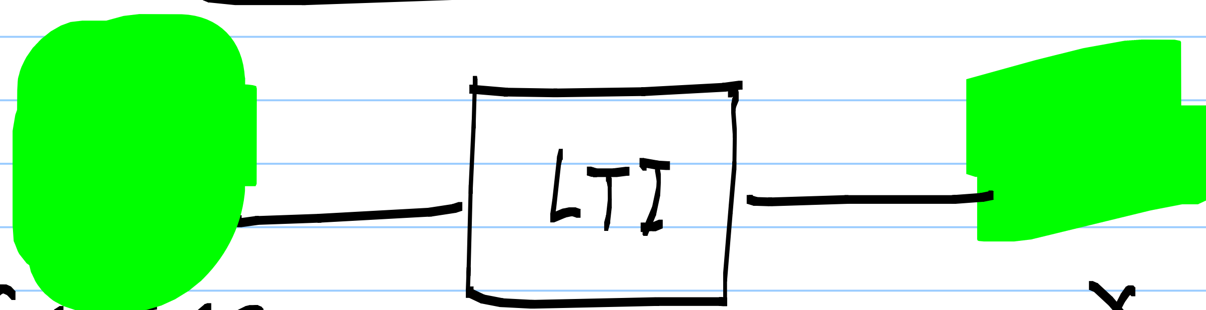
EE 2015

Computing the response of a linear system using the Fourier transform

6/11/2017



zero for t < 0

circuit w/ R, L, C

$X_{jw}(j\omega) \cdot H(j\omega)$

$\frac{1}{2\pi}\int X.(j\omega)\exp(j\omega t)\,d\omega$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_{j\omega}(j\omega) \exp(j\omega t)\, d\omega$$

$t > -\infty$

$\overbrace{\text{Steady-state}}$ response

to $\frac{1}{2\pi} X_{j\omega}(j\omega) \exp(j\omega t)$: $\frac{1}{2\pi} H(j\omega) \cdot X_{j\omega}(j\omega) \exp(j\omega t)$

$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(j\omega)\, X_{j\omega}(j\omega) \exp(j\omega t)\, d\omega$$

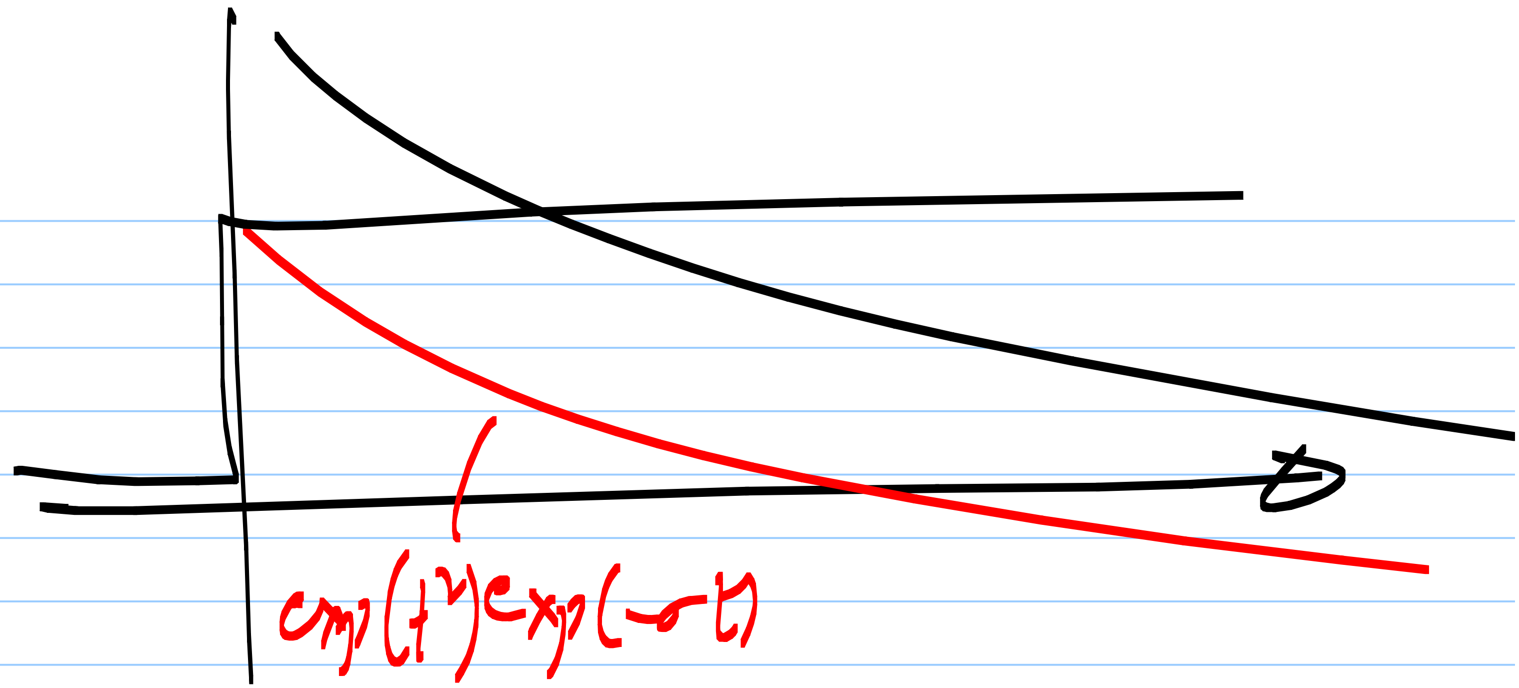
Compute $H(j\omega)$ using ckt analysis

Fourier transform doesn't exist for many common signals.

$x(t) = u(t)$ $\qquad$ $\int_{-\infty}^{\infty} x(t) \exp(-j\omega t) \cdot dt$

Calculate F.T. of $x(t) \exp(-$[illegible]$t)$

$$\int_{-\infty}^{\infty} x(t) \exp(-\sigma t) \exp(-j\omega t) \cdot dt$$


exp(t²)exp(−σt)

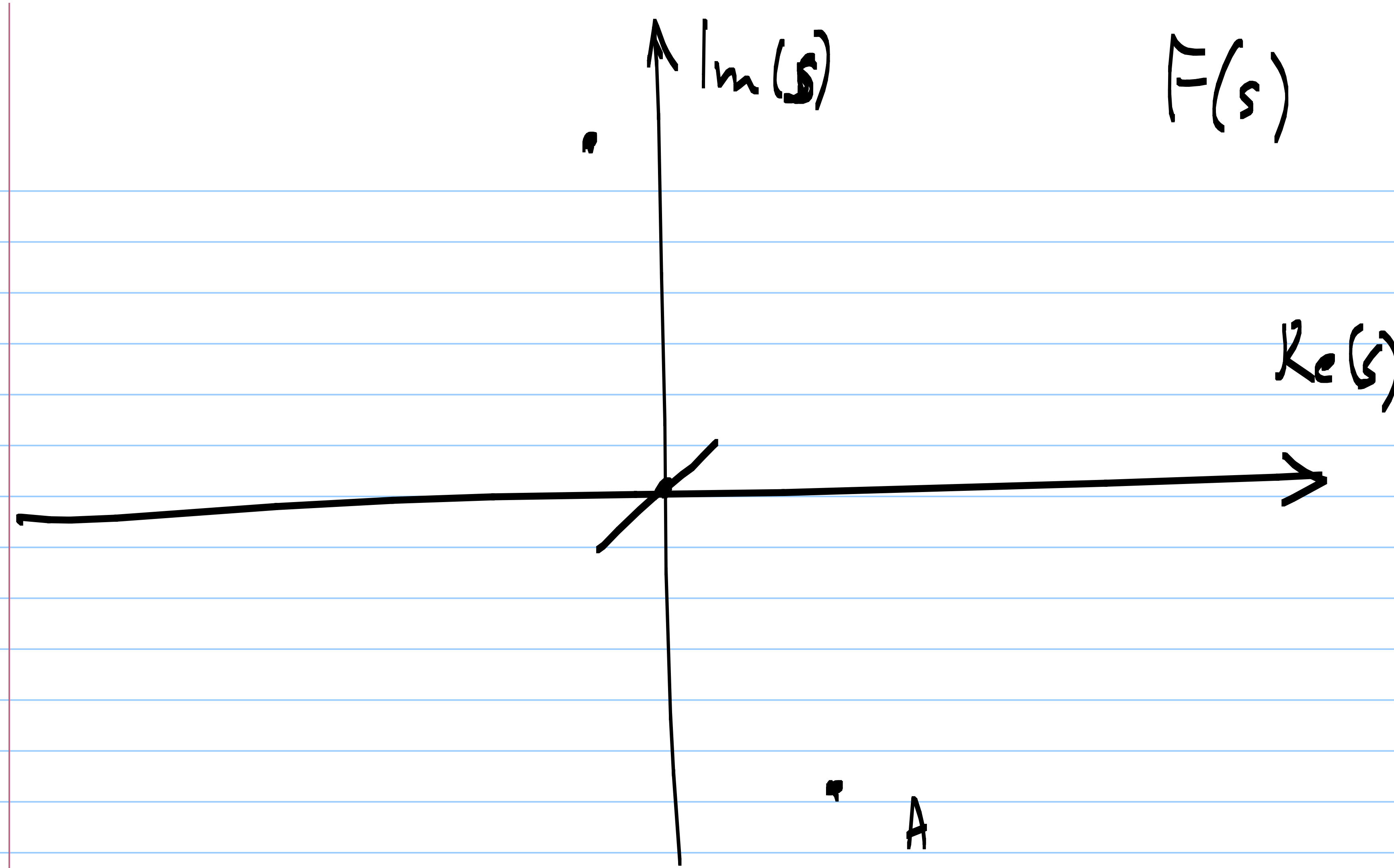
Inverse F.T of $X_{\sigma,j\omega}(j\omega)$

$$x(t) = \frac{\exp(\sigma t)}{j2\pi} \int_{-j\infty}^{j\infty} X_{\sigma,j\omega}(j\omega) \cdot \exp(j\omega t) \cdot d(j\omega)$$

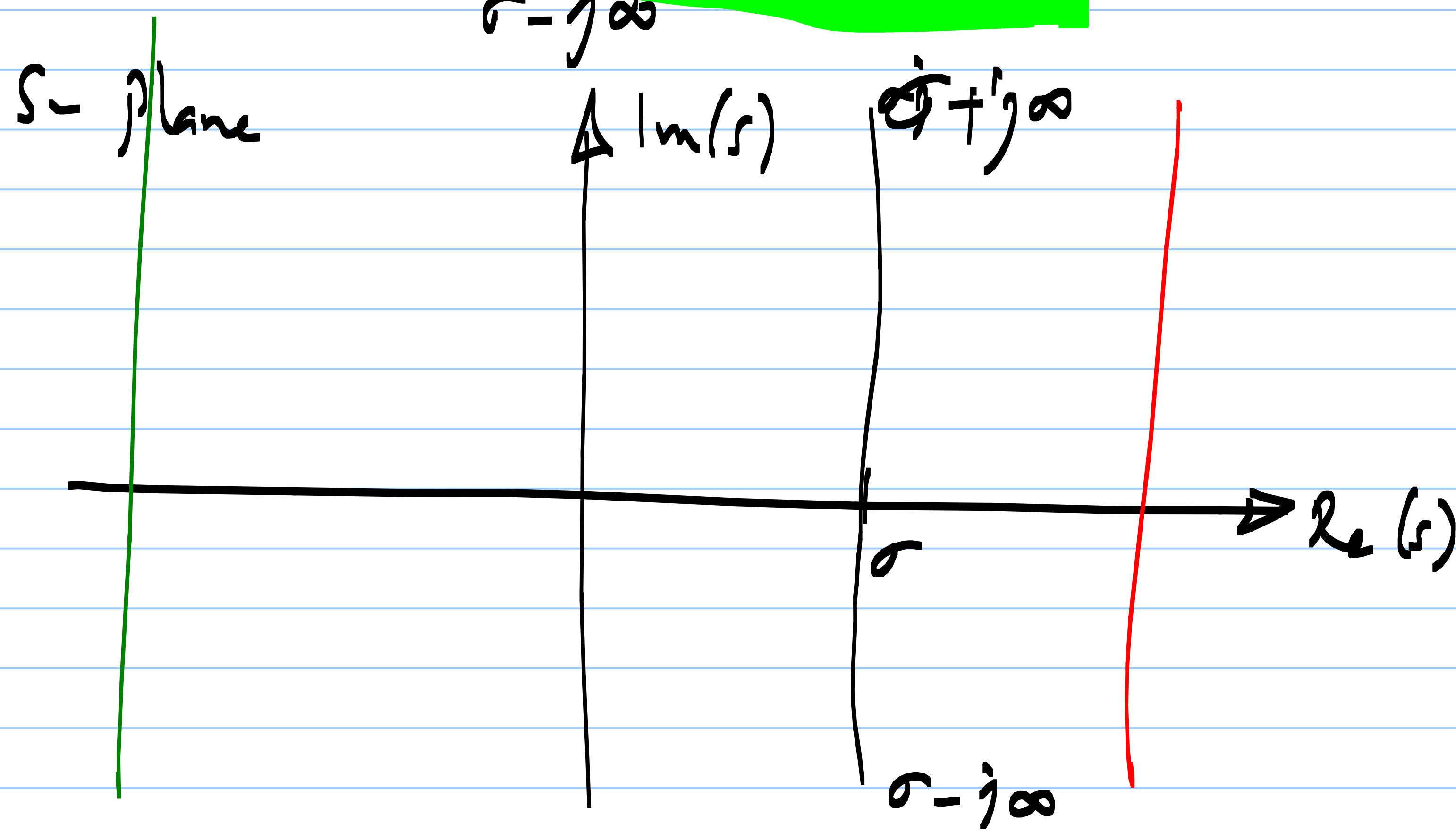
$s = \sigma + j\omega$

complex frequency s

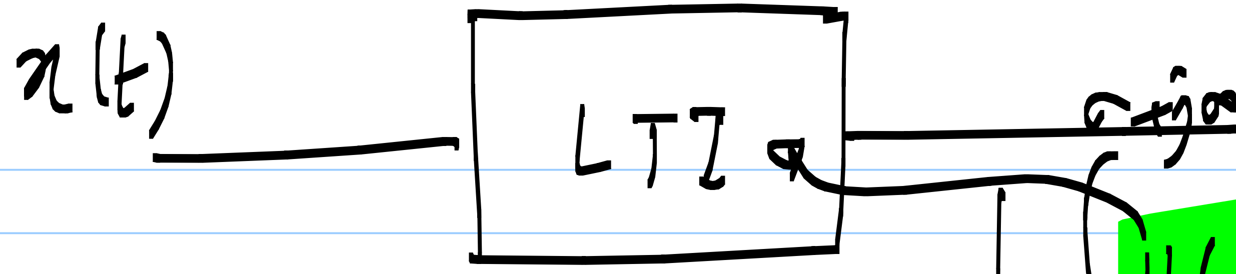
$$x(t) = \frac{1}{j2\pi} \int_{\sigma-j\infty}^{\sigma+j\infty} \cancel{X_{\sigma,j\omega}(j\omega)}\ X(s)\ \exp\big((\sigma + j\omega) \cdot t\big) \cdot \underbrace{d(j\omega)}_{}\ ds$$


F(s)
Im(s)
Re(s)
A

$$x(t) = \frac{1}{j2\pi} \int_{\sigma - j\infty}^{\sigma + j\infty} \text{[illegible]} \, ds$$

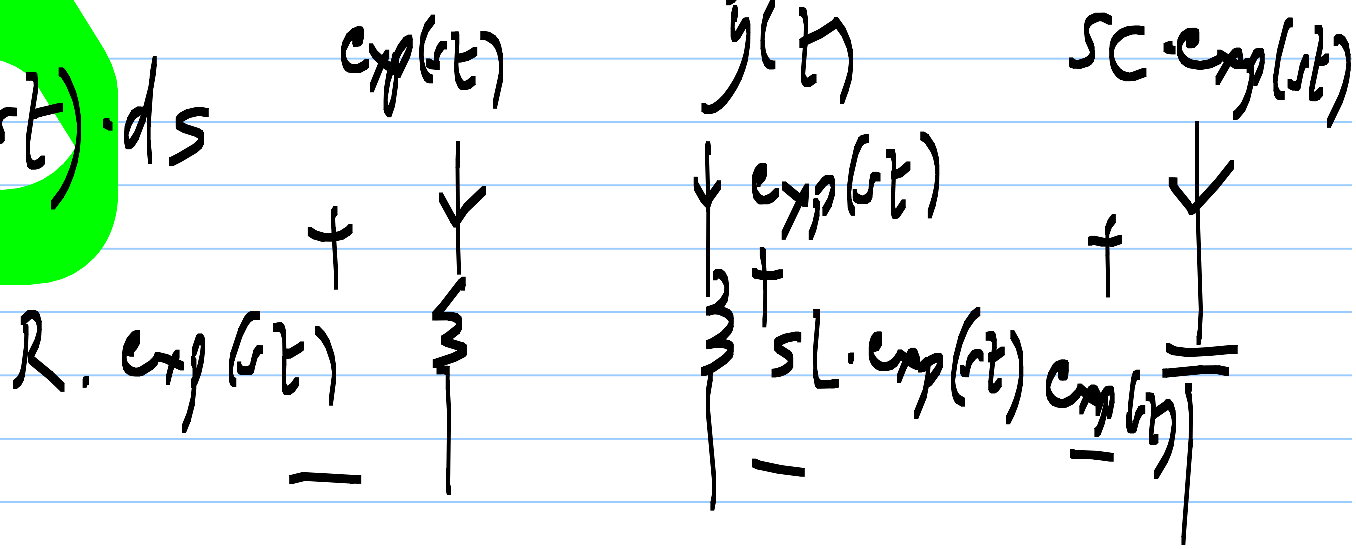


$$X(s) = \int_{0^-}^{\infty} x(t) \exp(-st)\, dt$$



$$\frac{1}{j2\pi}\int_{\sigma-j\infty}^{\sigma+j\infty} H(s)\cdot X(s)\exp(st)\,ds$$

$$\frac{1}{j2\pi}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)\exp(st)\cdot ds$$



$x(t)$: input to the LTI system

$X(s)$: Laplace transform of $x(t)$

$y(t)$: Output of the LTI system

$Y(s)$: L.T. of $y(t)$

$$Y(s) = X(s) \cdot \overbrace{H(s)}^{\text{Transfer fn. from i/p to o/p}}$$

$$\left\{ a_N \cdot \frac{d^N y}{dt^N} + a_{N-1} \frac{d^{N-1} y}{dt^{N-1}} + \cdots \; a_1 \frac{dy}{dt} + a_0 y = b_M \frac{d^M x}{dt^M} + \cdots + b_1 \frac{dx}{dt} + b_0 x \right.$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{b_M s^M + \cdots + b_1 s + b_0}{a_N s^N + a_{N-1} s^{N-1} + \cdots + a_1 s + a_0}$$

$$x(t) = X(s) \cdot \exp(st) \rightarrow y(t) = Y(s) \cdot \exp(st)$$

zeros

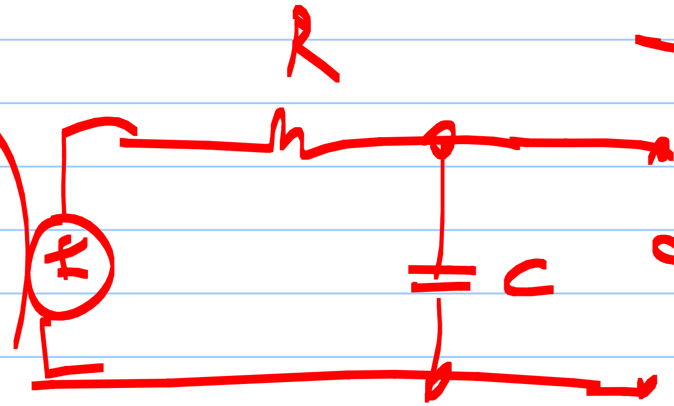
$$H(s) = \frac{Y(s)}{X(s)} = \frac{b_M s^M + \cdots + b_1 s + b_0}{a_N s^N + a_{N-1} s^{N-1} + \cdots + a_1 s + a_0} = \frac{N(s)}{D(s)}$$

poles s_i

N, D: polynomials in s

$$\frac{1}{s^2 LC + s\frac{L}{R} + 1} \qquad \frac{1}{1 + sCR} \qquad \frac{sCR}{}$$

$$p_1 = \frac{-1}{RC}$$



$\exp\left(-\frac{t}{RC}\right) \propto t \exp(-t/RC)$

$\cos(\omega_0 t)u(t)$ — $\boxed{\frac{1}{s^2+\omega_0^2}}$ — $t \cdot \cos(\omega_0 t)$

$$\frac{N(s)}{D(s)} = \frac{b_M s^M + \cdots + b_1 s + b_0}{a_N s^N + \cdots + a_1 s + a_0}$$

$\exp(-sT)$

$\exp(x)$

$$\frac{N_1(s) \cdot N_2(s) \cdots}{[\text{illegible}](s) \cdot [\text{illegible}]_2(s) \cdots}$$