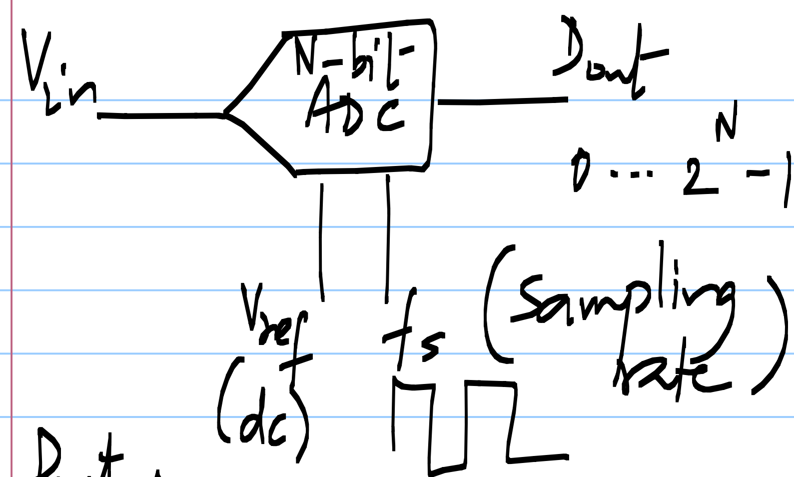
EE2019 — 19/4/2017

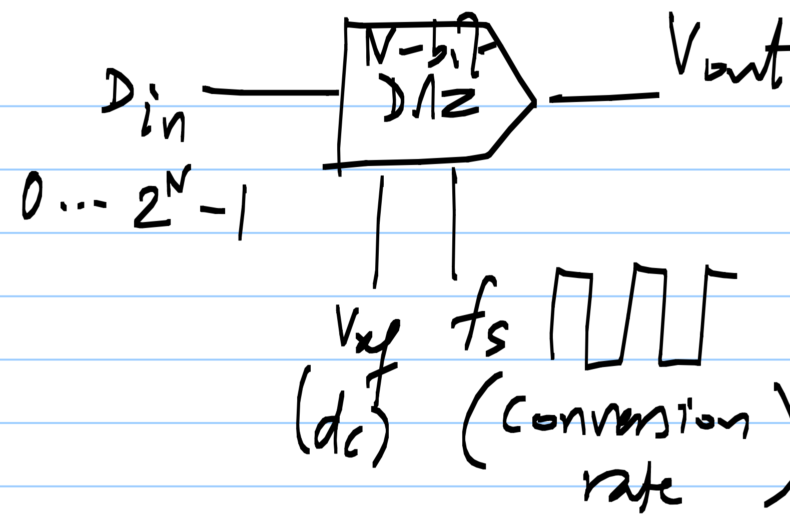
Data Converters

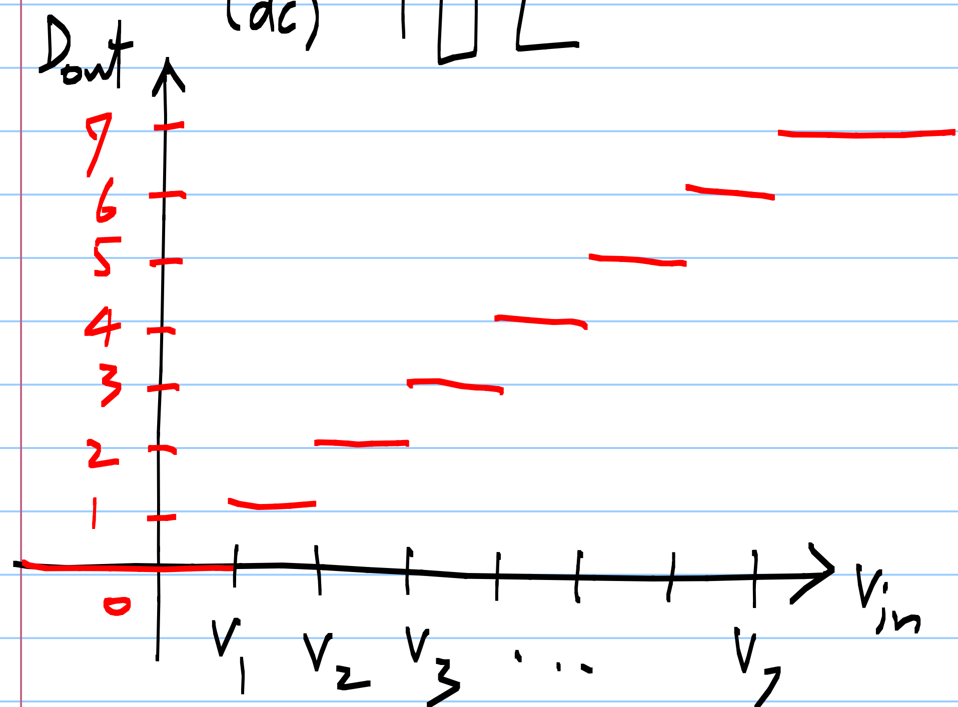
- Analog-to-Digital converters (ADC)
- Digital-to-Analog converters (DAC)

ADC



DAC





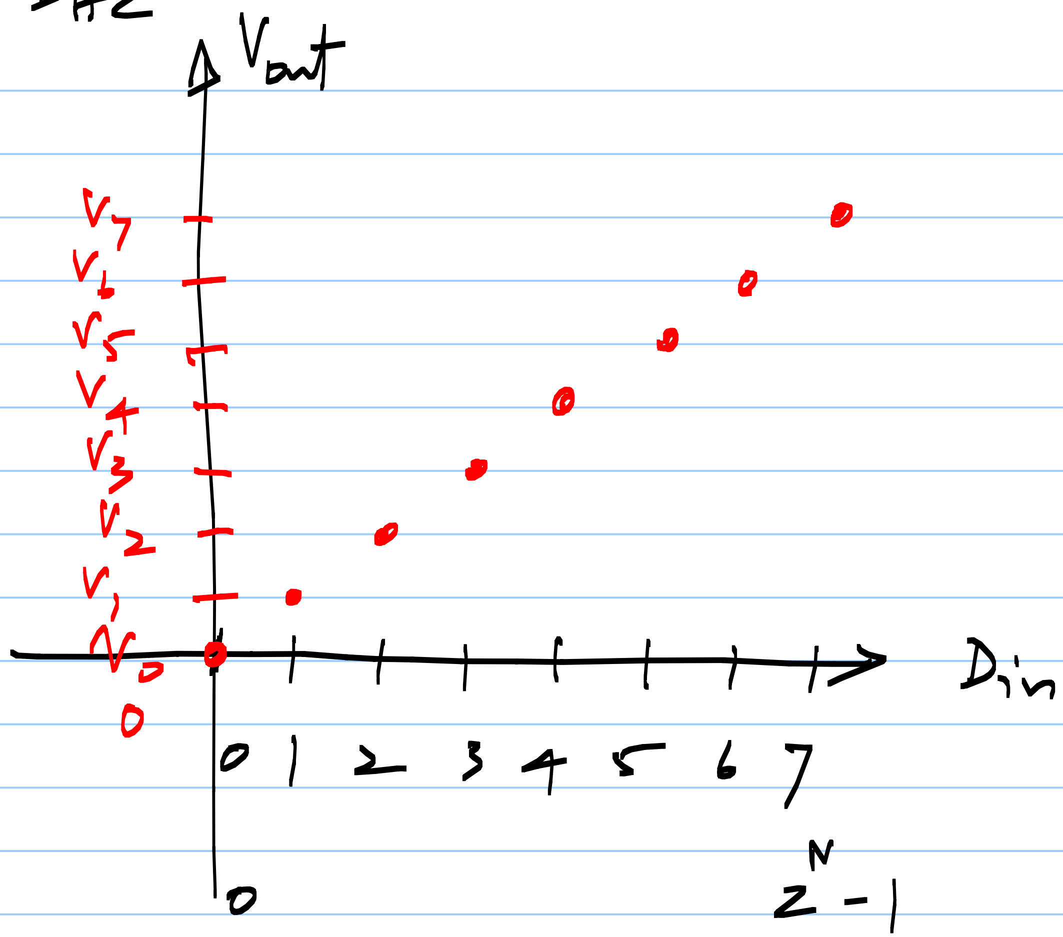
Transition points

$V_1, V_2 \dots V_7$

$V_1, V_2 \dots V_{2^N-1}$

Ideally $V_K = \frac{K}{2^N} V_{ref}$

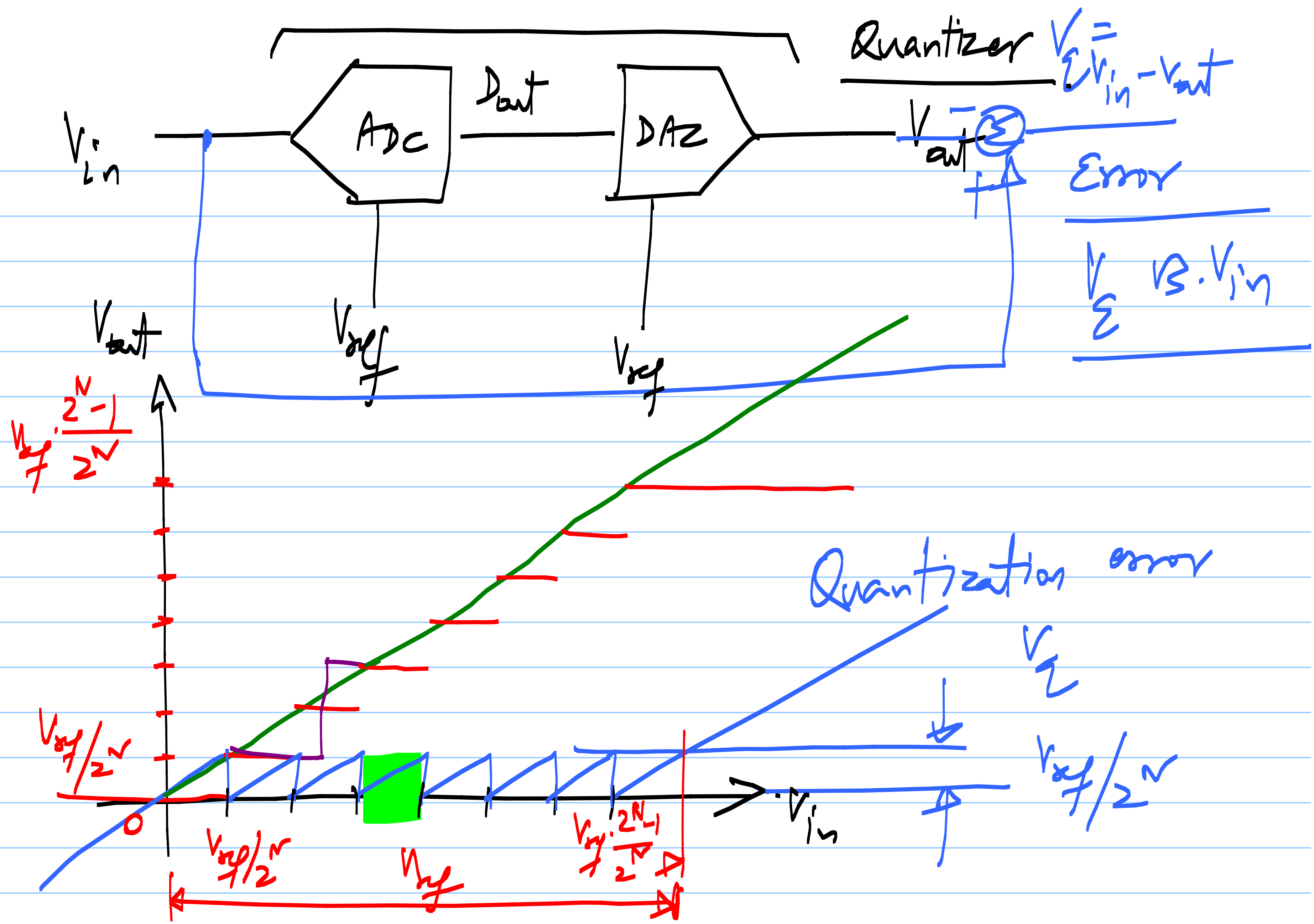
DAC



2^N possible outputs

$\{ V_0 \; \dots \; V_{2^N - 1}$

$$V_k = \frac{k}{2^N} \cdot V_{ref}$$



Quantizer
V_{in}
ADC
D_{out}
DAC
V_{out}
V_{ref}
V_{ref}
$V_\varepsilon = V_{in} - V_{out}$
Error
V_ε vs. V_{in}
V_{out}
$V_{ref} \cdot \frac{2^N - 1}{2^N}$
$V_{ref}/2^N$
0
$V_{ref}/2^N$
V_{ref}
$V_{ref} \cdot \frac{2^N - 1}{2^N}$
V_{in}
Quantization error
V_ε
$V_{ref}/2^N$

$\frac{V_{ref}}{2^N}$: V_{LSB} of the ADC/DAC

V_{ref} : Range of the ADC/DAC

Range over which q. error peak-peak remains within V_{LSB}

ADC/DAC Errors

$V_k \neq \frac{k}{2^N} V_{ref}, \quad 1 < k < 2^N - 1$

$\left[V_1 = V_{ref}/2^N \right.$, $\left. V_{2^N-1} = V_{ref} \cdot \frac{2^N - 1}{2^N} \right]$

ADC: Errors in the transition points

DAC: Errors in the output levels — $\left[V_0 = 0 \right.$, $\left. V_{2^N-1} = V_{ref} \cdot \frac{2^N-1}{2^N} \right]$

		ADC	DAC
Integral nonlinearity error (INL)	$\dfrac{V_k - \frac{k}{2^N} \cdot V_{ref}}{(V_{ref}/2^N)}$	$2 \leq k \leq 2^N - 2$;	$1 \leq k \leq 2^N - 2$
			$\lvert INL \rvert < 1/2$
Differential nonlinearity error (DNL)	$\dfrac{(V_k - V_{k-1}) - \frac{V_{ref}}{2^N}}{(V_{ref}/2^N)}$		$\lvert DNL \rvert < 1/2$

$$INL[k] = \frac{V_k - \frac{k}{2^N} \cdot V_{ref}}{\frac{V_{ref}}{2^N}}$$

$$DNL[k] = \frac{\left(V_k - V_{k-1}\right) - \frac{V_{ref}}{2^N}}{V_{ref}/2^N}$$

$$DNL[k] = INL[k] - INL[k-1]$$

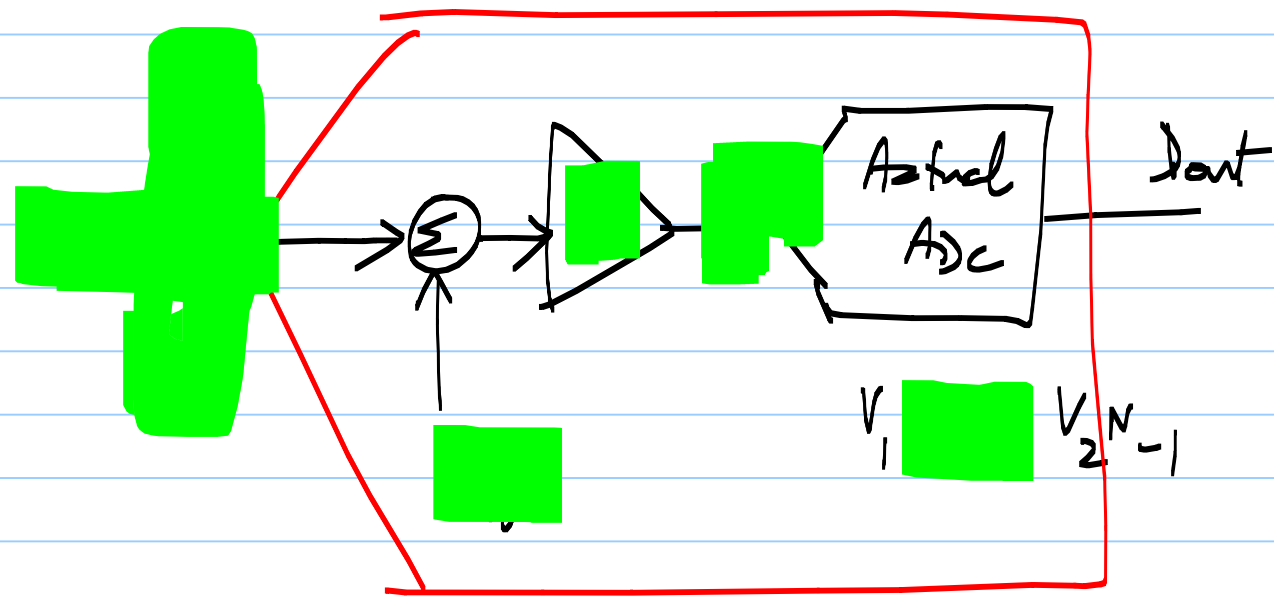
$$V_1 \neq V_{ref}/2^N$$

$$V_{2^N-1} \neq V_{ref} \cdot \frac{2^N-1}{2^N}$$

$$\left(V_{ref}/2^N + V_{off}\right) G = V_1$$

$$\left(V_{ref} \cdot \frac{2^N-1}{2^N} + V_{off}\right) \cdot G = V_{2^N-1}$$

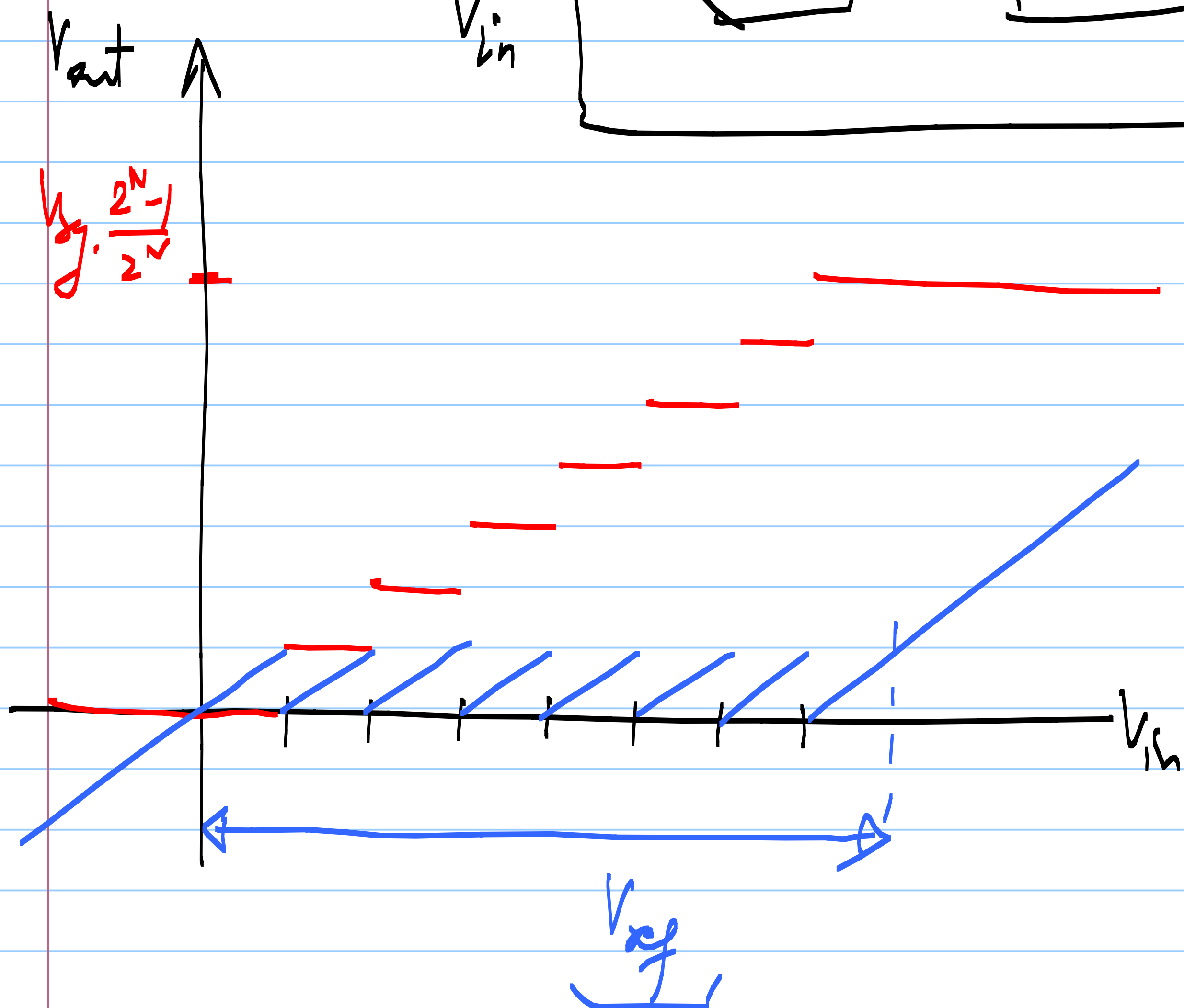
Actual transition points

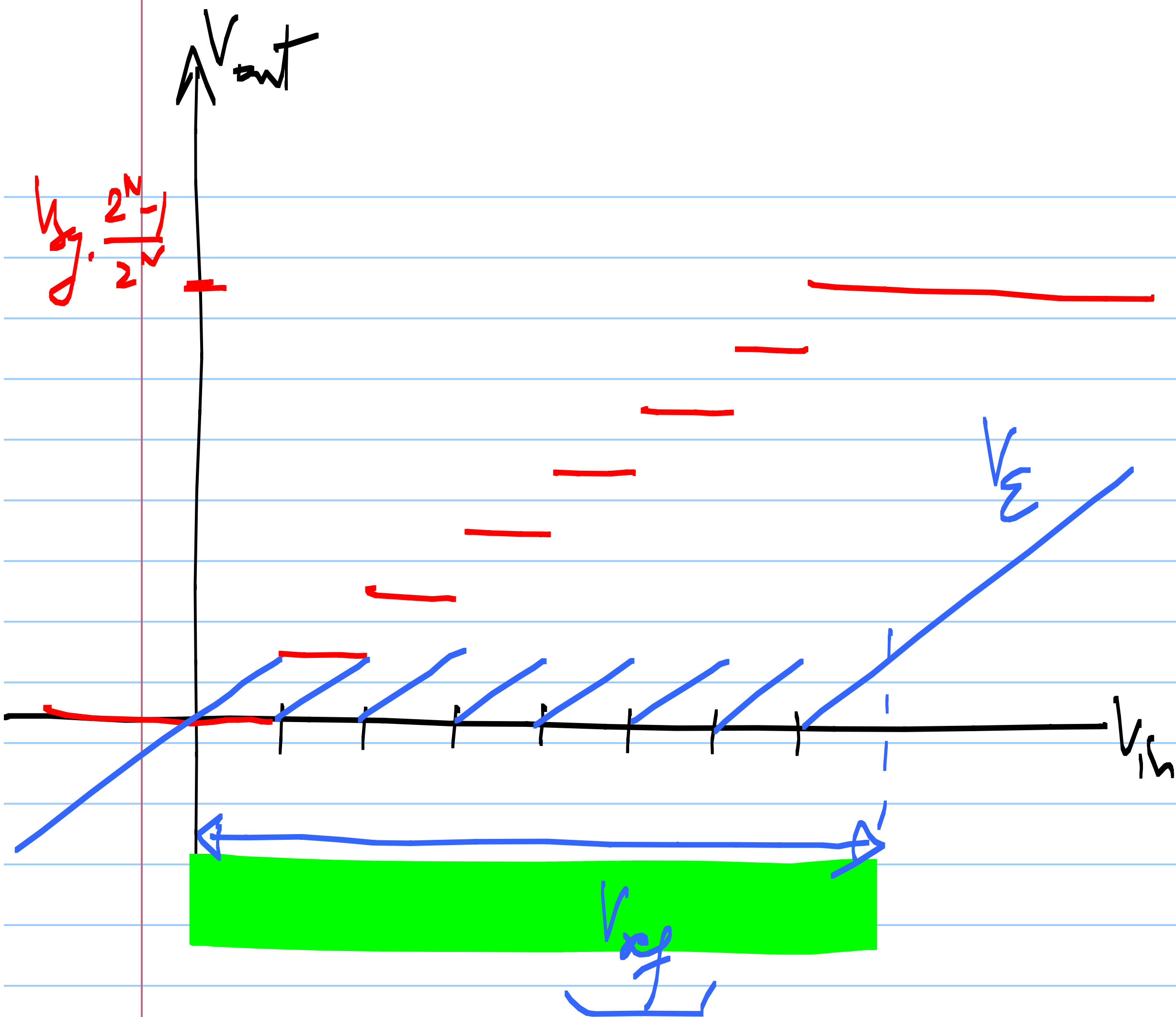


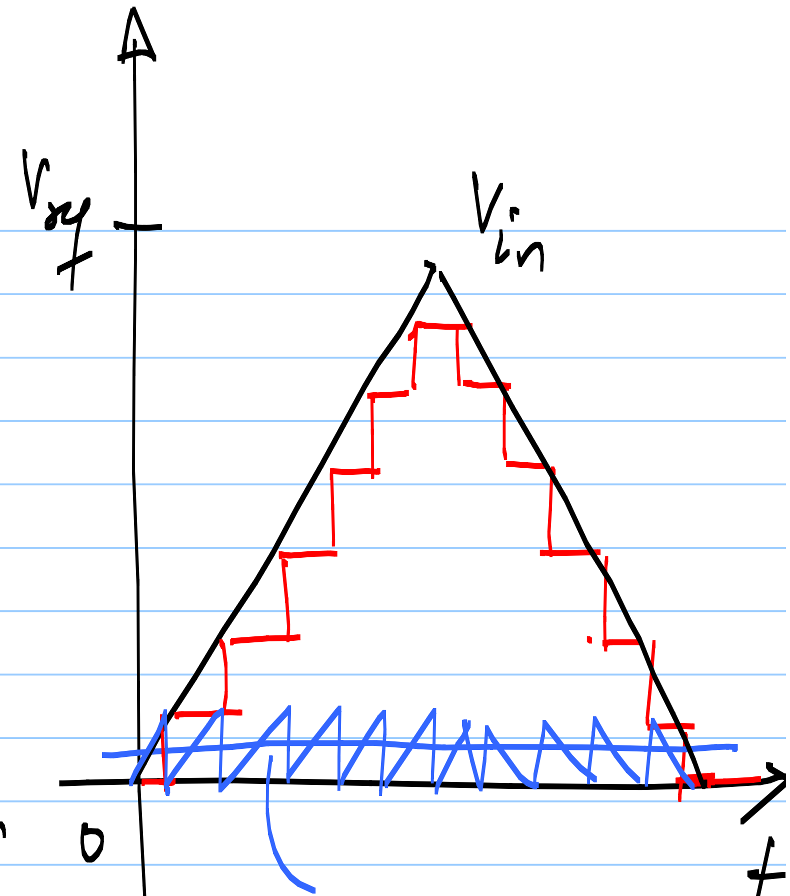
Gain; Offset, INL, DNL

ADC :

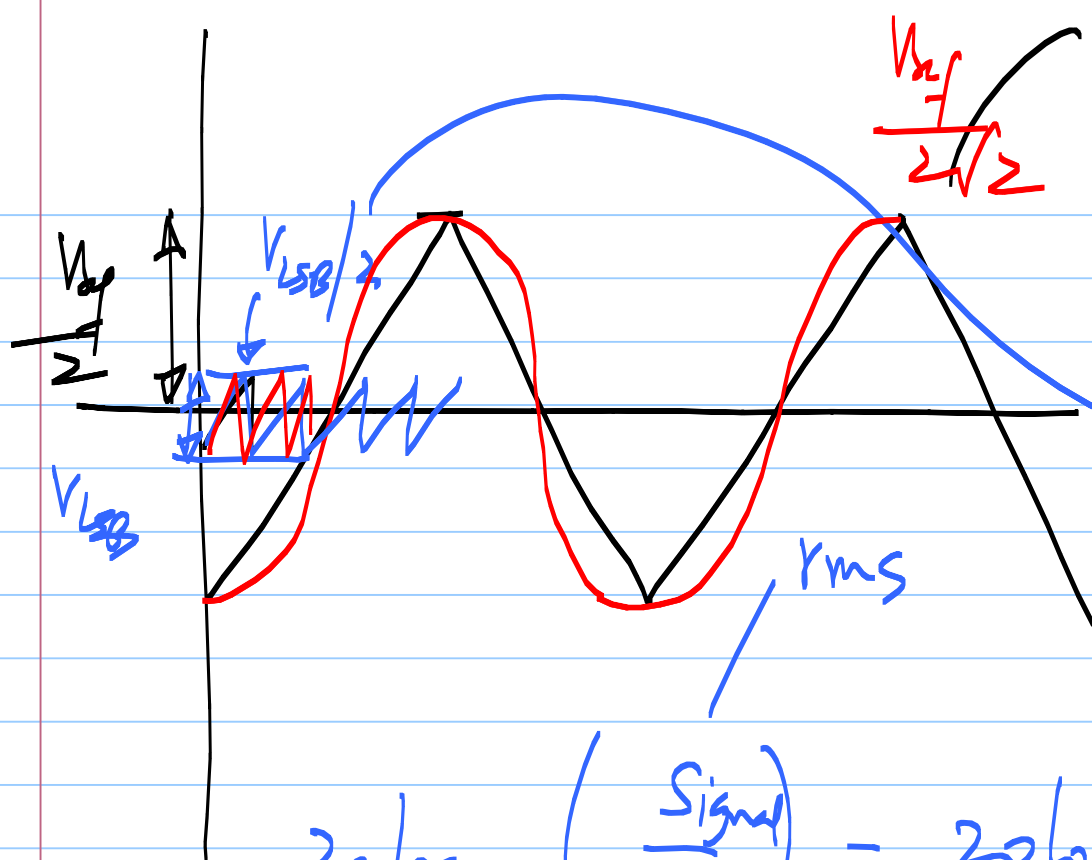

V_{in}
Ideal ADC
Ideal DAC
V_{out}
−
+
Σ
V_q


V_{out}
$V_{ref} \cdot \frac{2^N - 1}{2^N}$
V_{in}
V_{ref}





$$\text{Average}(V_q) = \frac{1}{2} \cdot V_{LSB}$$



$$rms = \frac{V_{ref}}{2\sqrt{3}} = S$$

Quantization error

$$rms = \frac{V_{LSB}}{2\sqrt{3}} = N$$

Triangular $\quad 20\log_{10}\left(\frac{\text{Signal}}{\text{Noise}}\right) = 20\log_{10}\left(2^N\right) = 6.02N \text{ dB}$

Sine wave $\quad 20\log_{10}\left(\frac{\text{Signal}}{\text{Noise}}\right) = 20\log_{10}\left(2^N \cdot \sqrt{\frac{3}{2}}\right) = 6.02N + 1.8 \text{ dB}$

SNR of an ideal N-bit quantizer: $6N + 1.8\ dB$.

10-bit ADC : $V_{pp} = 1V \longrightarrow$ SNR

$V_{sig} = 1V$

$62.$ [illegible] dB

Signal to noise ratio (SNR) of an ADC

N-bit quantizer w/ full-scale of V_{ref},

- Triangular wave of p-p $= V_{ref}$

  q. error = sawtooth wave pp $= \frac{V_{ref}}{2^N}$

  rms (after removing dc) $= V_{ref}/2\sqrt{3}$

  rms (") $= V_{ref}/2^N \cdot 2\sqrt{3}$

$$SNR = 20\log_{10}\left(\frac{V_{ref}/2\sqrt{3}}{V_{ref}/2^N \cdot 2\sqrt{3}}\right) = \underline{6.02N}\ dB$$

– Sine-wave of p-p = V_{ref}

q. error = distorted sawtooth wave p-p = $\frac{V_{ref}}{2^N}$

rms (after removing dc) = $V_{ref}/2\sqrt{2}$

[redacted]

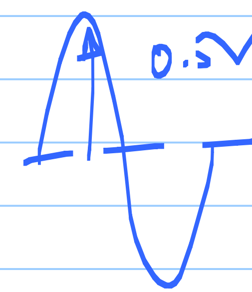
$$SNR = 20\log_{10}\left(\frac{V_{ref}/2\sqrt{2}}{V_{ref}/2^N \cdot 2\sqrt{3}}\right) = \underline{6.02N + 1.8\ dB}$$

N-bit (10-bits)

Ideal N-bit ADC: driven by a sinusoid whose $pp = V_{ref}$ (1V)

has an o/p $SNR = 6.02N + 1.8\,dB$

(56 dB) 10 bits ⟺ 62 dB



Real ADC: nominally N-bits

Measure the SNR with a sinusoid of $pp = V_{ref}$

$$6.02N_{eff} + 1.8\,dB = SNR' < 6.02N + 1.8\,dB$$

$$N_{eff} = \frac{SNR' - 1.8}{6.02} = \frac{56 - 1.8\,dB}{6.02} \quad (\sim 54) \quad = 9\,\text{bits}$$

INL, DNL, Offset, Gain

+

SNR, N_{eff}

} characterize ADCs