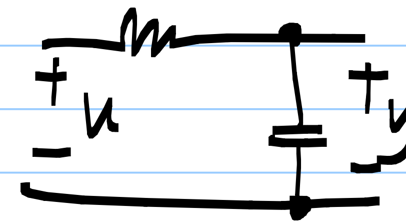
EE 2015 — Second order systems — 9/10/2017

exp(st)

u → $\boxed{T_1 = \frac{1}{\omega_1}}$ → y → $\boxed{T_2 = \frac{1}{\omega_2}}$ → v

First order system — First order system



$T_1 = T_2 = T$

$\exp(-t/T)$

$t\exp(-t/T)$

$$T_1 \cdot \frac{dy}{dt} + y = u \qquad T_2 \cdot \frac{dv}{dt} + v = y$$

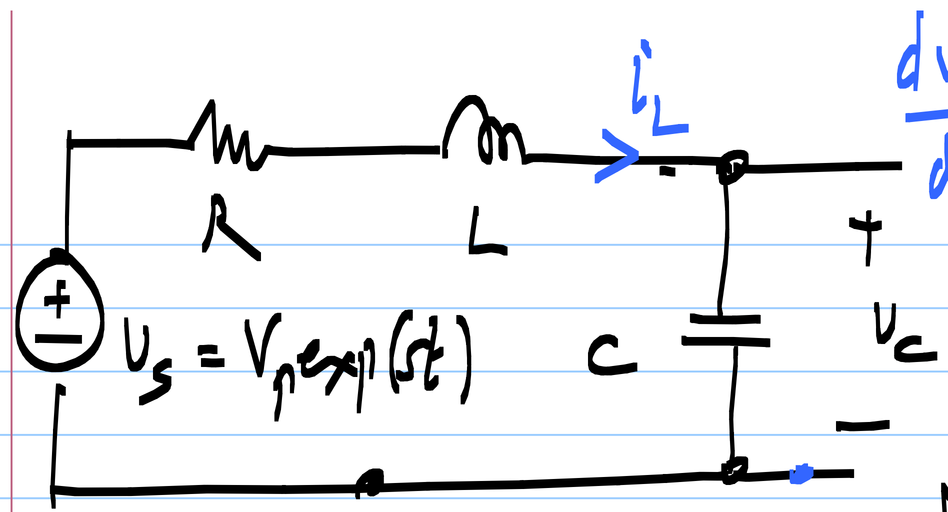
$$\frac{1}{s_1} \frac{dy}{dt} + y = u \qquad \frac{1}{s_2} \frac{dv}{dt} + v = y$$

Eliminating y, we get:

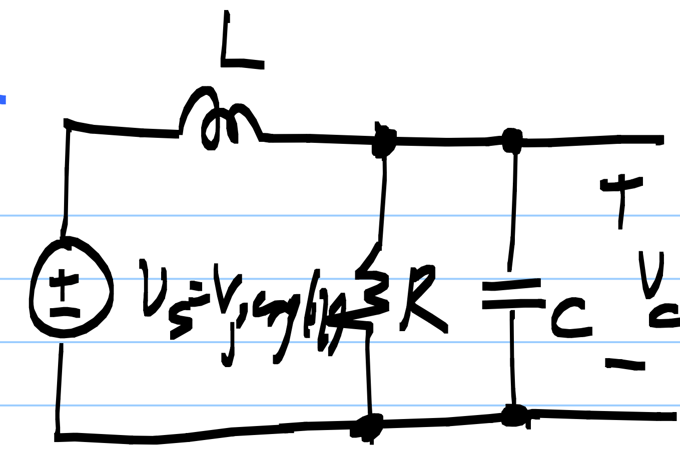
$$\frac{d^2v}{dt^2} + (s_1+s_2)\frac{dv}{dt} + (s_1 s_2)\cdot v = (s_1 s_2)u$$

$$T_1 T_2 \frac{d^2v}{dt^2} + (T_1+T_2)\frac{dv}{dt} + v = u$$

2nd order differential equation



$$\frac{dv_c}{dt} = \frac{i_L}{C}$$



$$LC\frac{d^2v_c}{dt^2} + RC\frac{dv_c}{dt} + \underbrace{v_c = v_s}$$

$$T_1T_2\frac{d^2v_c}{dt^2} + (T_1+T_2)\frac{dv_c}{dt} + v_c = v_s$$

$$LC\frac{d^2v_c}{dt^2} + \frac{L}{R}\frac{dv_c}{dt} + v_c = v_s$$

$T_1 = T_2^* \Rightarrow A_1 = A_2^*$

Natural response of a second-order system

$$\sim \quad A_1 \cdot \exp(-t/T_1) + A_2 \cdot \exp(-t/T_2)$$

$$\sim \quad A_1 \exp(-t/T) + A_2\, t \exp(-t/T) \quad T_1 = T_2 = T$$

$T_1 \cdot T_2 = LC$	$T_1 \cdot T_2 = LC$
$T_1 + T_2 = C \cdot R$	$T_1 + T_2 = \frac{L}{R}$

$A_1 \exp(-s_1 t) + A_2 \exp(-s_2 t)$

$H(s)$

$V_p \exp(st)$ Input: $\underbrace{\dfrac{1}{s^2 LC + \underbrace{CR}s + 1} V_p \exp(st)}_{\text{Forced response}}$

ckt #1

$+ A_1 \exp(-s_1 t)$

$+ A_2 \exp(-s_2 t)$

$H(s)$ $\underbrace{\dfrac{1}{s^2 LC + s\frac{L}{R} + 1} \cdot V_p \exp(st)}$

Input : $\underbrace{\exp(st)}$

Natural modes : $\exp(-s_1 t)$, $\exp(-s_2 t)$ s_1, s_2 : real, distinct

$s_1 = s_2$: real