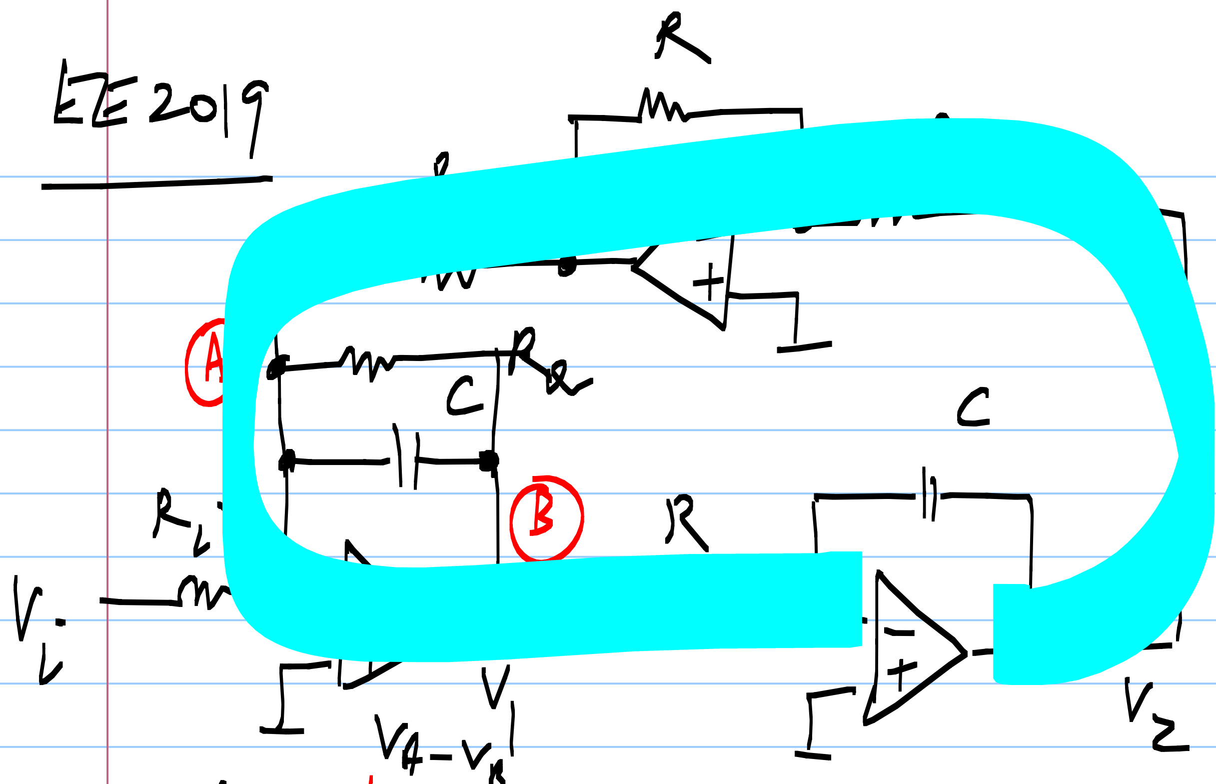
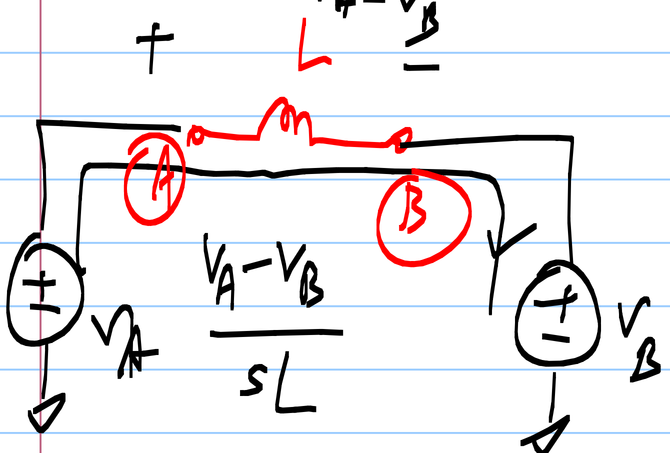
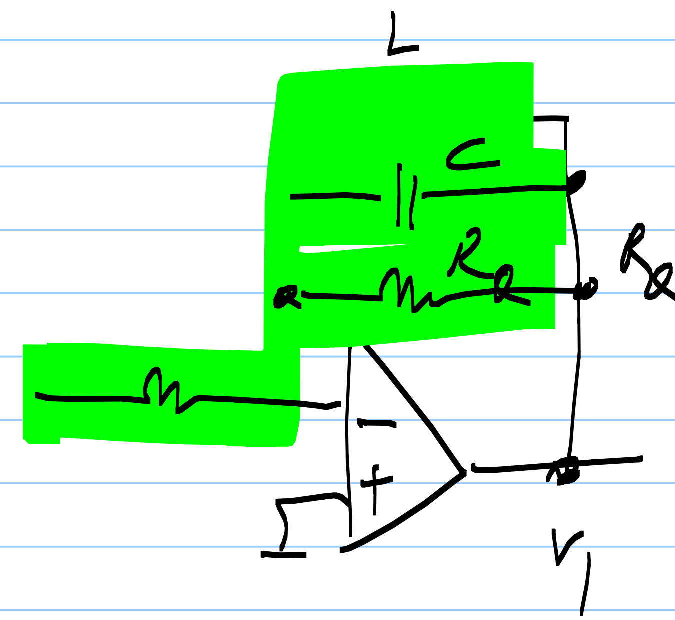
EE 2019

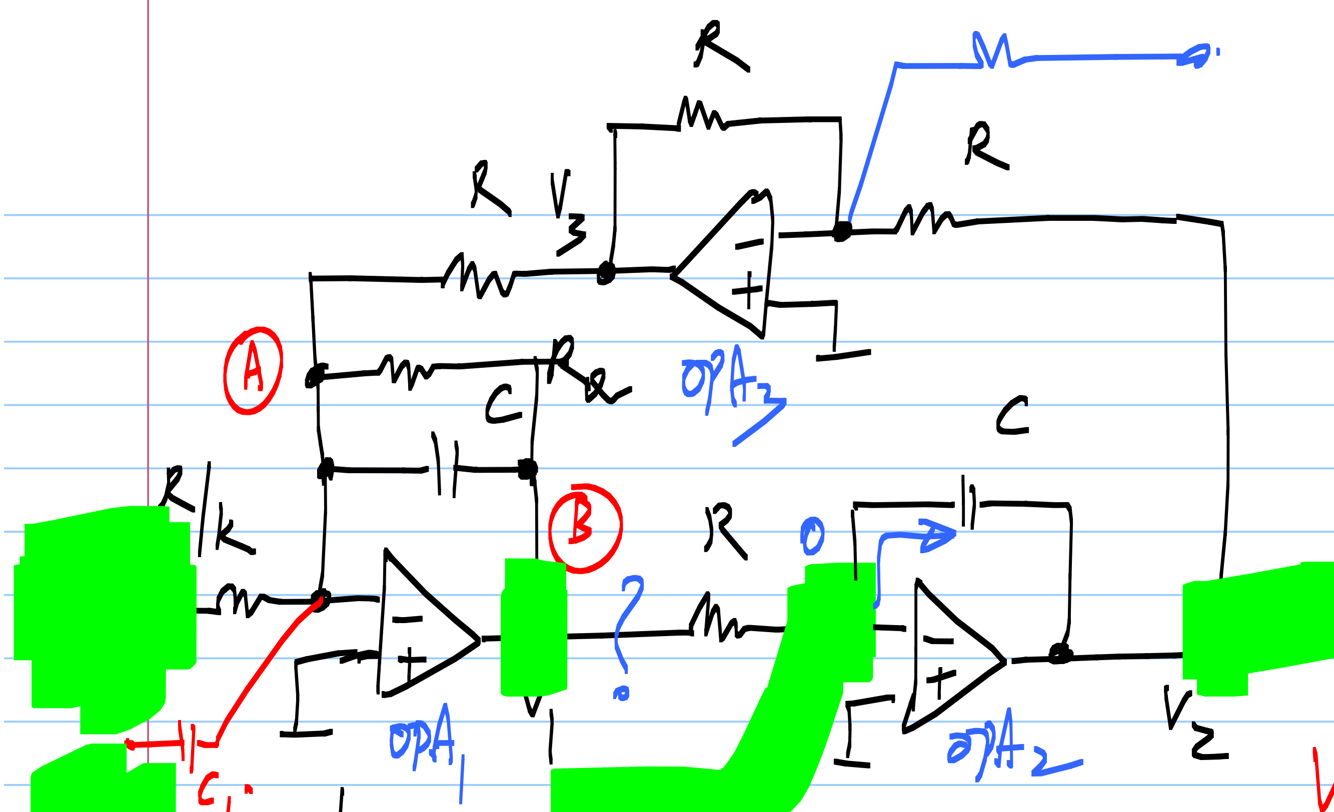
25/3/2017





$$Q = \frac{R_Q}{R}$$





$$\frac{V_2}{V_{i,R}} = \frac{k}{1 + sCR\cdot\frac{R}{R_Q} + (sCR)^2}$$

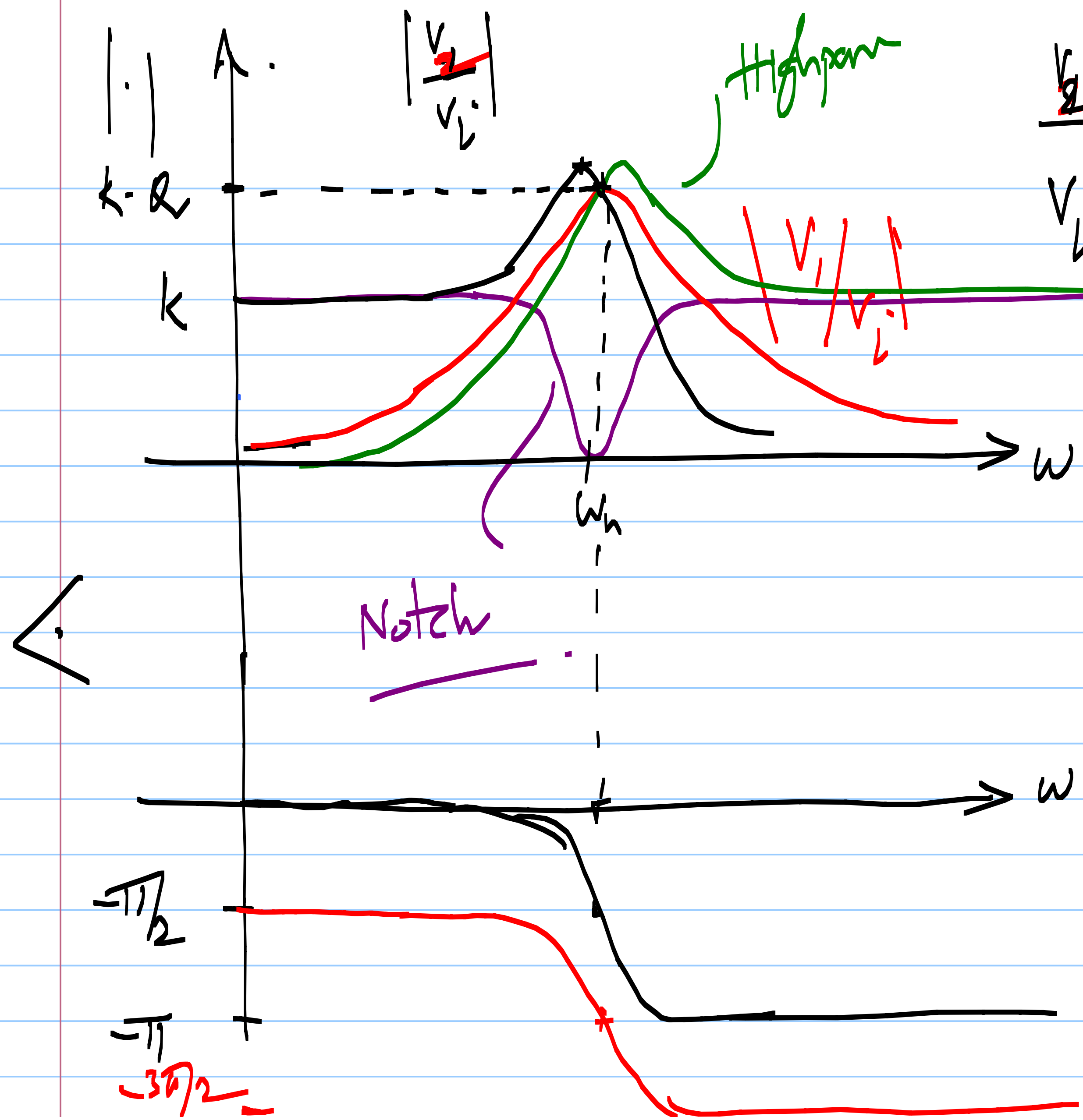
$$\frac{V_1}{V_{i,R}} = \frac{-\,sCR^2/(R/k)}{1 + sCR\cdot\frac{R}{R_Q} + (sCR)^2}$$

$$\frac{V_1}{V_{i,C}} = \frac{-s^2C^2\cdot R^2\cdot\frac{C_i}{C}}{1 + sCR\frac{R}{R_Q} + (sCR)^2}$$

$$\omega_n = \frac{1}{CR}$$

$$\frac{V_2}{V_1} = -\frac{1}{sCR}$$

$$Q = \frac{R_Q}{R} \qquad \zeta = \frac{R}{2R_Q}$$



$$\frac{V_2}{V_i} = \frac{K}{1+\frac{sCR}{Q}+(sCR)^2}$$

$$\frac{V_1}{V_i} = \frac{-K\cdot sCR}{1+\frac{sCR}{Q}+(sCR)^2}$$

$$\frac{V_2}{V_1} = -\frac{1}{sCR}$$

$$V_1 = -(sCR)\,V_2$$

Lowpass:

$$\frac{k}{\left(\frac{s}{\omega_n}\right)^2 + \frac{s}{\omega_n Q} + 1}$$

Bandpass:

$$\frac{k' \frac{s}{\omega_n}}{\left(\frac{s}{\omega_n}\right)^2 + \frac{s}{\omega_n Q} + 1}$$

Highpass:

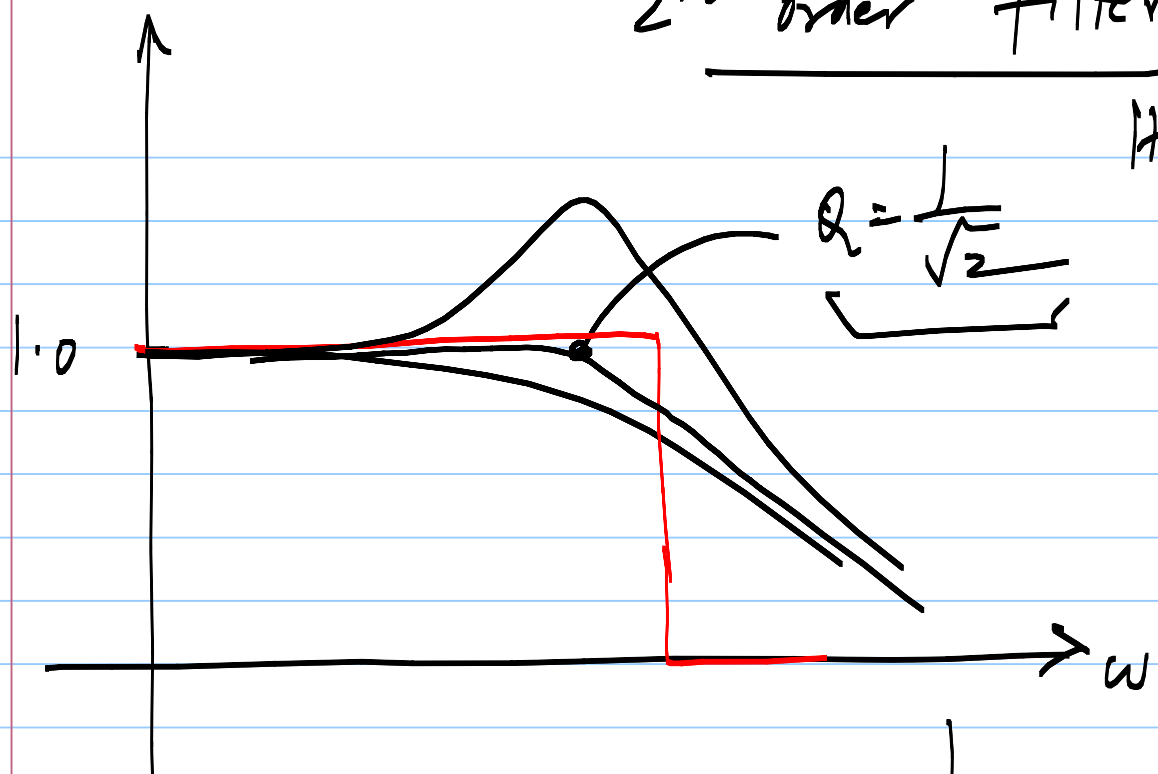
$$\frac{k\left(\frac{s}{\omega_n}\right)^2}{\left(\frac{s}{\omega_n}\right)^2 + \left(\frac{s}{\omega_n Q}\right) + 1}$$

Bandstop.
Notch

$$\longrightarrow \frac{k\left[1 + \left(\frac{s}{\omega_n}\right)^2\right]}{D(s)}$$

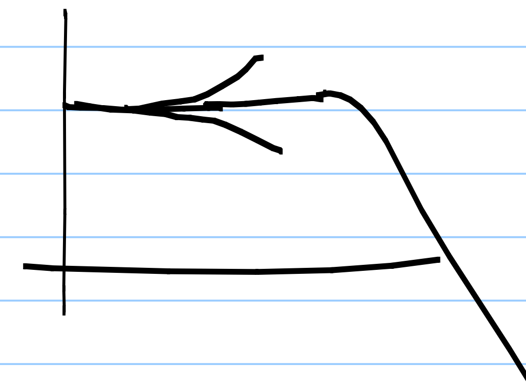
<u>2nd order filter.</u>

$$H(s) = \frac{1}{\left(\frac{s}{\omega_n}\right)^2 + \frac{s}{\omega_n Q} + 1}$$



$$\underbrace{|H(j\omega)|^2} = \frac{1}{\left(1 - \frac{\omega^2}{\omega_N^2}\right)^2 + \left(\frac{\omega}{\omega_n Q}\right)^2}$$

$$\underbrace{\left|H(j\omega)\right|^2} = \frac{1}{\left(1-\frac{\omega^2}{\omega_N^2}\right)^2 + \left(\frac{\omega}{\omega_n Q}\right)^2}$$



$$\underbrace{\left|H(j\omega)\right|^2} = \frac{1}{1 + \underbrace{\frac{\omega^2}{\omega_N^2}\left(\frac{1}{Q^2} - 2\right)}_{=0} + \frac{\omega^4}{\omega_N^4}}$$

<u>Maximally flat magnitude response</u>

$$H(s) = \frac{1}{1 + a_1 s + \dots \left(\frac{s}{\omega_N}\right)^N}$$

N^{th} order lowpass

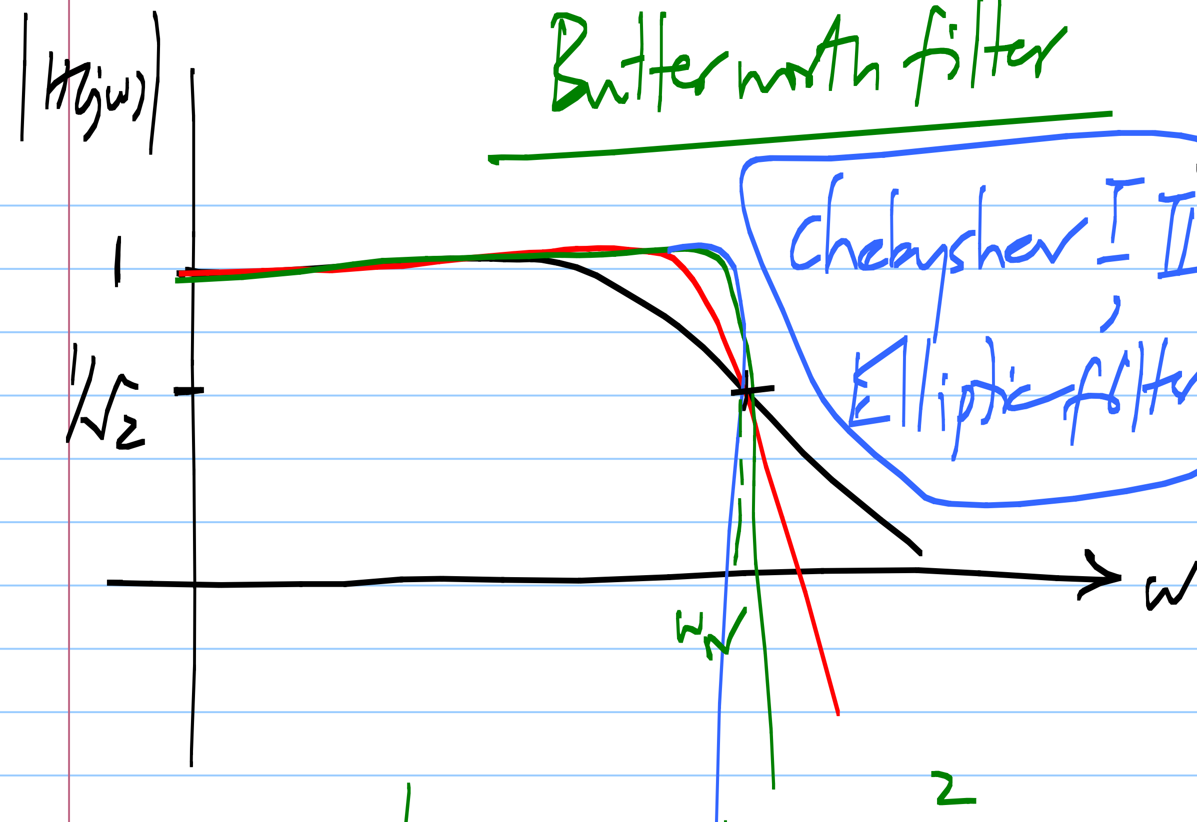
$$|H(j\omega)|^2 = \frac{1}{1 + c_2 \left(\frac{\omega}{\omega_n}\right)^2 + \dots + c_{2N-2}\left(\frac{\omega}{\omega_n}\right)^{2N-2} + \left(\frac{\omega}{\omega_n}\right)^{2N}}$$

Only even order terms; order $2N$

$= 0$ for maximally flat magnitude

Butterworth filter

N^{th} order max. flat magnitude filter



Chebyshev I, II

Elliptic filter

$$|H(j\omega)|^2 = \frac{1}{1+\left(\frac{\omega}{\omega_N}\right)^{2N}}$$

$$H(s) = \frac{1}{D(s)} \qquad |H(j\omega)|^2 = \frac{1}{|D(j\omega)|^2} = \frac{1}{1+\left(\frac{\omega}{\omega_N}\right)^{2N}}$$

Need $D(s)$ such that

$$|D(j\omega)|^2 = 1 + \left(\frac{\omega}{\omega_N}\right)^{2N}$$

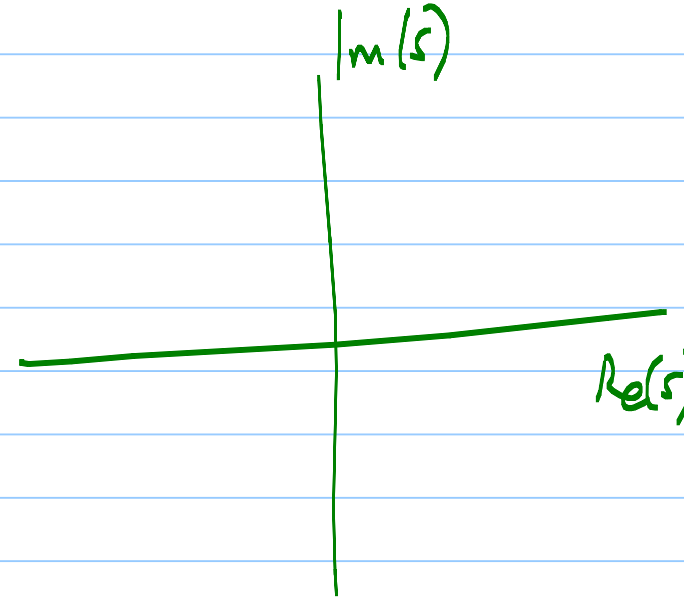
$$s = j\omega$$

$$\omega = \frac{s}{j}$$

$$D(j\omega) \cdot D^*(j\omega)$$

$$D(s) \cdot D(-s)\Big|_{s=j\omega} = 1 + \left(\frac{\omega}{\omega_N}\right)^{2N}$$

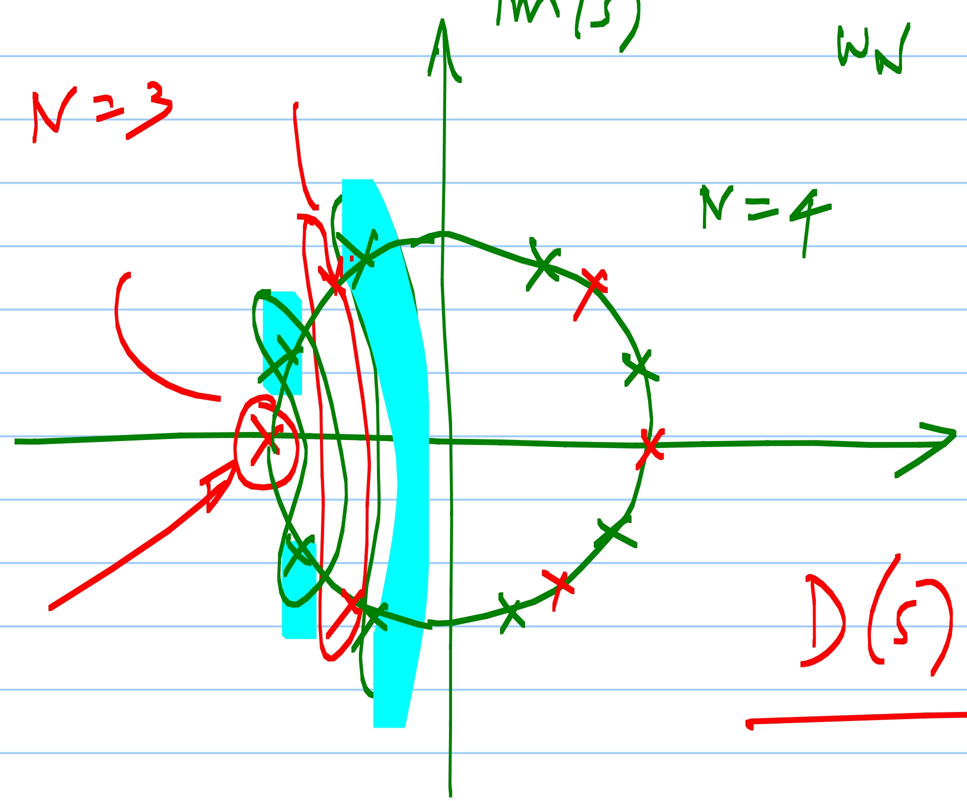
$$D(s) \cdot D(-s) = 1 + \left(\frac{s}{j\omega_N}\right)^{2N}$$



$$1+\left(\frac{s}{j\omega_N}\right)^{2N}=0$$

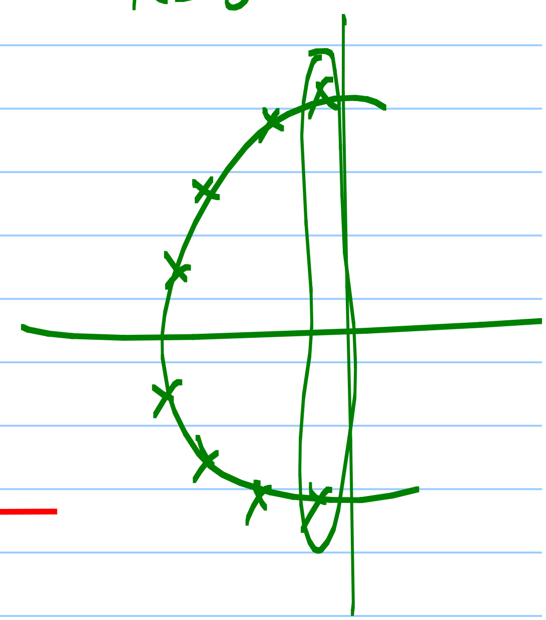
$$\left(\frac{s}{j\omega_N}\right) = \exp\left(j\left(\frac{2\pi k+\pi}{2N}\right)\right)$$

$$\frac{s}{\omega_N} = \exp\left(j\left(\frac{2\pi k+\pi}{2N}+\frac{\pi}{2}\right)\right)$$

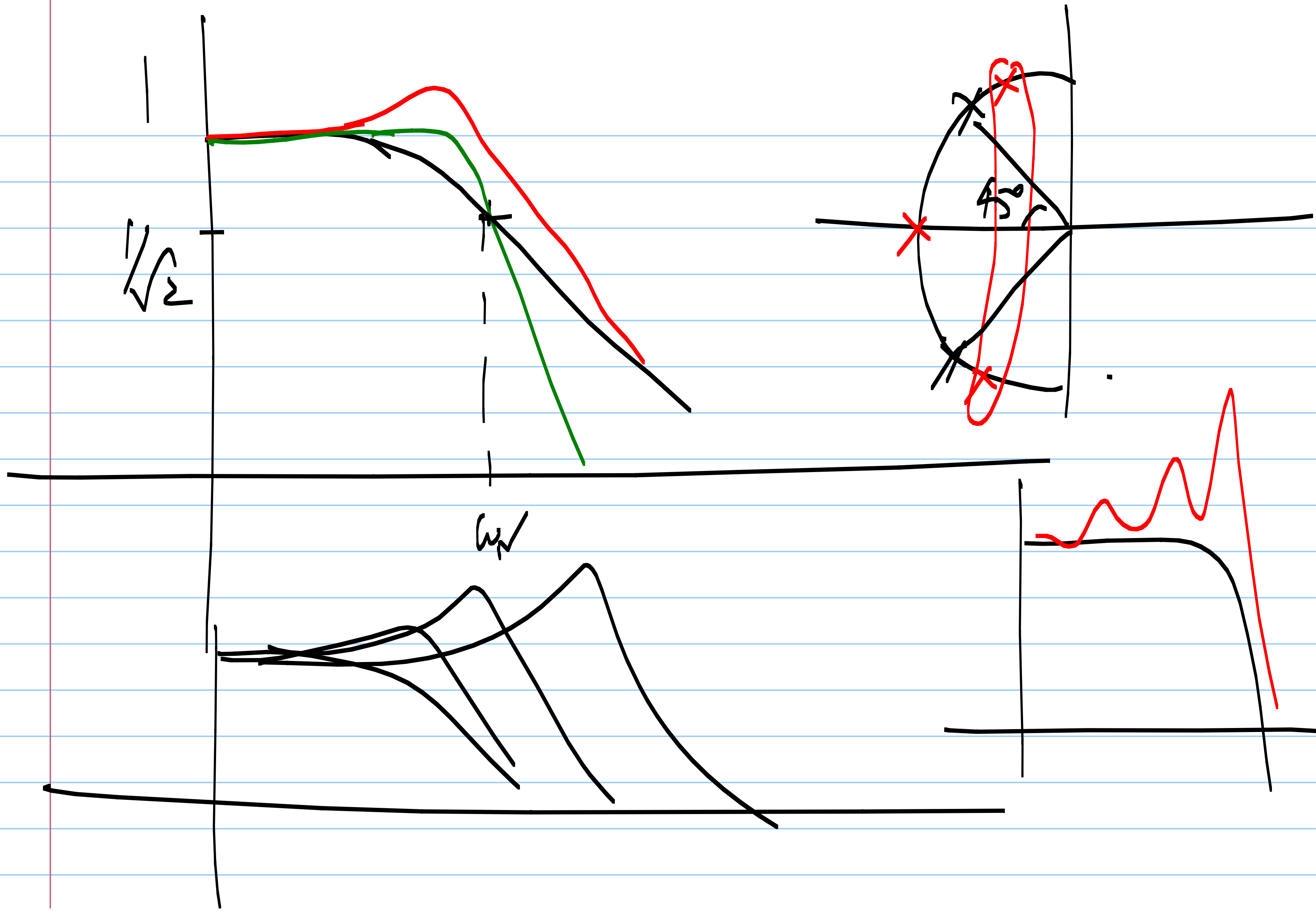


N=4

N=4 k=0

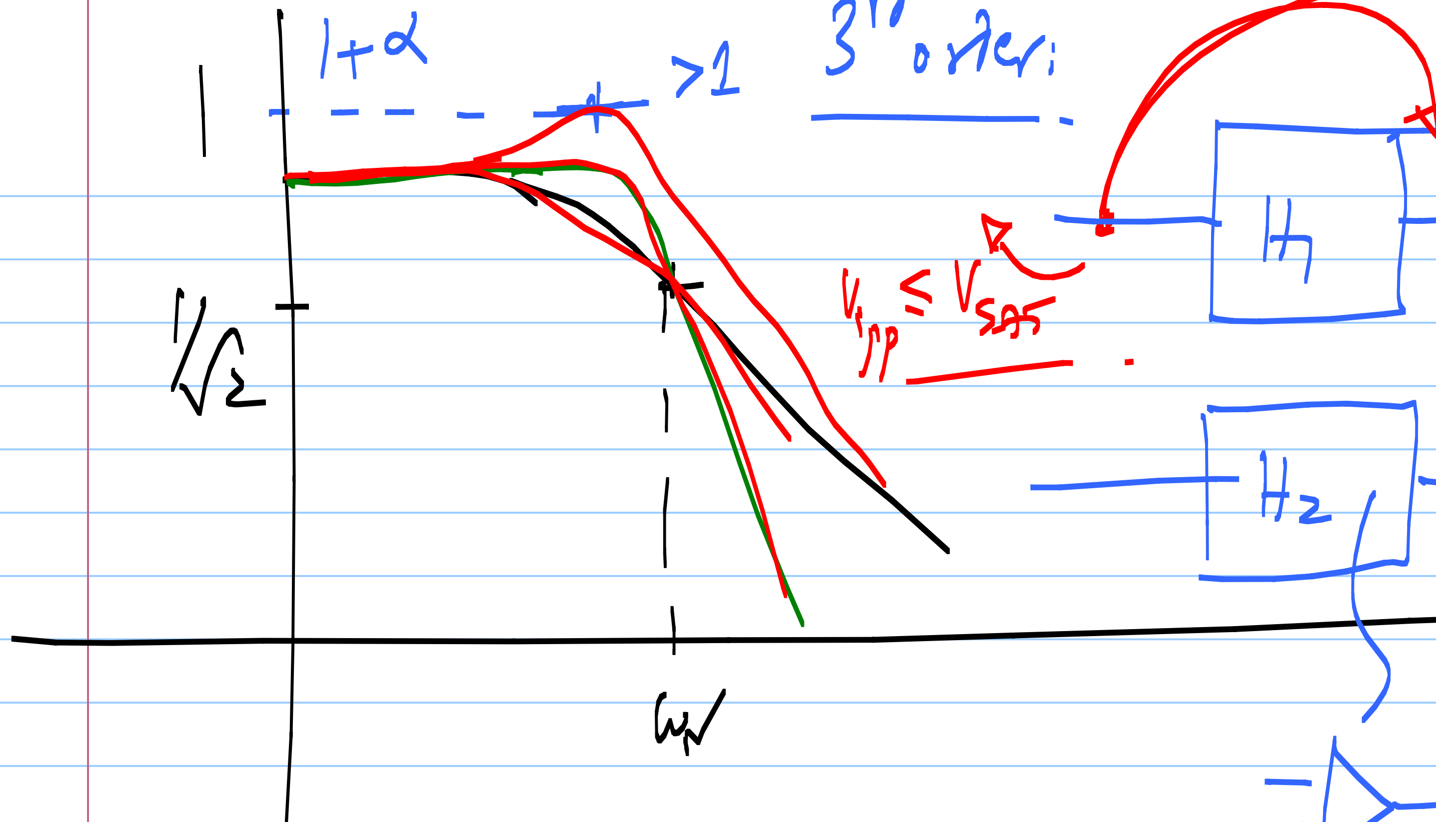


$D(s)\,D(-s)$

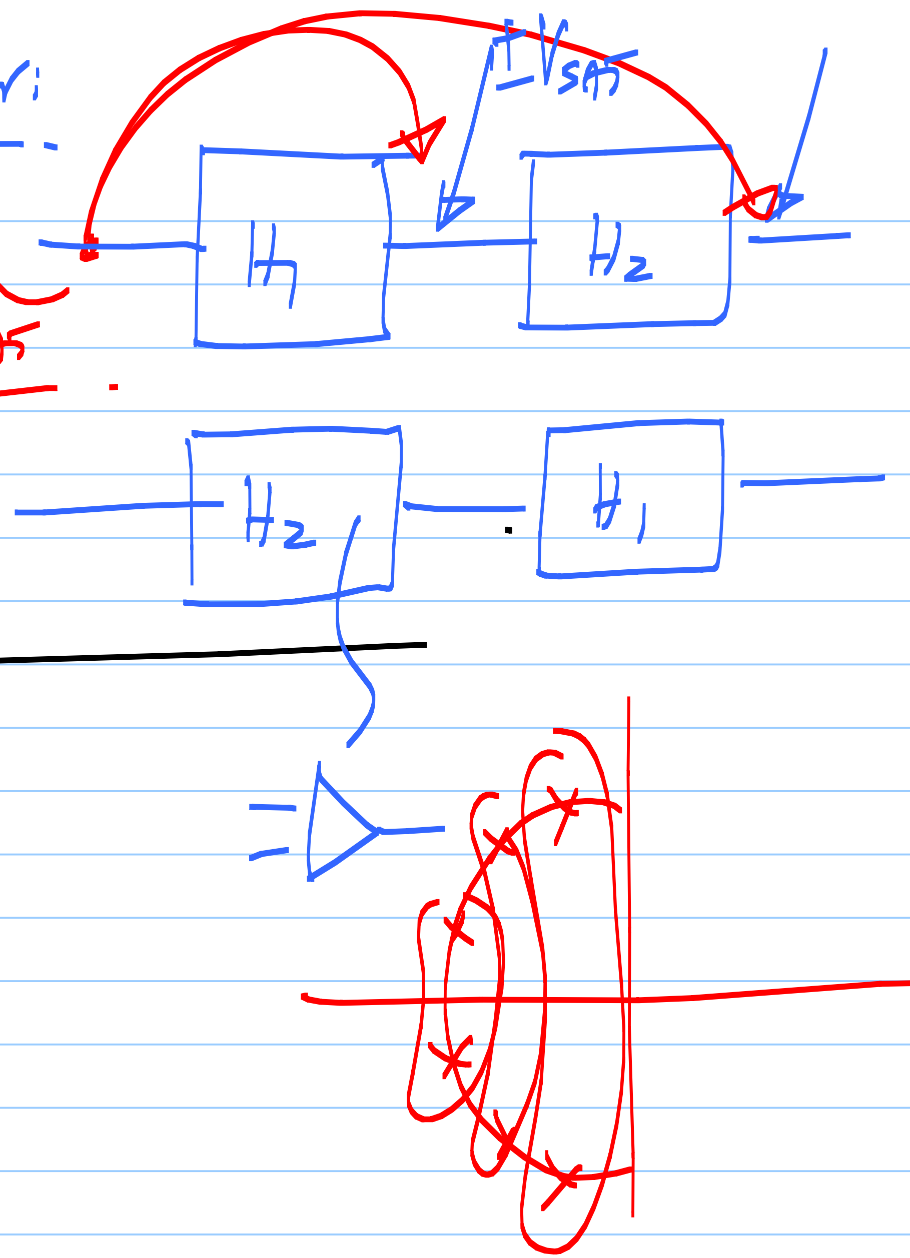


1/√2
ω_r
45°

3rd order:



$V_{i,pp} \le V_{SAT}$

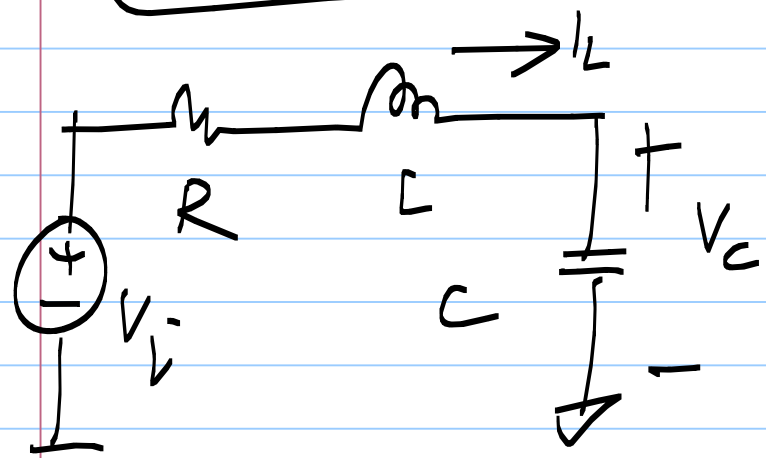


$$V_{i,ppk} < \frac{V_{SAT}}{1+\alpha}$$

Ordering of sections:

Increasing order of Q to maximize the input that can be applied (outputs limited to V_{SAT})

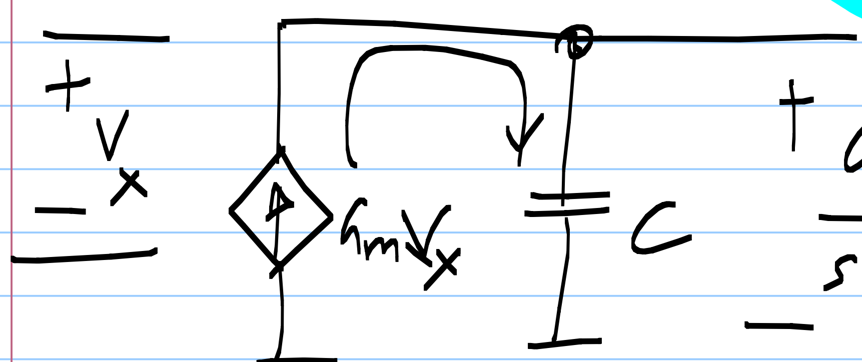
Filters using Opamp. Integrators — Active-RC filter



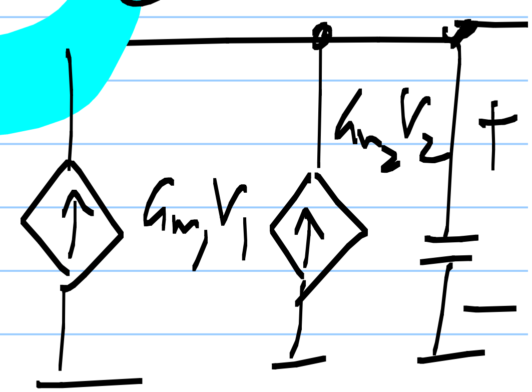
$$R \cdot I_L = \left(V_i - I_L \cdot R - V_c\right)\frac{R}{sL}$$

$$V_c = \frac{I_L}{sC}$$

$$\frac{G_{m1}}{sC} V_1 + \frac{G_{m2}}{sC} V_2$$



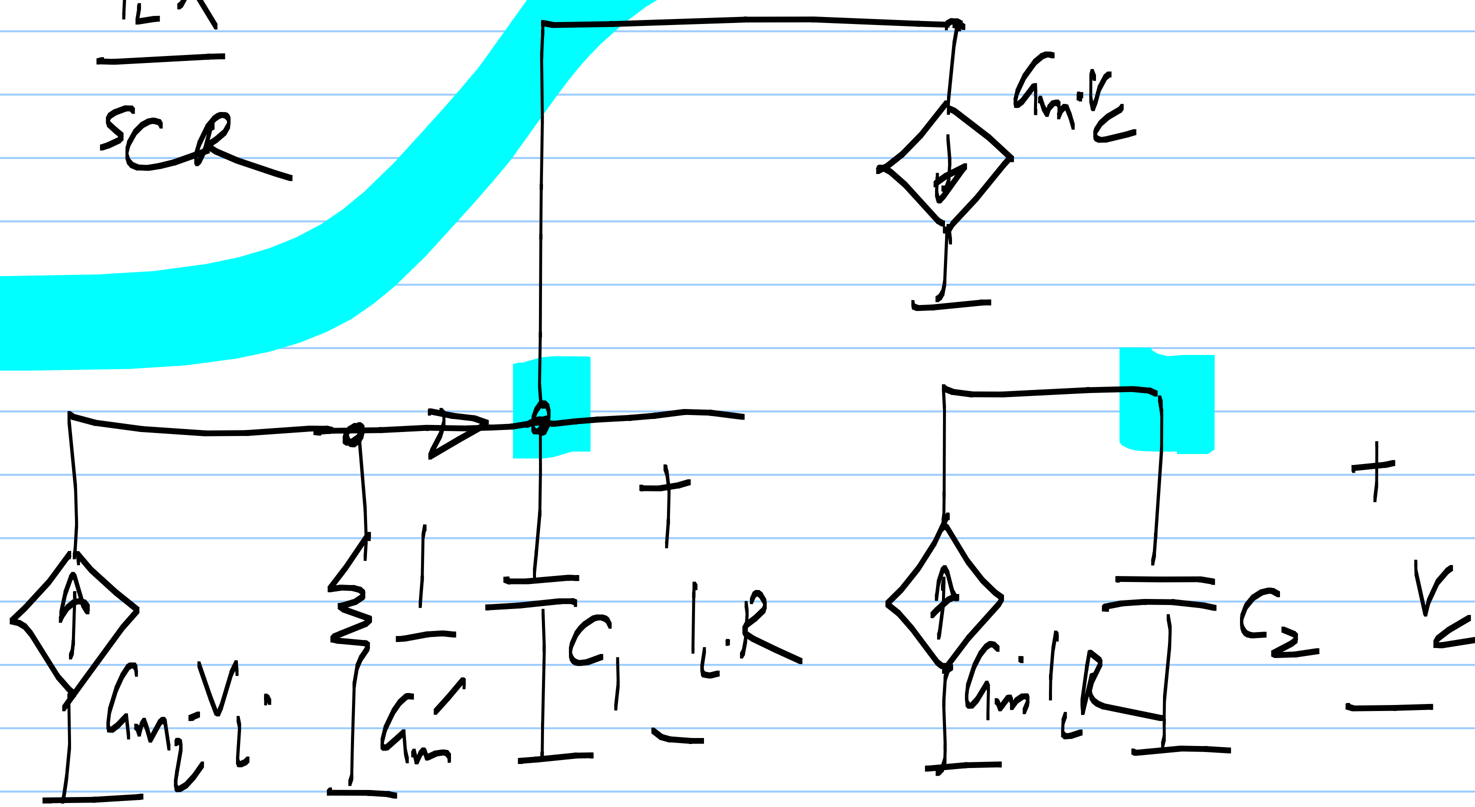
$$\frac{G_m}{sC} \cdot V_x$$



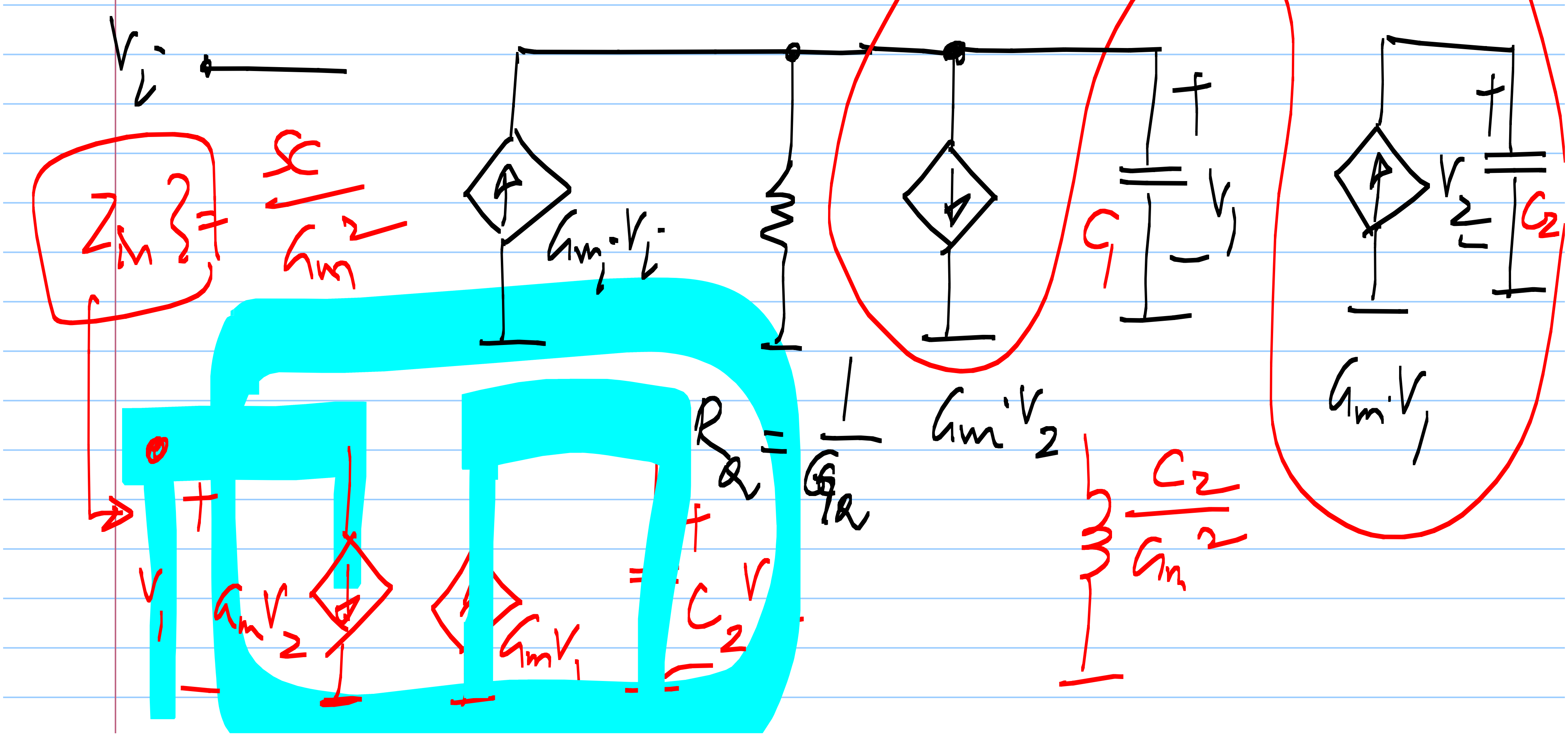
$R \cdot I_L = \left(V_i \text{[illegible]} \right) \frac{R}{s_1}$

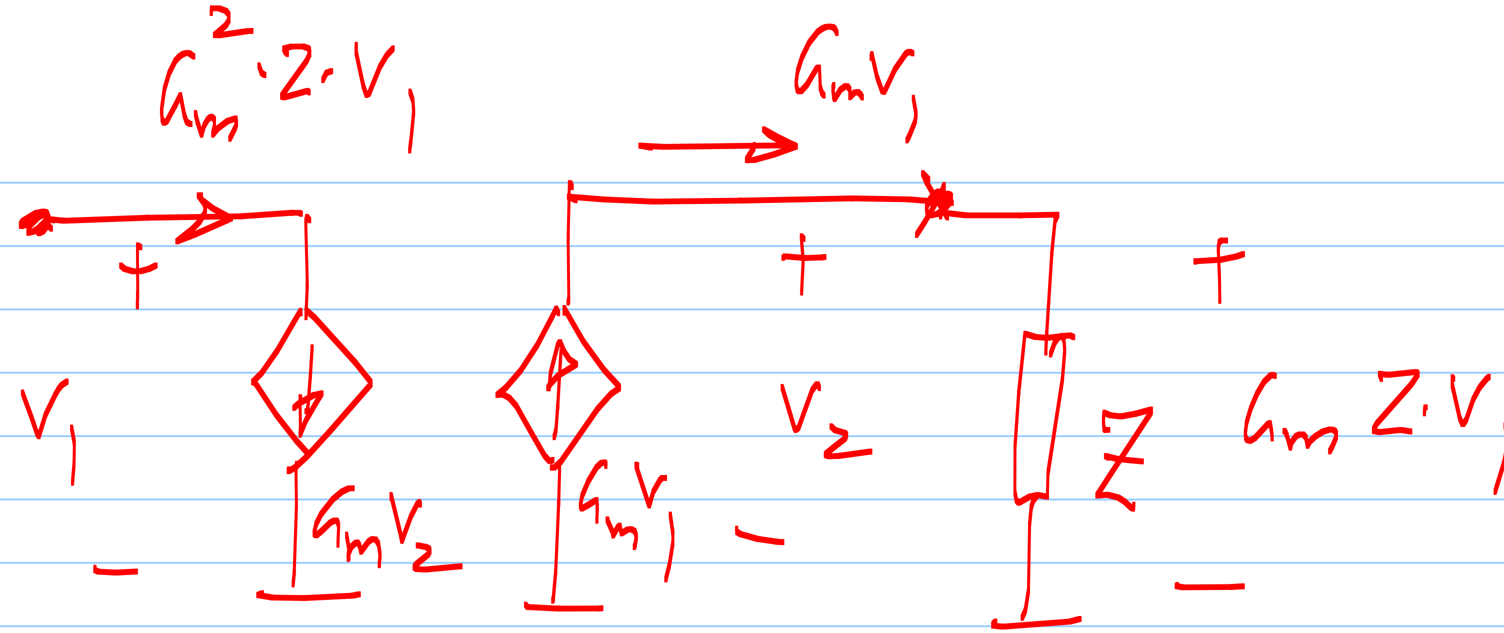
$V_c = \frac{I_L R}{sCR}$

G_m-C filter



Calculate $\frac{V_1}{V_i}$, $\frac{V_2}{V_i}$





$$\frac{1}{G_m^2 Z}$$

Gyrator