EE2019 4/4/2018

$$H(s) = \frac{1}{D(s)}$$ N^{th} order

$$H(s) = \frac{1}{1 + a_1 s + a_2 s^2 + \dots a_N s^N}.$$

$$\left| H(j\omega) \right|^2 = \frac{1}{1 + c_2 \cdot \left(\frac{\omega}{\omega_n}\right)^2 + c_4 \left(\frac{\omega}{\omega_n}\right)^4 + \dots + c_{2N} \left(\frac{\omega}{\omega_n}\right)^{2N}}$$

Maximally flat magnitude response

$$C_2 = C_4 = \ldots = C_{2N-2} = 0 \qquad C_{2N} = 1$$

N^{th} order

$$|H(j\omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{\omega_n}\right)^{2N}};$$

3-dB BW $= \omega_n$

2^{nd} order: $\underline{Q = 1/\sqrt{2}}$

$$|H(j\omega)|^2 = H(j\omega)\cdot H^*(j\omega) = H(j\omega)\,H(-j\omega)$$

$$= H(s)\cdot H(-s)\Big|_{s=j\omega} = \frac{1}{1+\left(\frac{\omega}{\omega_n}\right)^{2N}}$$

$\omega = s/j$

$$H(s)\cdot H(-s) = \frac{1}{1+\left(\frac{s}{j\omega_n}\right)^{2N}}$$

plot the poles on the s-plane

$N = 2, 3, 4$

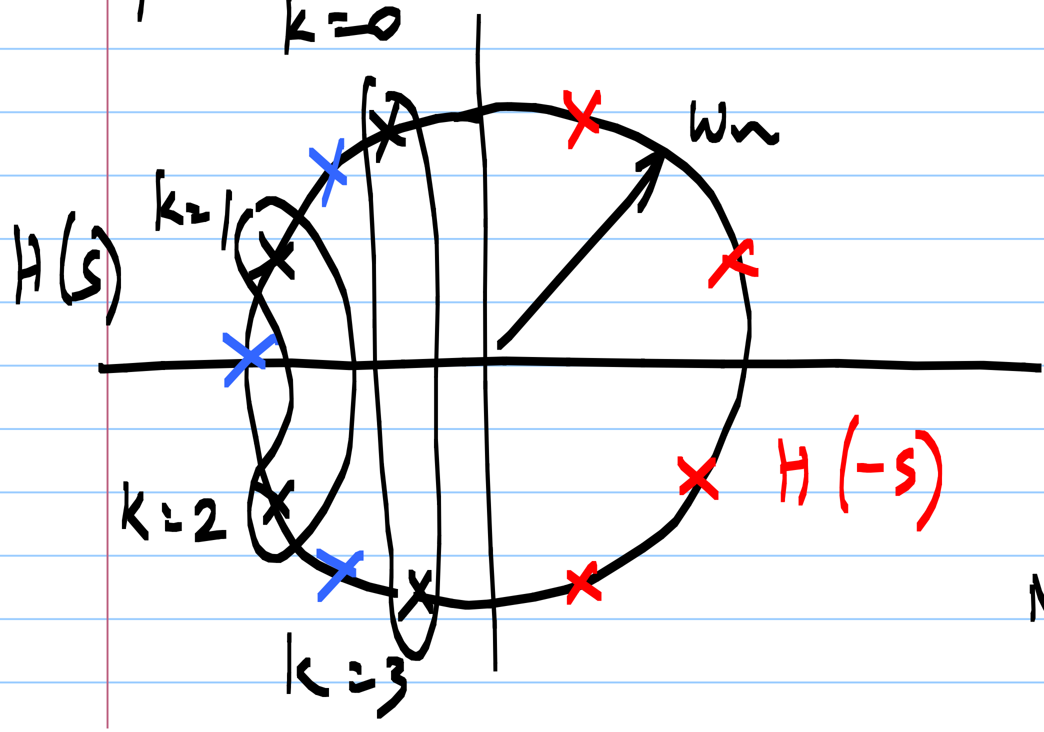
$$\boxed{1+\left(\frac{s}{j\omega_n}\right)^{2N} = 0}$$

N poles $\{p_k\}$ N poles $\{-p_k\}$

$$1+\left(\frac{s}{j\omega_n}\right)^{2N}=0 \Rightarrow \left(\frac{s}{j\omega_n}\right)^{2N}=-1=\exp\left(j(2k+1)\pi\right)$$

$$s=\omega_n\cdot\exp\left(\frac{j(2k+1)\cdot\pi}{2N}+\frac{j\pi}{2}\right) \quad k:0\ldots 2N-1$$

$N=4$



$k=0:\ \pi/2+\pi/2N$

$\vdots$

$\pi/2+\frac{\pi}{2N}+\frac{\pi\cdot k}{N}$

$\underline{N-1}$

(LHP)

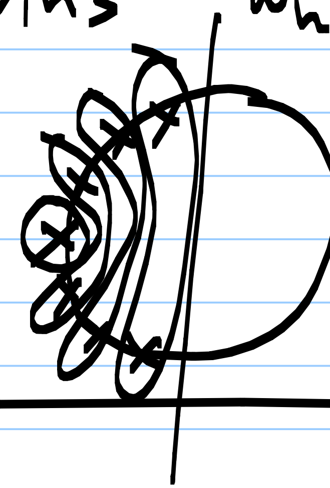
$N\ldots 2N-1$

(RHP)

N^{th} order

Butterworth filter: Poles lie on a circle of radius ω_n ($-3dB$ BW)

Equally spaced by $\frac{2\pi}{2N}$

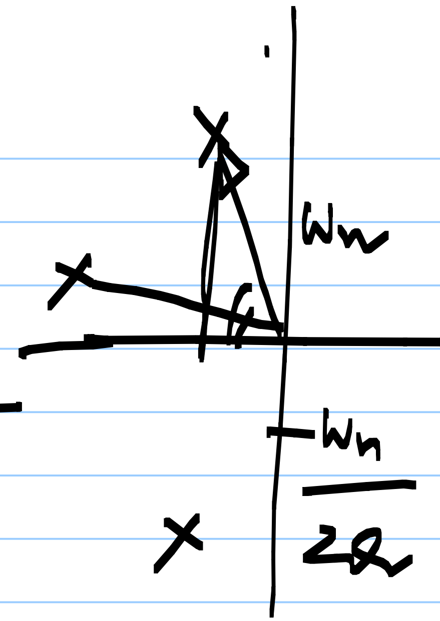


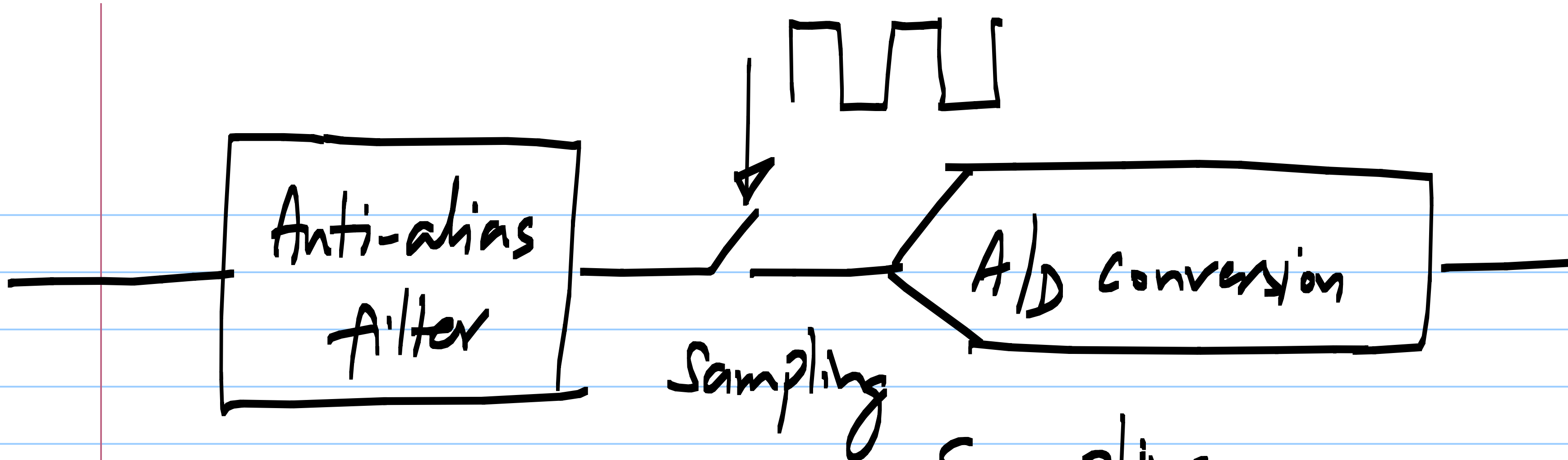
Ordering:

First-order; Increasing order of quality factors

$$\left(\frac{s}{\omega_n}\right)^2 + \frac{s}{Q\omega_n} + 1 = 0$$

$$\frac{s}{\omega_n} = \frac{-\frac{1}{Q} \pm j\sqrt{4 - 1/Q^2}}{2}$$



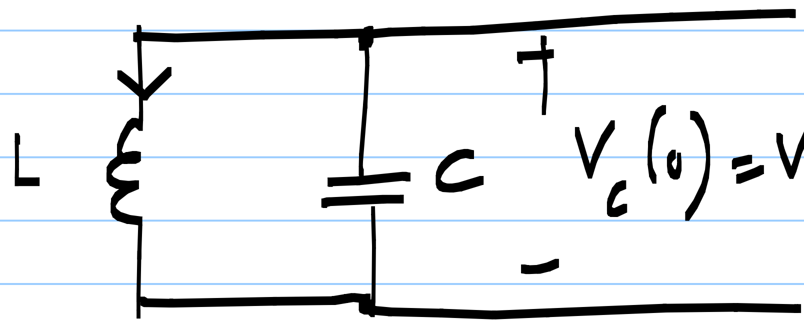


Periodic clocks:

- Sampling
- Digital circuits (state machines)
- Modulation

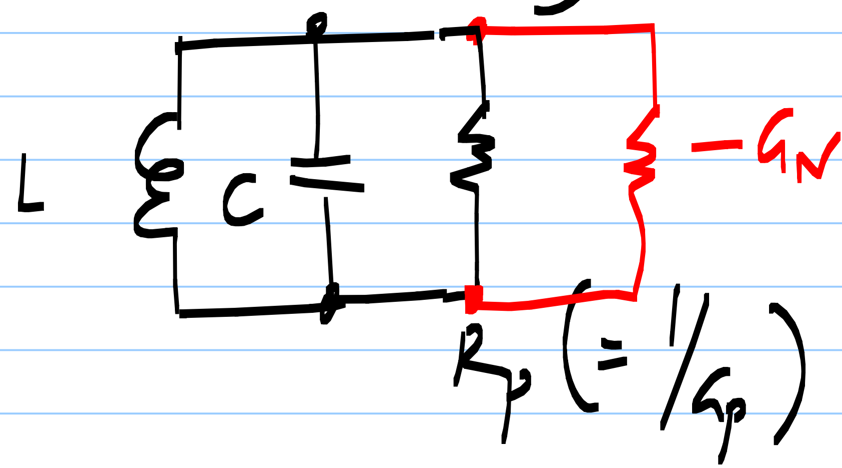
Oscillator

LC oscillator:



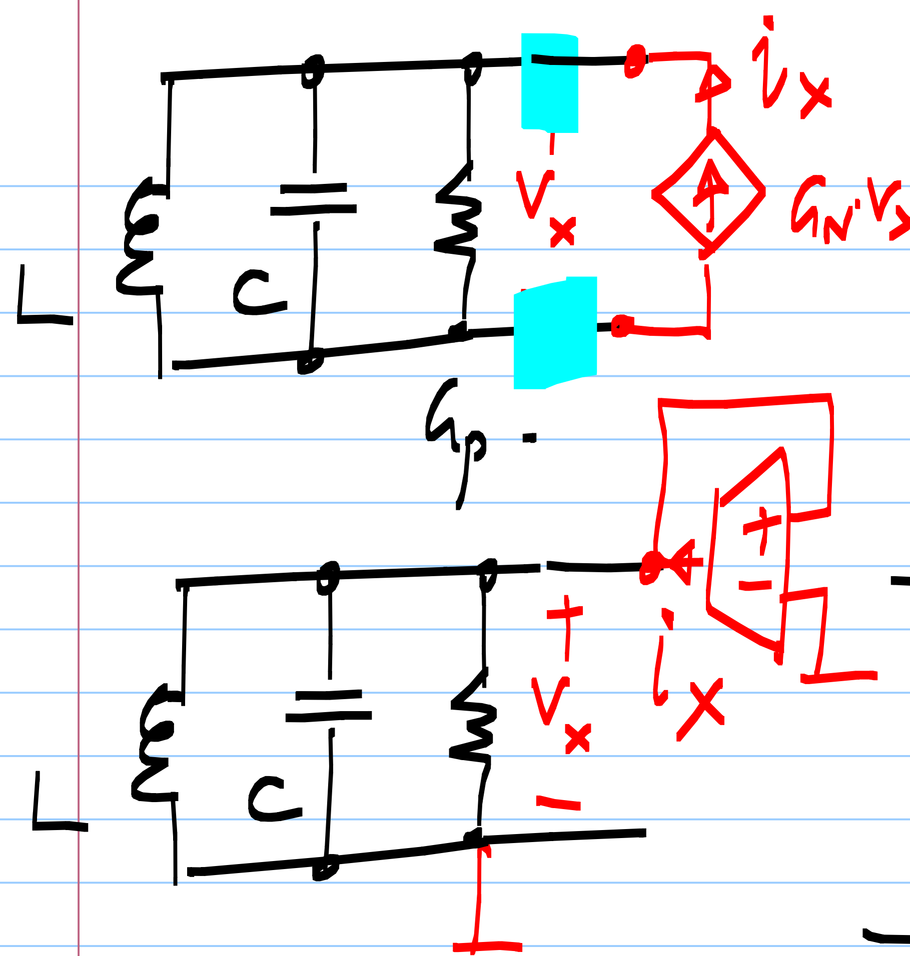
$V_p \cdot \cos(\omega t)$

$G_N > 0$

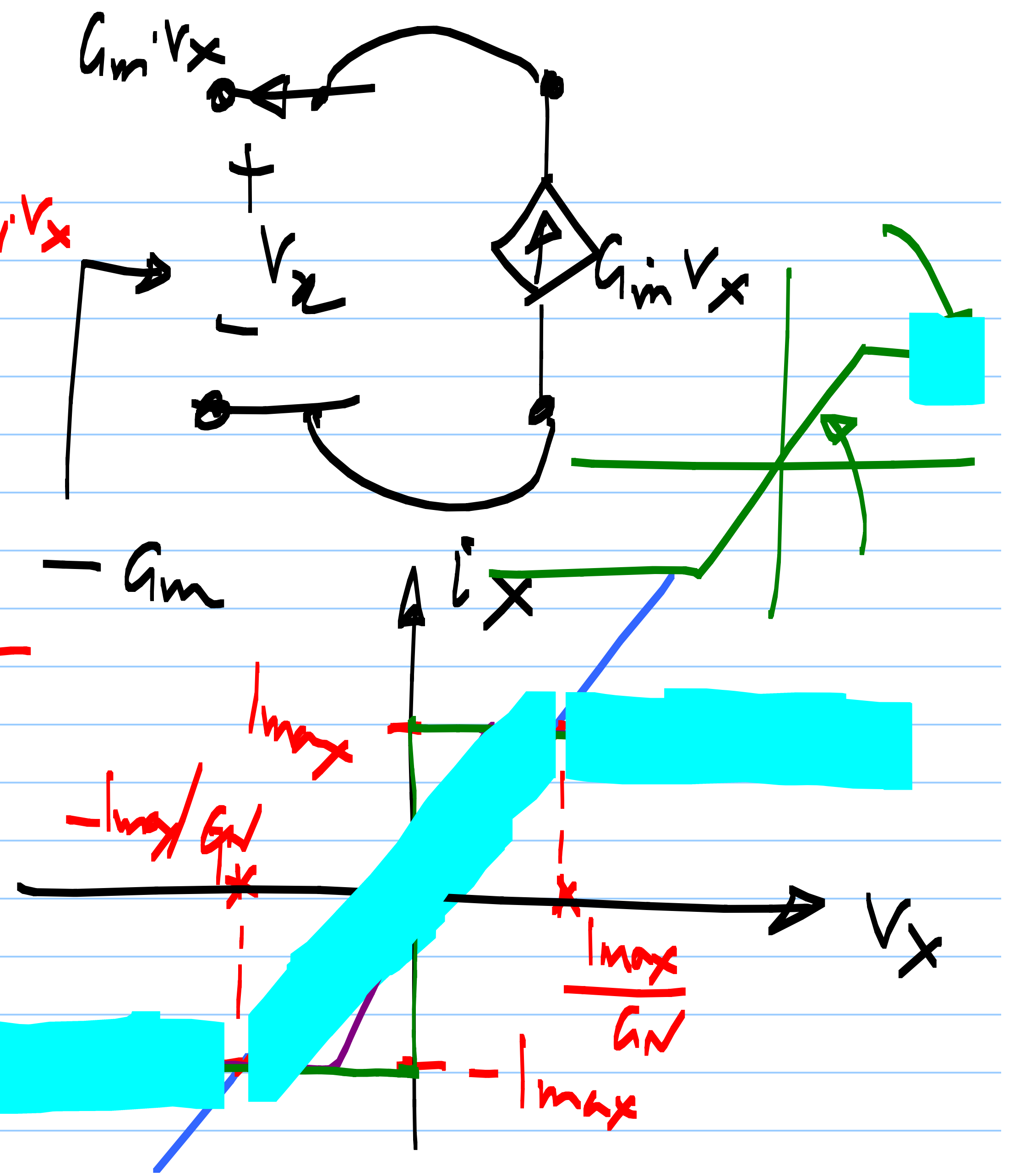


If $G_N = G_p$, the result is a lossless LC parallel circuit

$G_N > G_p$

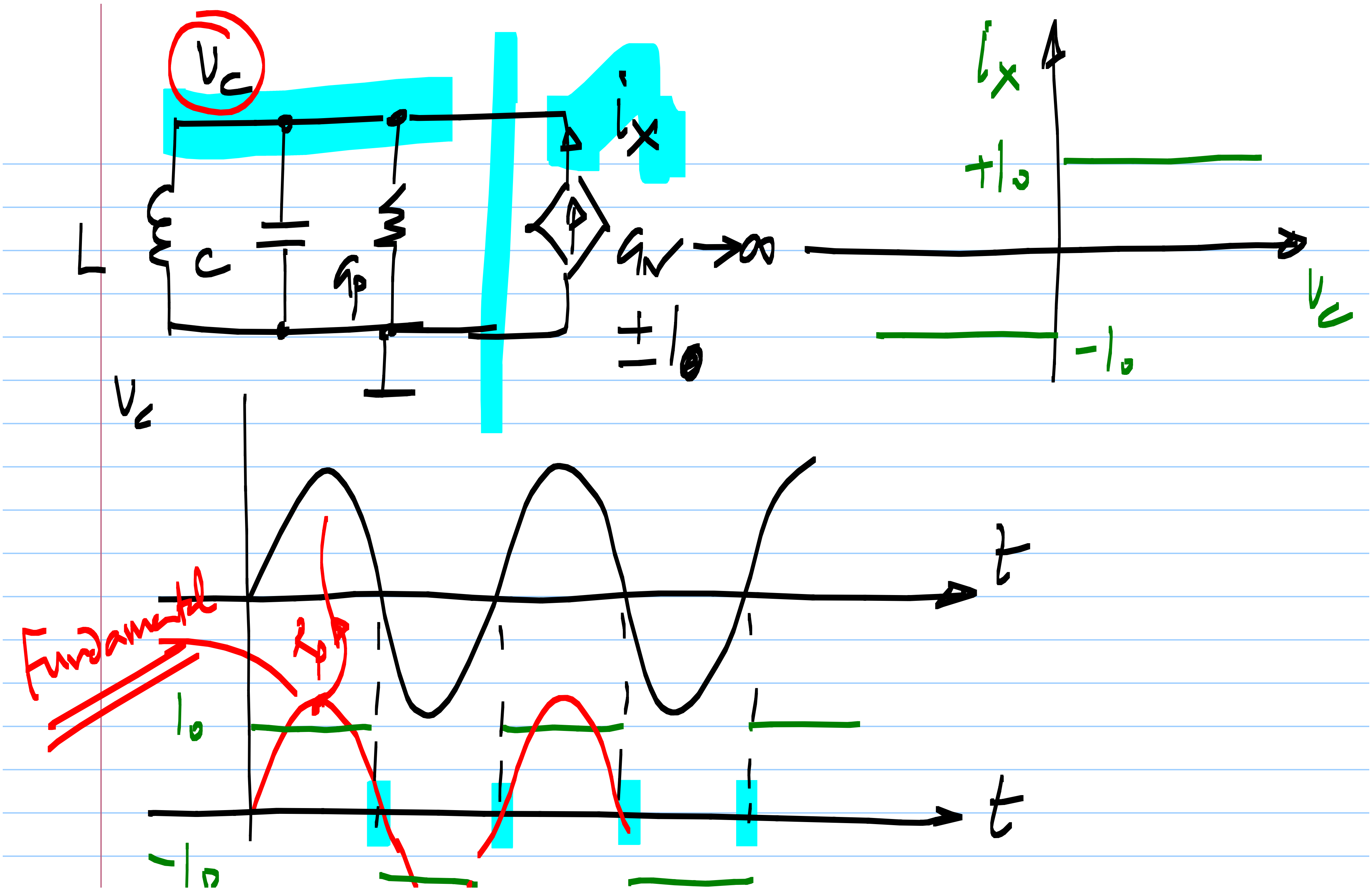


L
C
i_x
V_x
$G_N' V_x$
G_p
L
C
+
V_x
−
i_x
+
−

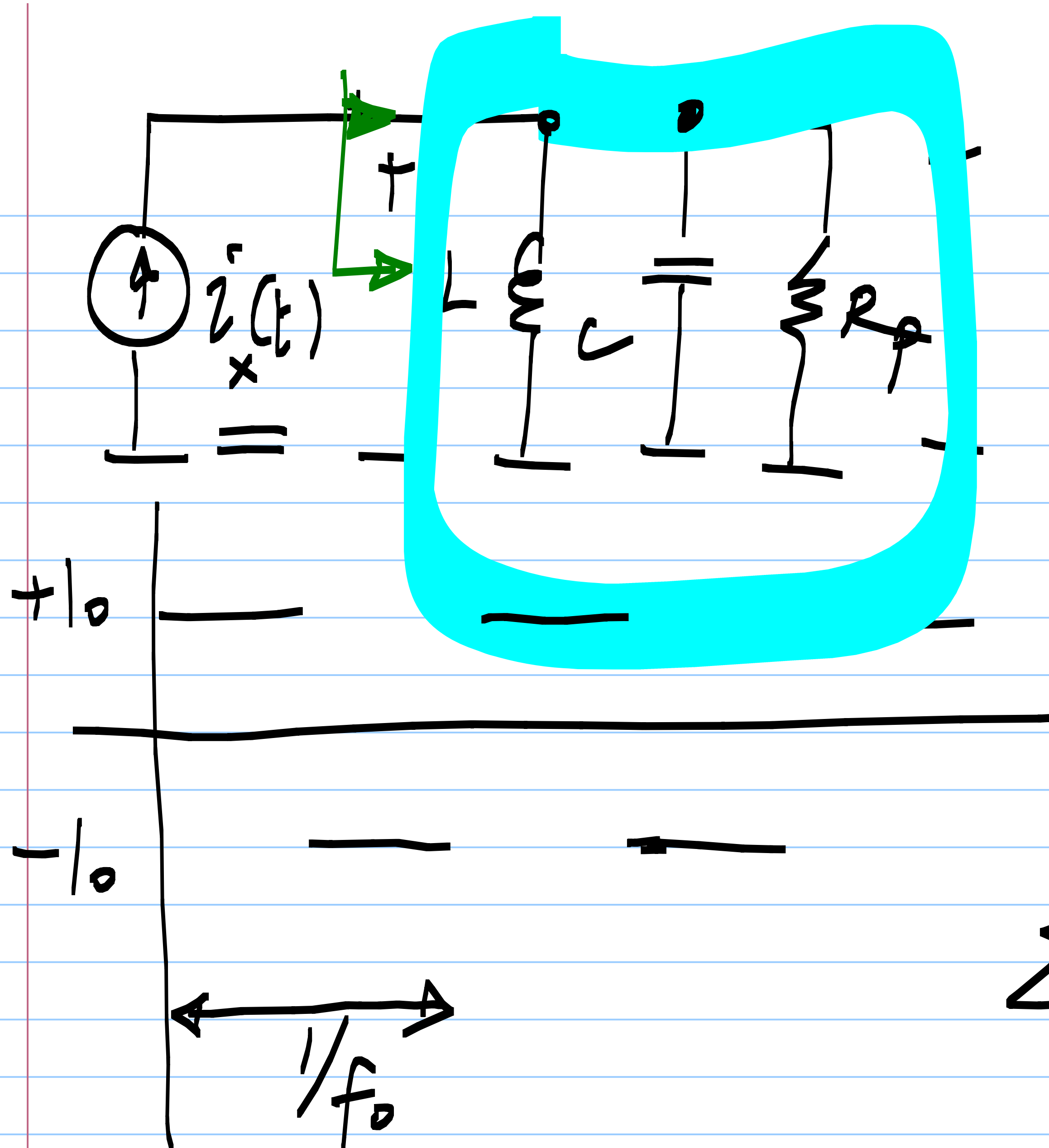


$G_m' V_x$
+
V_x
−
$G_m V_x$
$-G_m$
i_x
I_{max}
$-I_{max}/G_N$
V_x
$\frac{I_{max}}{G_N}$
$-I_{max}$

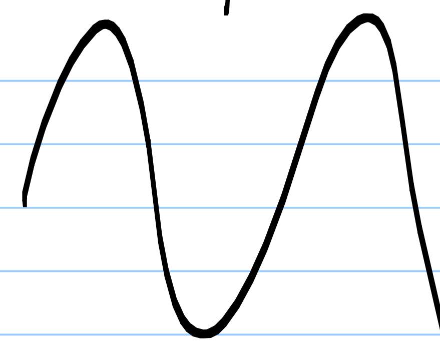
$G_N > G_p$



v_c
L
C
g_p
i_x
$g_N \to \infty$
$\pm I_0$
i_x
$+I_0$
$-I_0$
v_c
v_c
t
Fundamental
i_p
I_0
$-I_0$
t



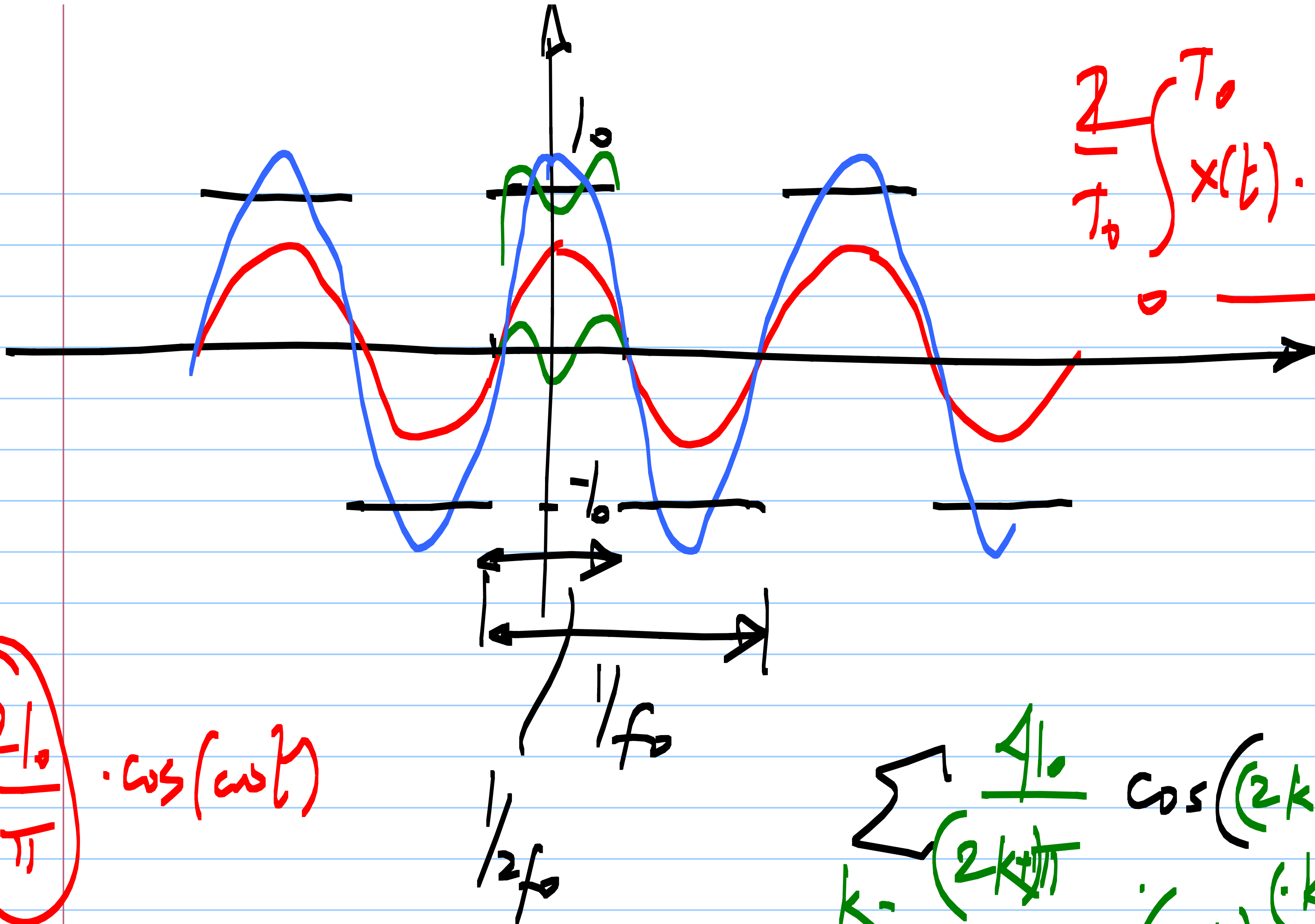
@ f_0



$$\sum_k I_k \cos(k\omega_0 t + \phi_k)$$

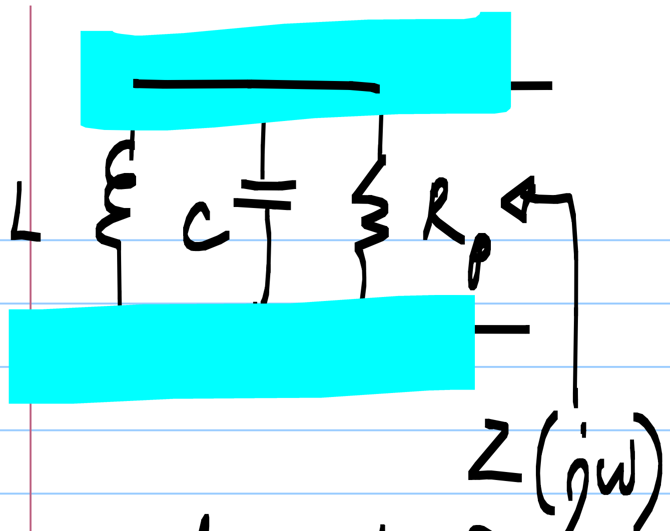
$$\sum_k I_k \cdot |Z(jk\omega_0)| \cdot \cos(k\omega_0 t + \phi_k + \angle Z(jk\omega_0))$$

$$\frac{2}{T_0}\int_0^{T_0} x(t)\cdot\cos(k\omega_0 t)\,dt$$

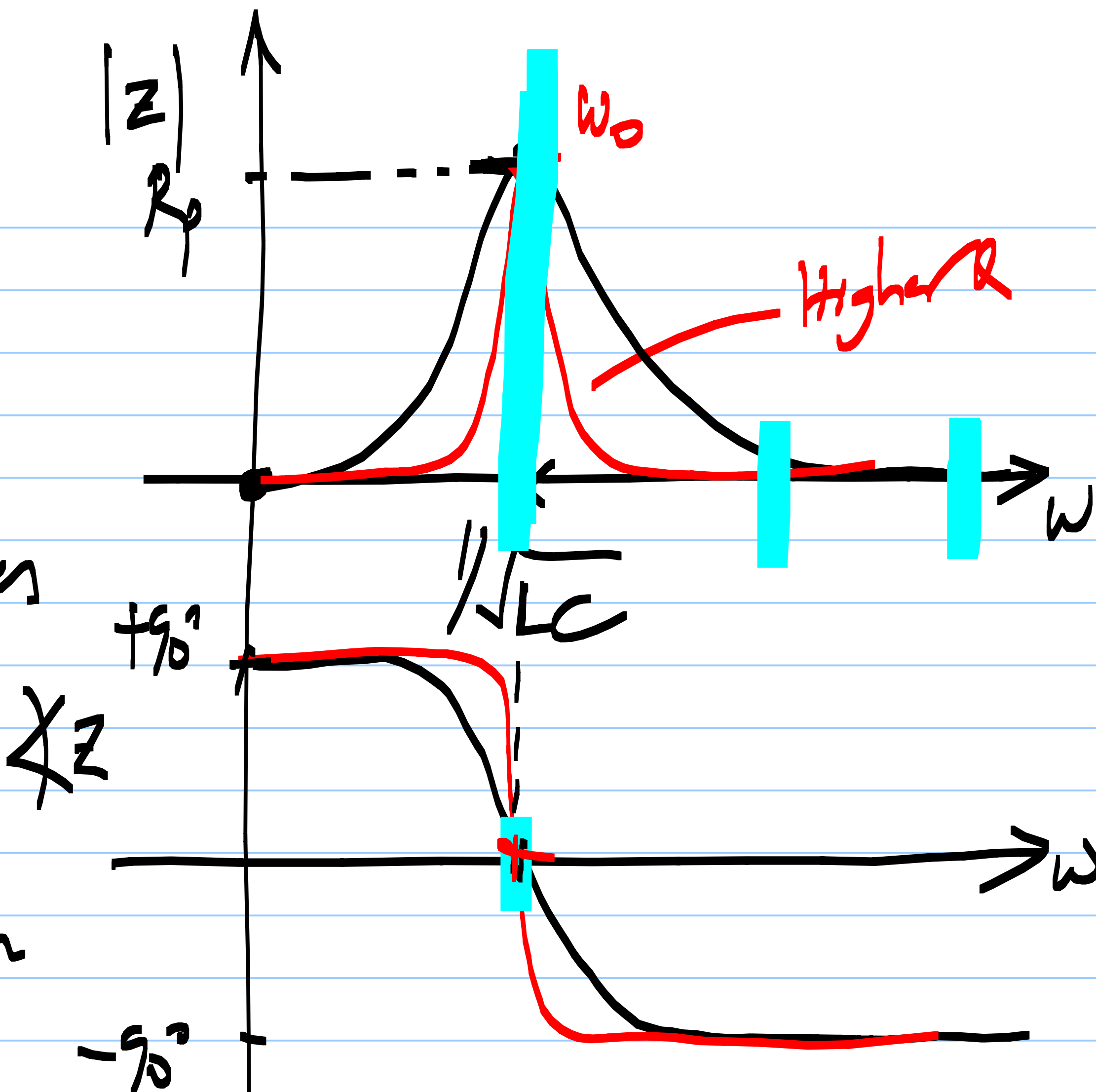


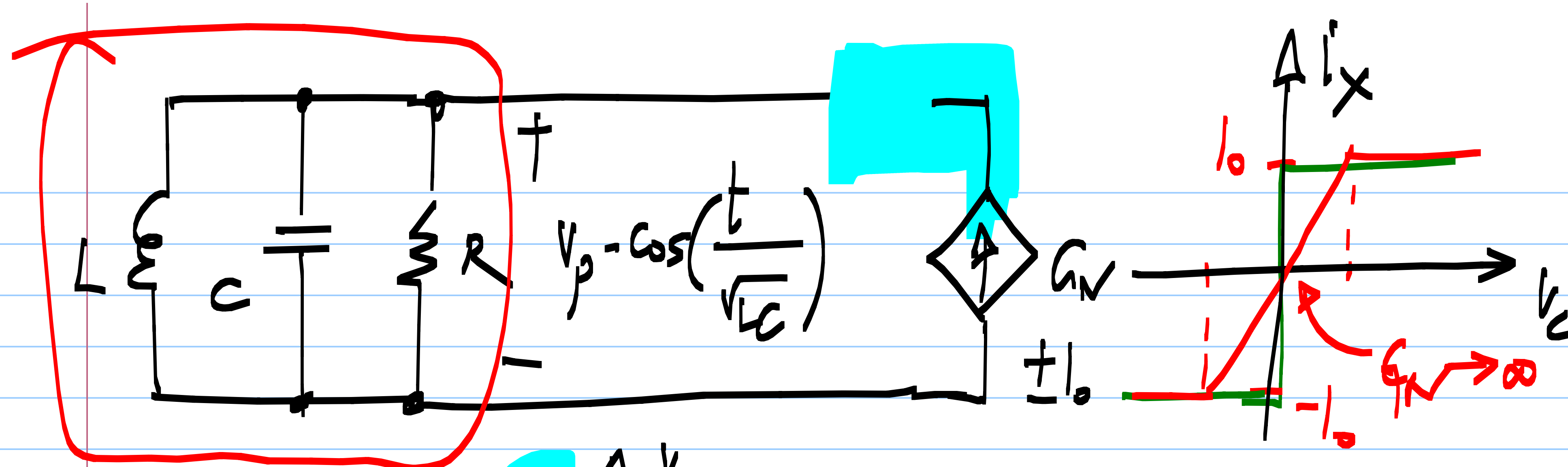
$\frac{2 I_0}{\pi}\cdot\cos(\omega_0 t)$

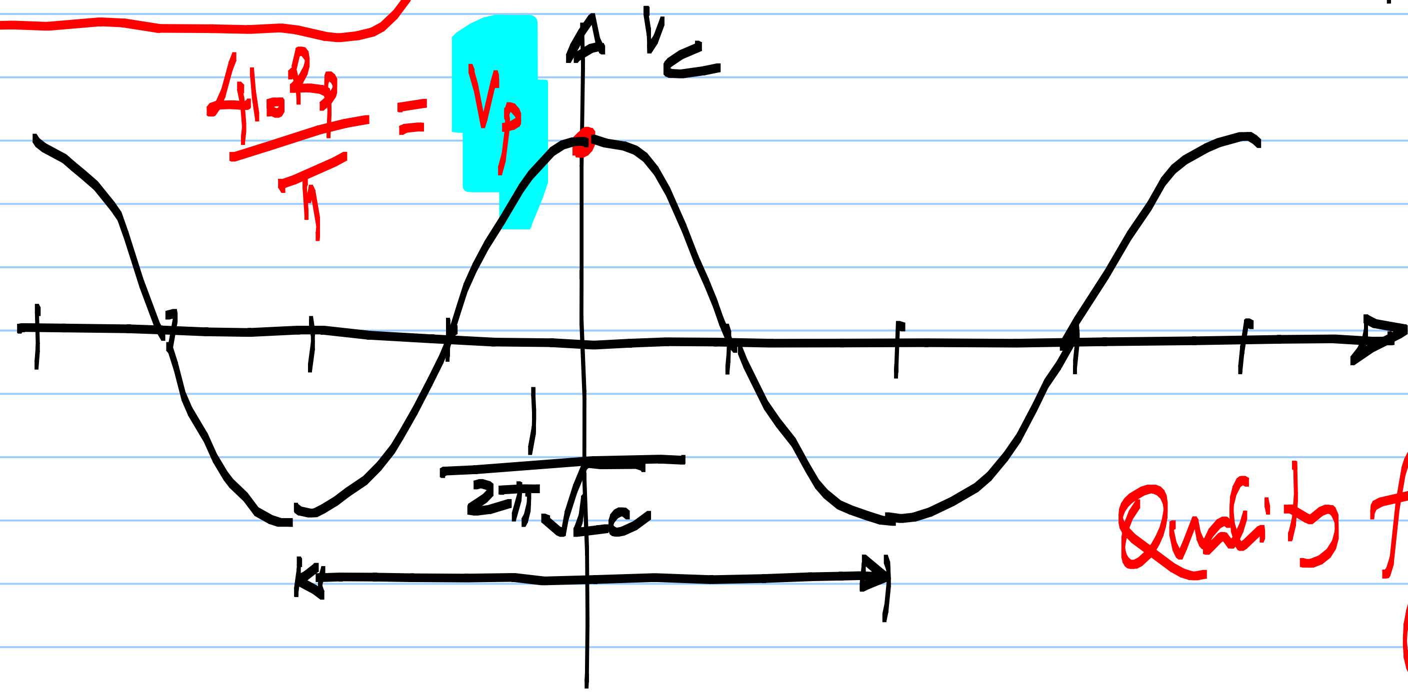
$$\sum_{k} \frac{4 I_0}{(2k+1)\pi} \cos((2k+1)t)\cdot(-1)^{(k-1)}$$



* Signals at frequency far from $1/\sqrt{LC}$ shorted out
* signal @ $1/\sqrt{LC}$ has no phaseshift
* far away large phaseshift







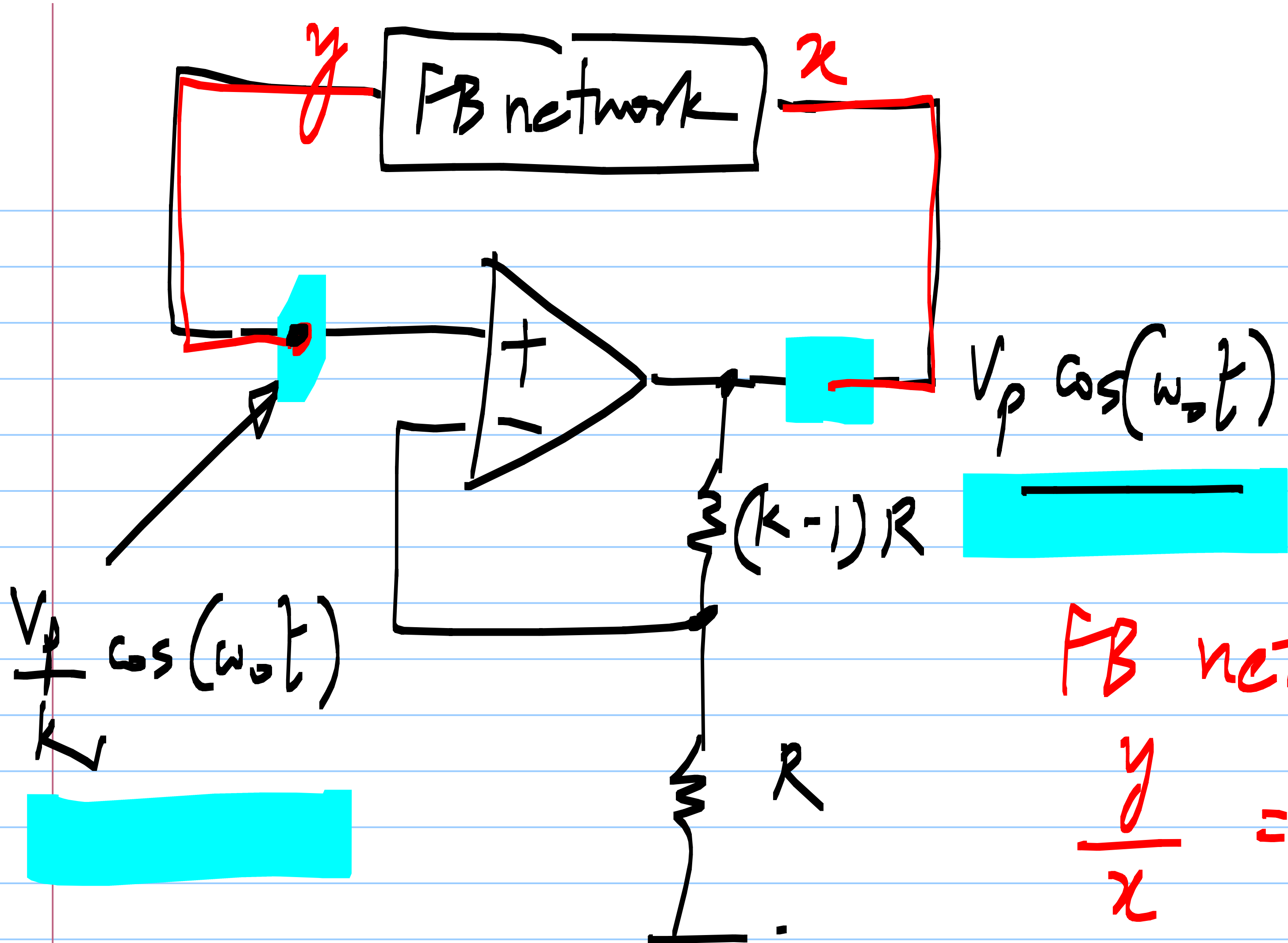
Quality factor is high (~10)

LC oscillators (L, C)

Best (periodicity)

Ring oscillator

Relaxation oscillators
(schmitt-trigger oscillator)

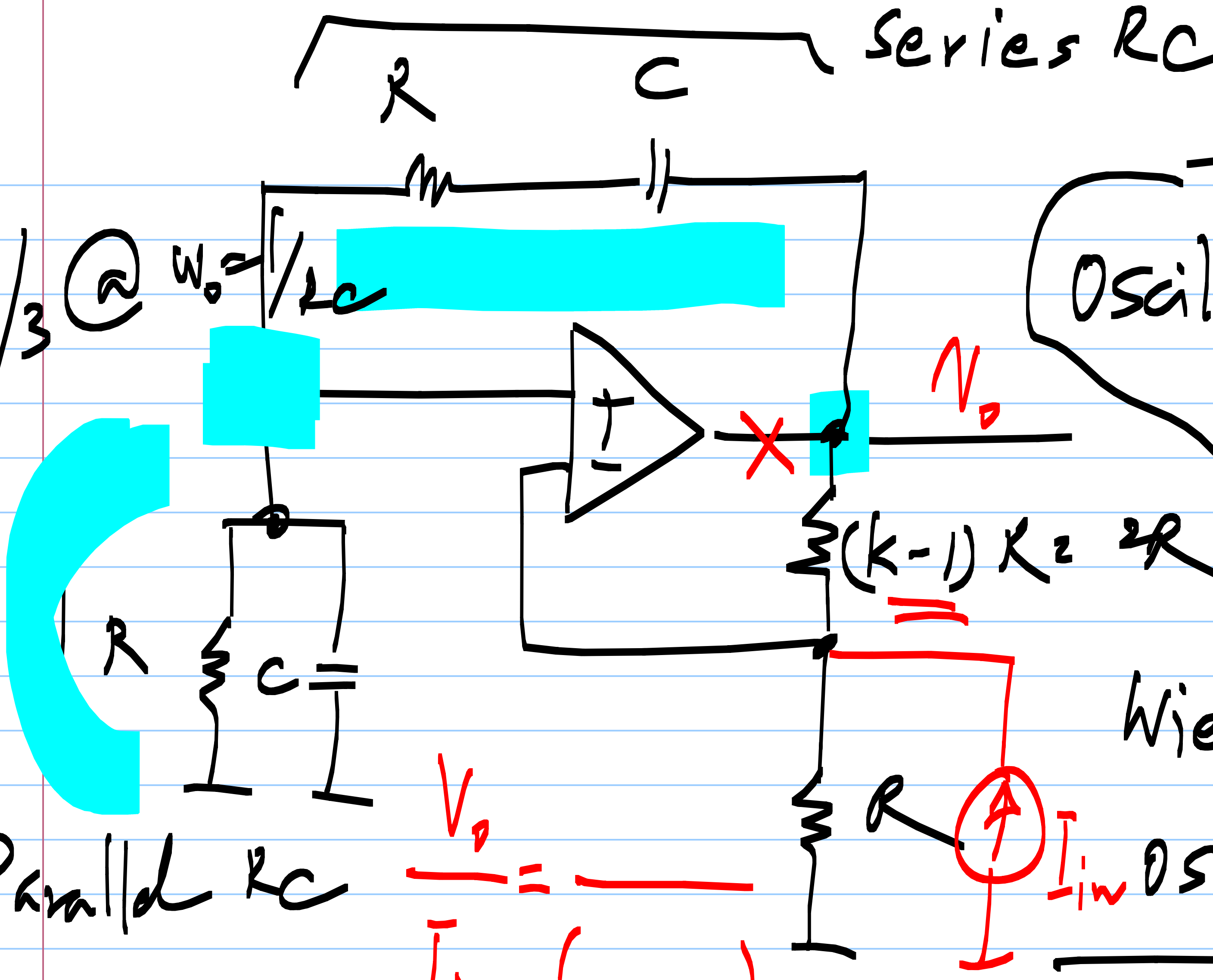


FB network:

$$\frac{y}{x} = \frac{1}{K} \text{ only @ } \omega_o$$

Series RC

$1/3$ @ $\omega_0 = 1/RC$



Parallel RC

Oscillator @ $\omega_0 = 1/RC$

Wien-Bridge Oscillator

$$\frac{V_o}{I_{in}} = \underline{\qquad} \quad (\qquad)$$