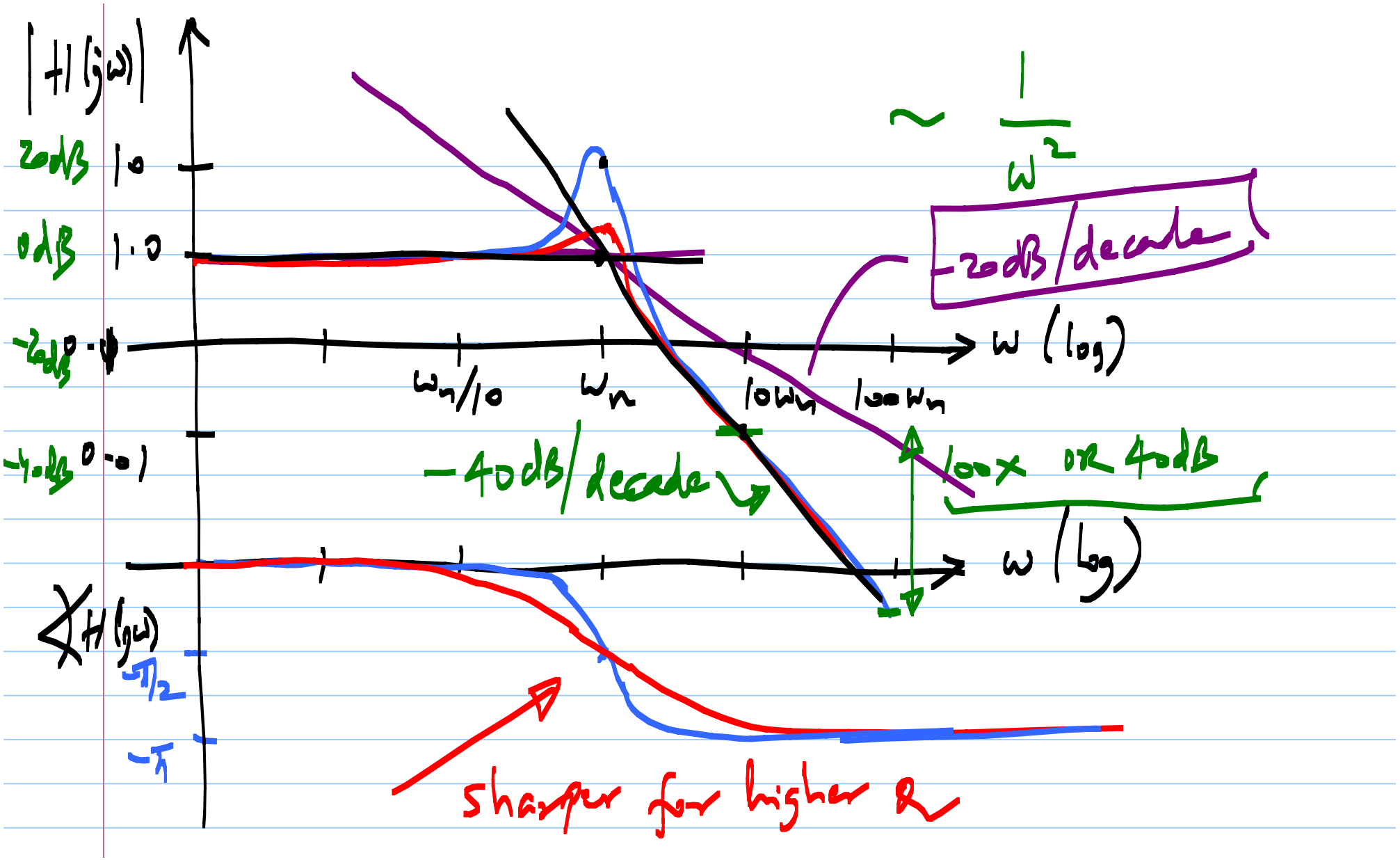
EE2015 — Sinusoidal steady state response of second order systems — 16/10/2017

$$H(j\omega) = \frac{1}{1 - \frac{\omega^2}{\omega_n^2} + j\cdot\frac{\omega}{Q\omega_n}}$$

$\omega \ll \omega_n$: $|H(j\omega)| \simeq 1$, $\angle H(j\omega) = 0$

$\omega \gg \omega_n$: $|H(j\omega)| \simeq \frac{\omega_n^2}{\omega^2}$, $\angle H(j\omega) = -\pi$

$\omega = \omega_n$: $|H(j\omega)| = Q$, $\angle H(j\omega) = -\pi/2$


$|H(j\omega)|$
20dB
0dB
-20dB
-40dB
10
1.0
0.1
$\omega_n/10$
ω_n
10ω_n
100ω_n
ω (log)
$\sim \frac{1}{\omega^2}$
-20dB/decade
-40dB/decade
100x OR 40dB
$\angle H(j\omega)$
$\pi/2$
$-\pi$
sharper for higher Q

decibel unit

$20 \log_{10} |H(j\omega)|$

$20 \log_{10} \left| \frac{V_2}{V_s} \right|$

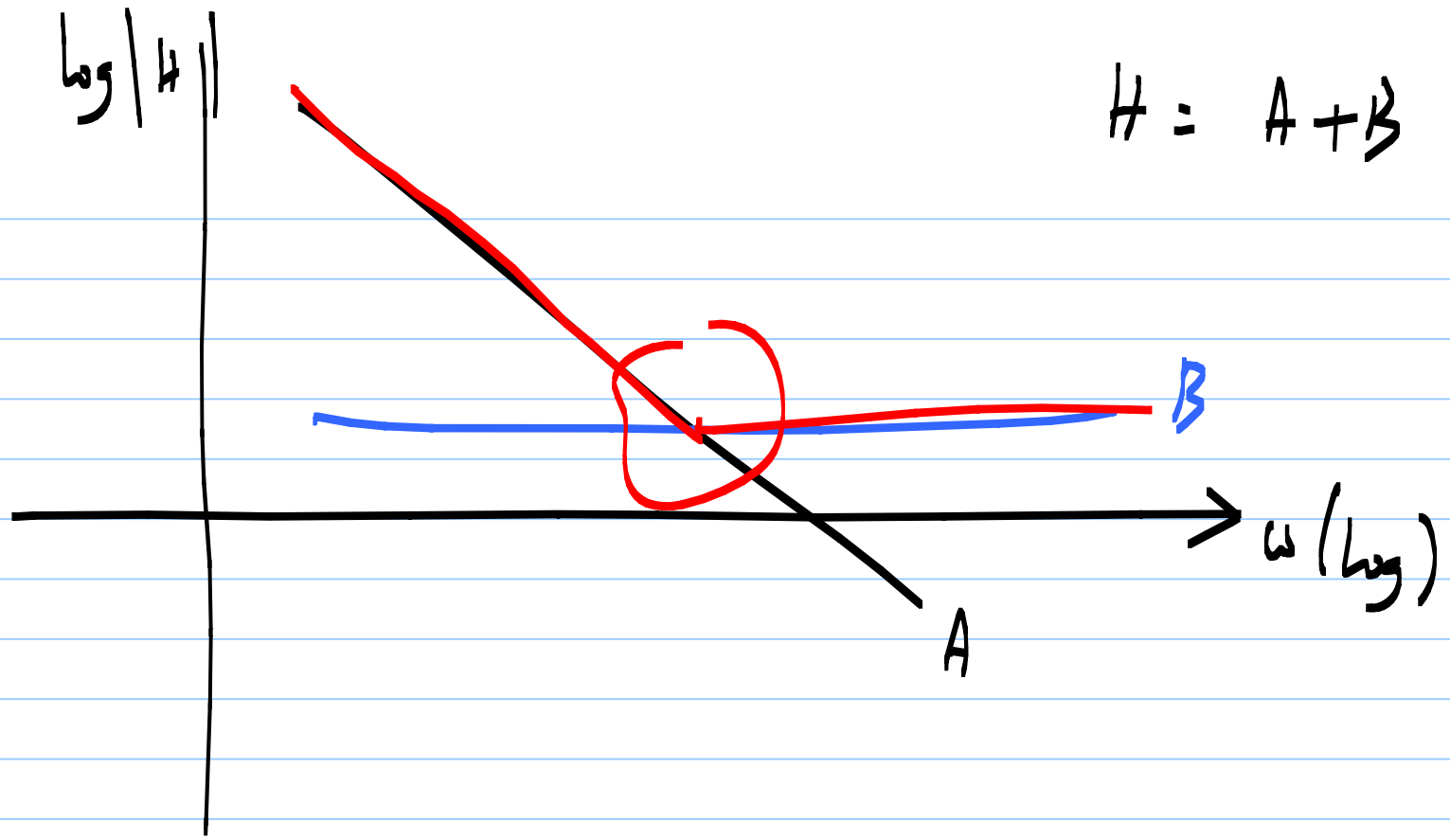
$10x \Rightarrow \underbrace{20\,dB}$

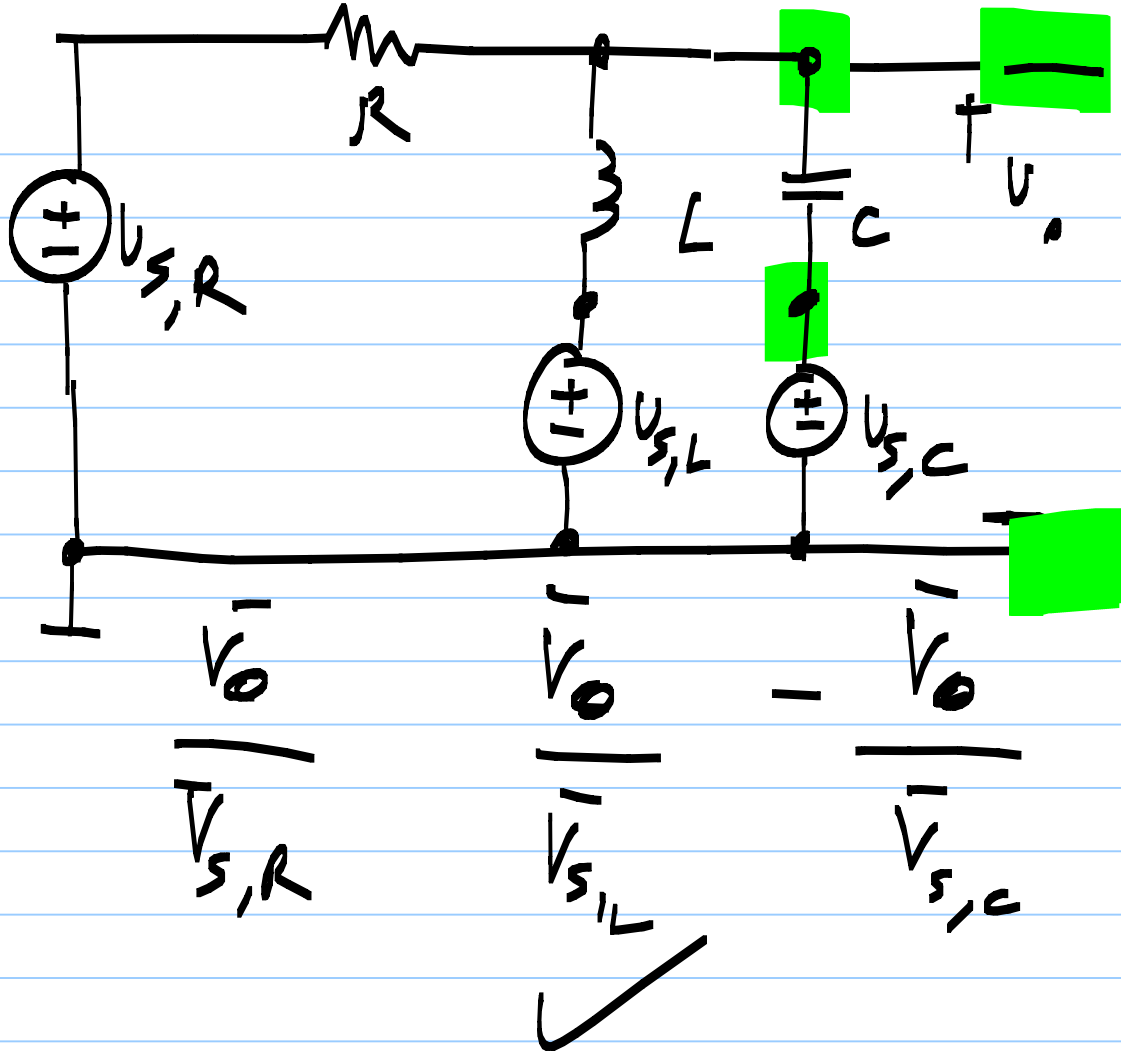
V_1 (+, −) across R: $P_1 = \frac{V_1^2}{R}$

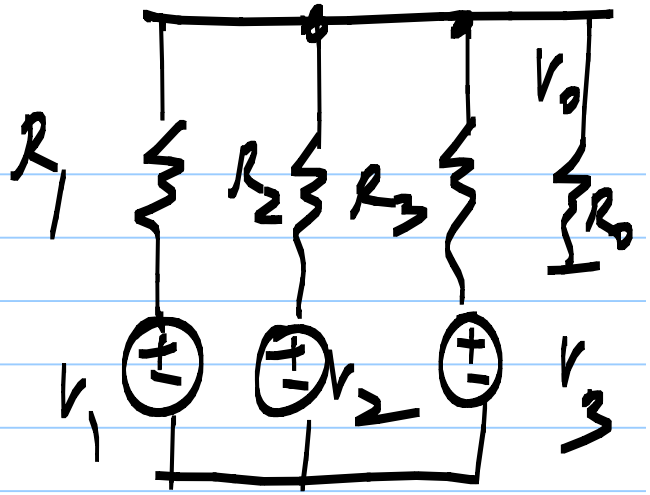
V_2 (+, −) across R: $P_2 = \frac{V_2^2}{R}$

$10 \log_{10} \frac{P_1}{P_2}$

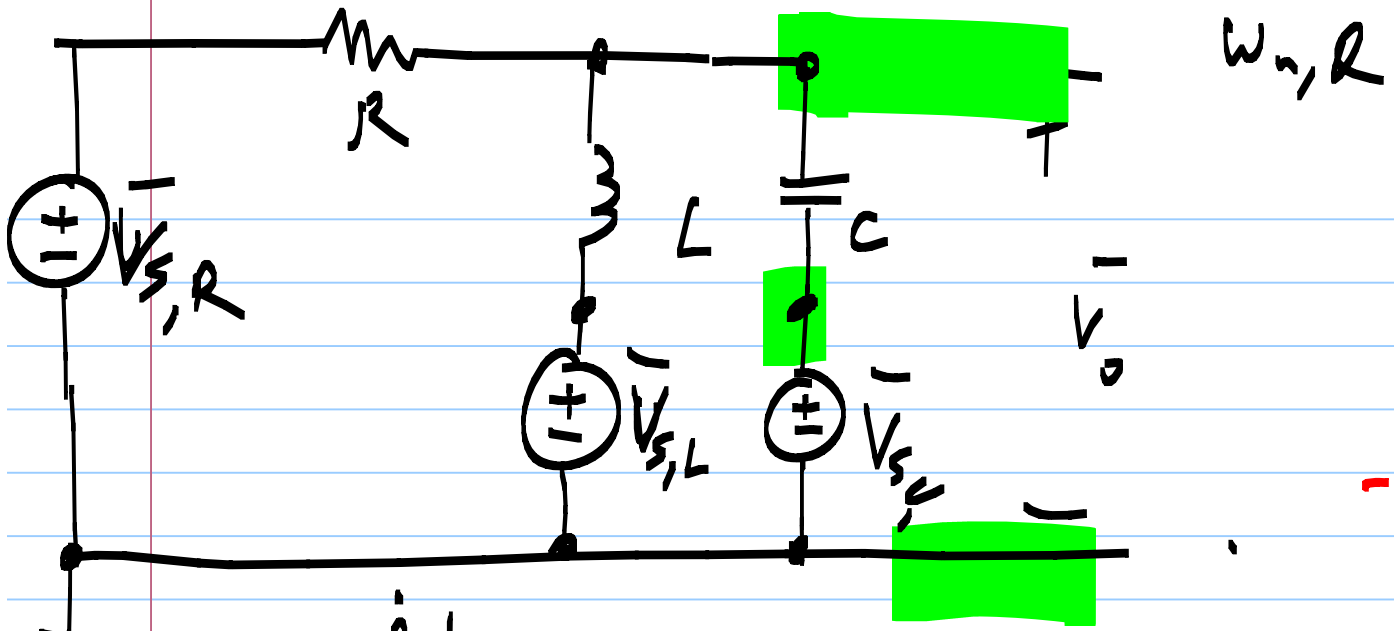
$= 20 \log_{10} \frac{V_1}{V_2}$


log|H|
H = A + B
B
ω (log)
A





$$v_0 = \frac{v_1 \cdot G_1 + v_2 \cdot G_2 + v_3 G_3}{G_1 + G_2 + G_3 + G_0}$$



ω_n, R

$$\bar{V}_o = \frac{j\frac{\omega}{Q\omega_n}\,\bar{V}_{s,R}\cdot\frac{j\omega L}{R} + \bar{V}_{s,L}\cdot 1 + \bar{V}_{s,C}\cdot(-\omega^2 LC)}{\left(\frac{j\omega L}{R} + 1 - \omega^2 LC\right)\left[\left(1-\frac{\omega^2}{\omega_n^2}\right) + j\frac{\omega}{Q\omega_n}\right]}$$

$-\frac{\omega^2}{\omega_n^2}$

$$\frac{V_o}{V_{s,L}} = \frac{1}{1 - \omega^2 LC + j\omega \frac{L}{R}} = \frac{1}{1 - \frac{\omega^2}{\omega_n^2} + j\frac{\omega}{Q\omega_n}}$$

$$\frac{V_o}{V_{s,R}} = \frac{j\omega L/R}{1 - \omega^2 LC + j\omega \frac{L}{R}} = \frac{j\cdot\frac{\omega}{Q\omega_n}}{1 - \frac{\omega^2}{\omega_n^2} + j\frac{\omega}{Q\omega_n}}$$

$$\frac{V_o}{V_{s,C}} = \frac{-\omega^2 LC}{1 - \omega^2 LC + j\omega \frac{L}{R}} = \frac{-\omega^2/\omega_n^2}{1 - \frac{\omega^2}{\omega_n^2} + j\frac{\omega}{Q\omega_n}}$$