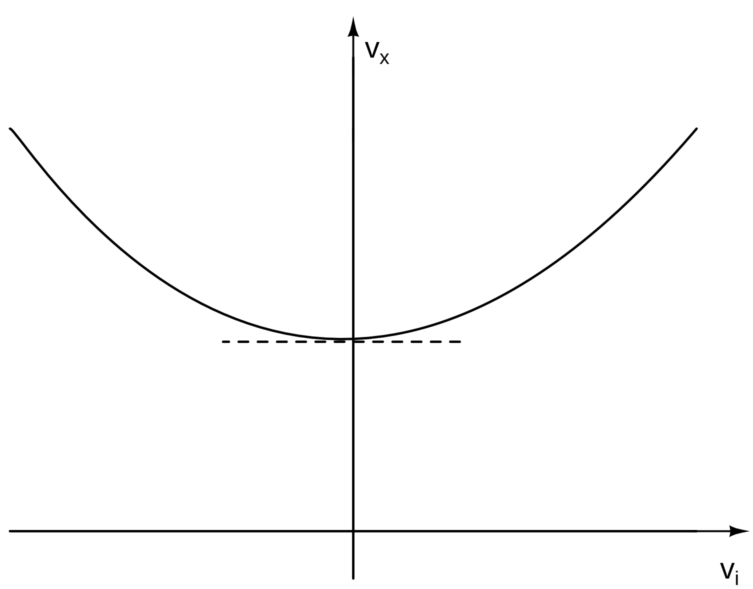


Figure 11: $v_x$ VS $v_i$

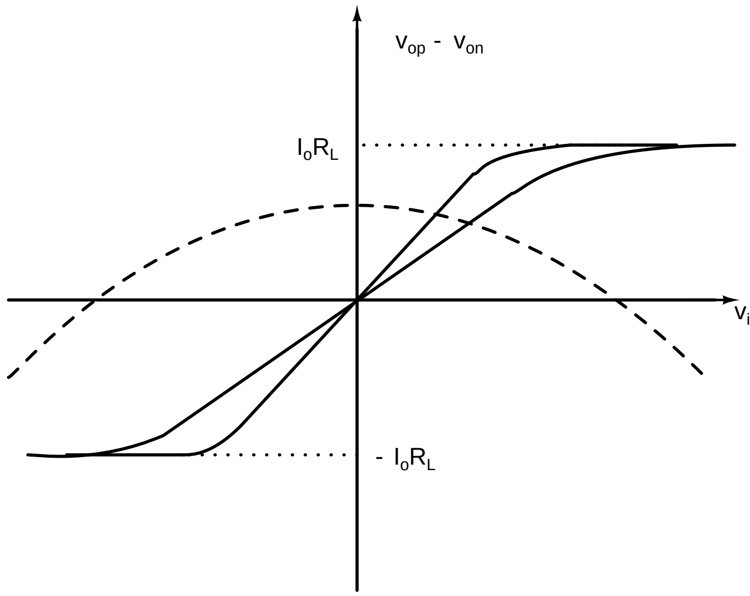


Figure 12: $V_{op} - V_{on}$ VS $v_i$

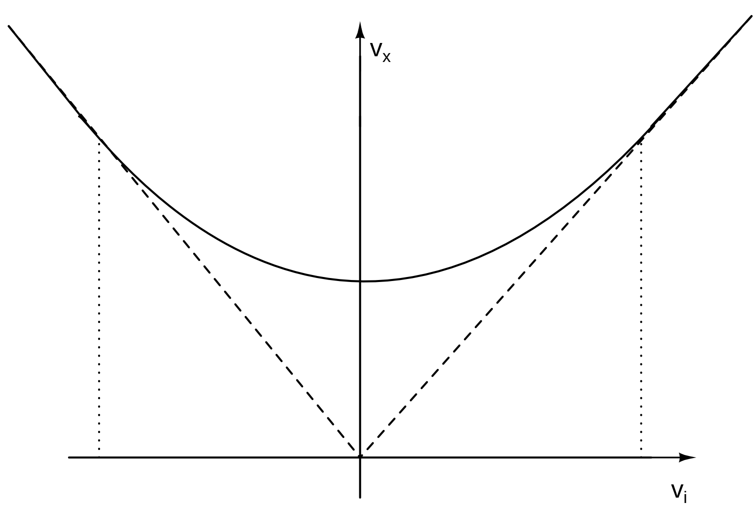


Figure 13: $V_x$ VS $v_i$