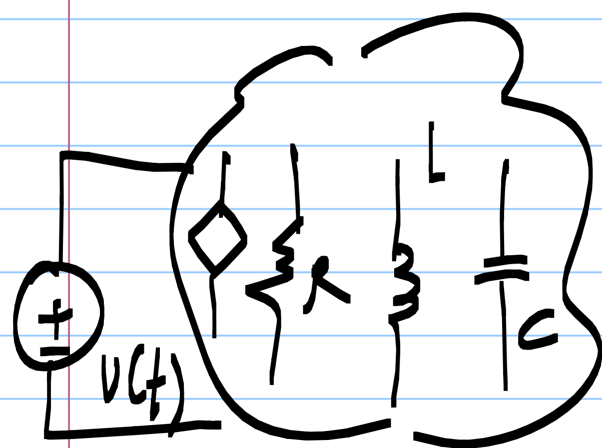
EE 2015

28/9/2017

R, L, C, Controlled sources



Linear diff. equation with
Constant coefficients

Order N = # States
(Cap. voltages,
inductor currents)

Natural response
(N. natural modes)

Forced response

Forced response

Constant input $\rightarrow$ constant (setting derivatives to zero)

Exponential input

$\exp(st) \rightarrow H(s) \cdot \exp(st)$

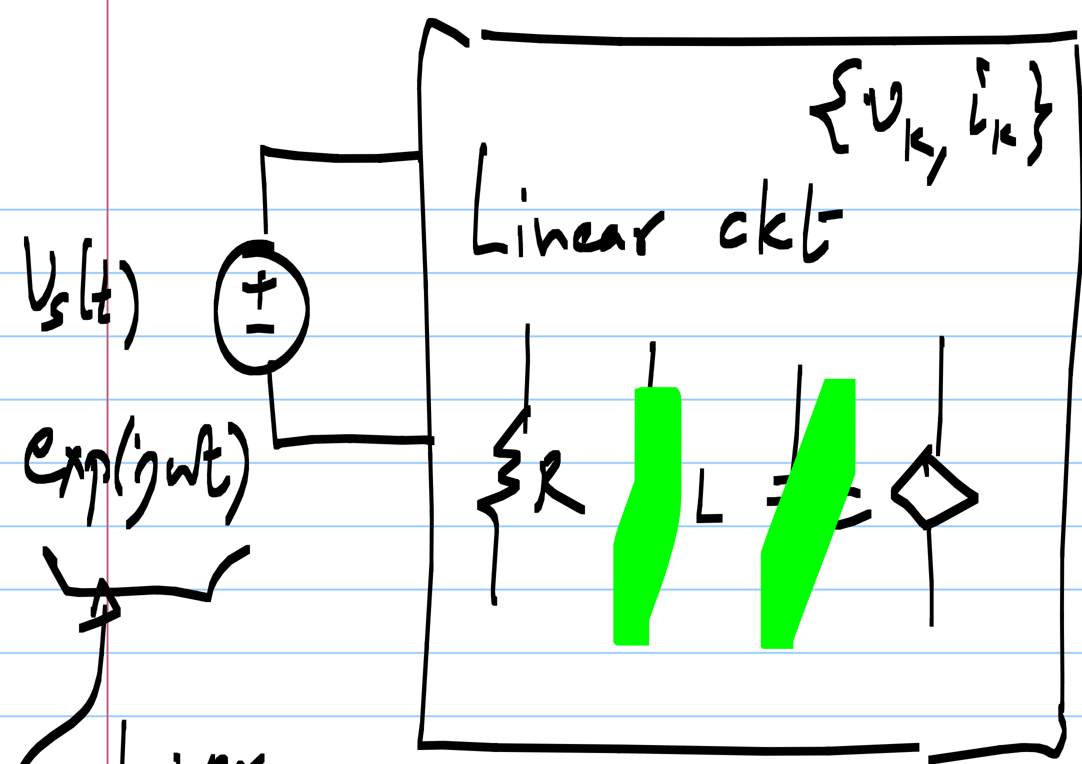
$\exp(j\omega t) \rightarrow H(j\omega)\exp(j\omega t)$

Re( )

Real valued components

$\cos(\omega t) \rightarrow \mathrm{Re}\left[H(j\omega)\exp(j\omega t)\right]$

$|H(j\omega)|\cos(\omega t + \measuredangle H(j\omega))$



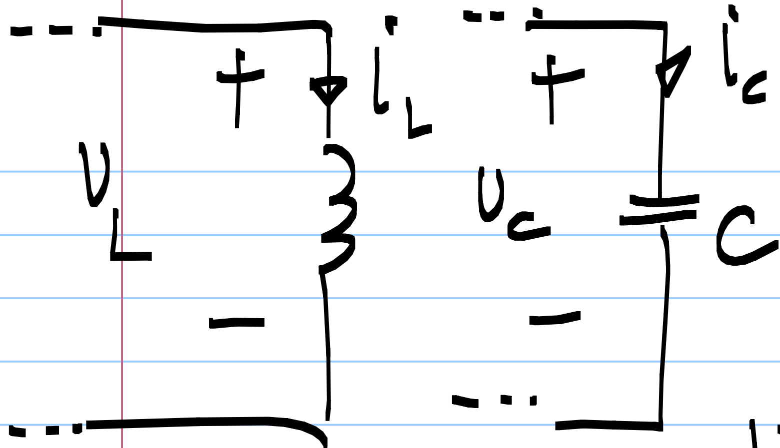
Lower case

steady state

$$\begin{cases} v_k(t) = H_{v_k v_s}(j\omega)\exp(j\omega t) \\ i_k(t) = H_{i_k v_s}(j\omega)\exp(j\omega t) \end{cases}$$

Transfer fn. from v_s to v_k

Transfer fn. from v_s to i_k



$$v_L(t) = H_{v_L, v_s}(j\omega)\exp(j\omega t)$$

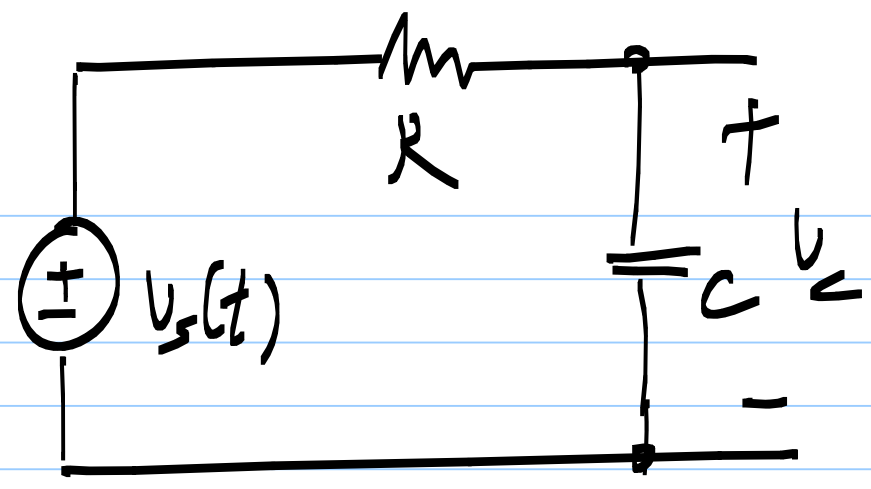
$$i_L(t) = H_{i_L, v_s}(j\omega)\cdot\exp(j\omega t)$$

$$v_L = L\cdot\frac{di_L}{dt} = H_{i_L, v_s}(j\omega)\cdot j\omega L\cdot\exp(j\omega t)$$

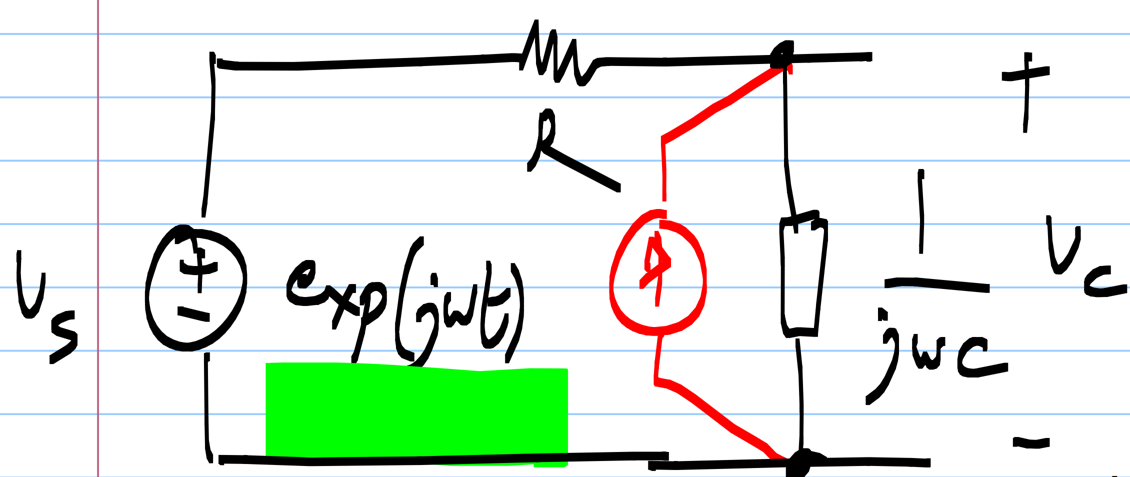
ohm's law

$$\frac{v_L(t)}{i_L(t)} = j\omega L \qquad \frac{v_c(t)}{i_c(t)} = \frac{1}{j\omega C}$$

constant



$$\boxed{RC \cdot \frac{dv_c}{dt} + v_c = v_s}$$



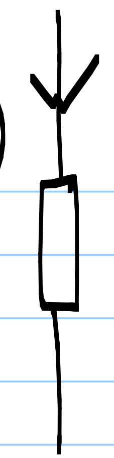
$\sim \exp(j\omega t$ $\quad i_s(t)$

$$v_c(t) = v_s(t)\frac{1/j\omega C}{R + 1/j\omega C} + i_s(t)\cdot\frac{R \cdot 1/j\omega C}{R + 1/j\omega C}$$

$$= v_s(t) \cdot \frac{1}{1 + j\omega CR}$$

only when $v_s = \exp(j\omega t)$

$$\frac{v(t)}{i(t)} \doteq \underbrace{\overset{\Omega}{\text{Impedance}}}_{\text{Complex}} \sim \underset{\downarrow \atop \text{Real}}{\text{Resistance}}$$



(steady-state w/ $\exp(j\omega t)$ input)

$j2\pi\ \Omega$

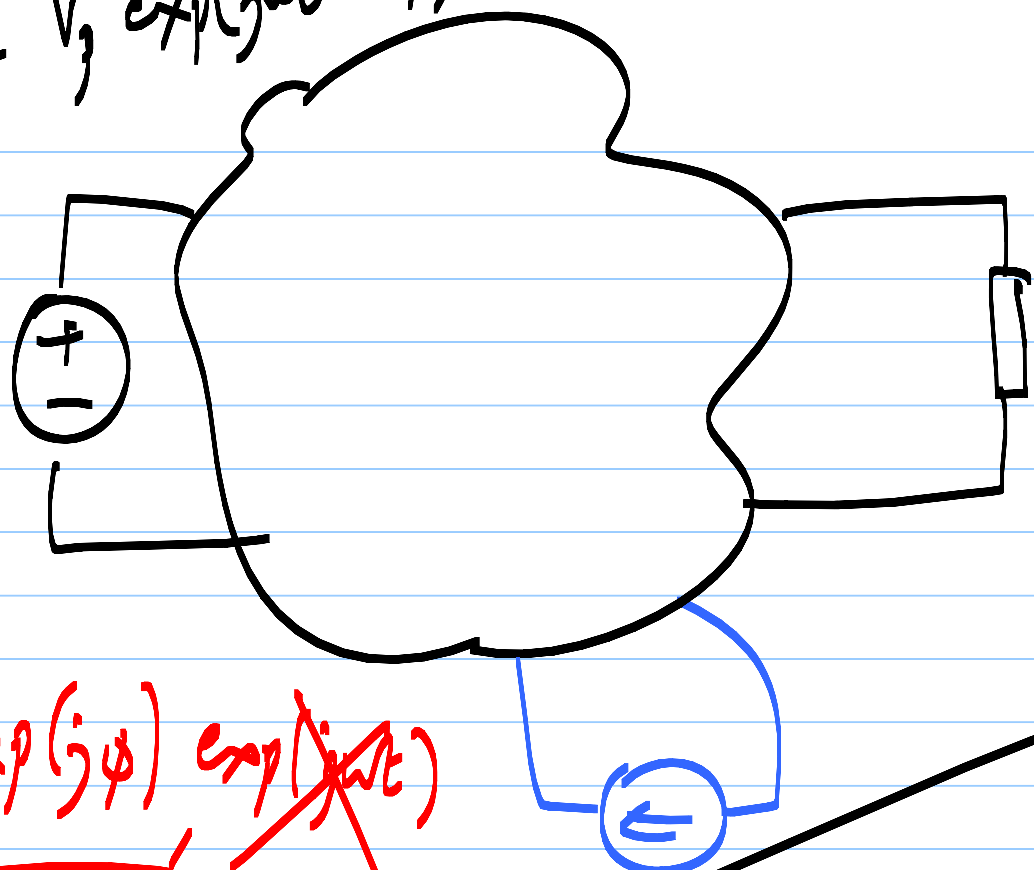
@ 1GHz

$-\frac{j}{2\pi}\ k\Omega$

$-j159\Omega$

@ 1GHz

$v_s(t) = V_p \exp(j\omega t + \phi)$



$\underbrace{V_p \exp(j\phi)}$ $\cancel{\exp(j\omega t)}$

$I_p \exp(j\omega t + \theta)$

$+$

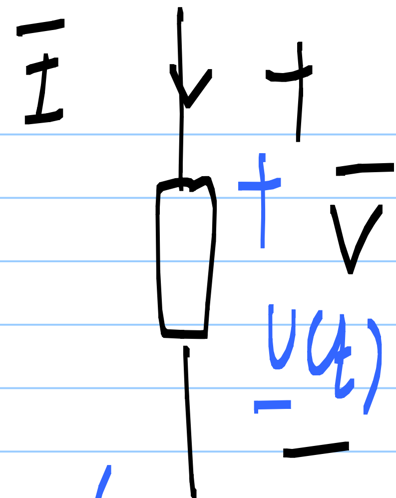
$\underbrace{(\quad)}$ $\cancel{\exp(j\omega t)}$

$-$

Phasor

$\underbrace{\bar{V}, \bar{I}}$

③



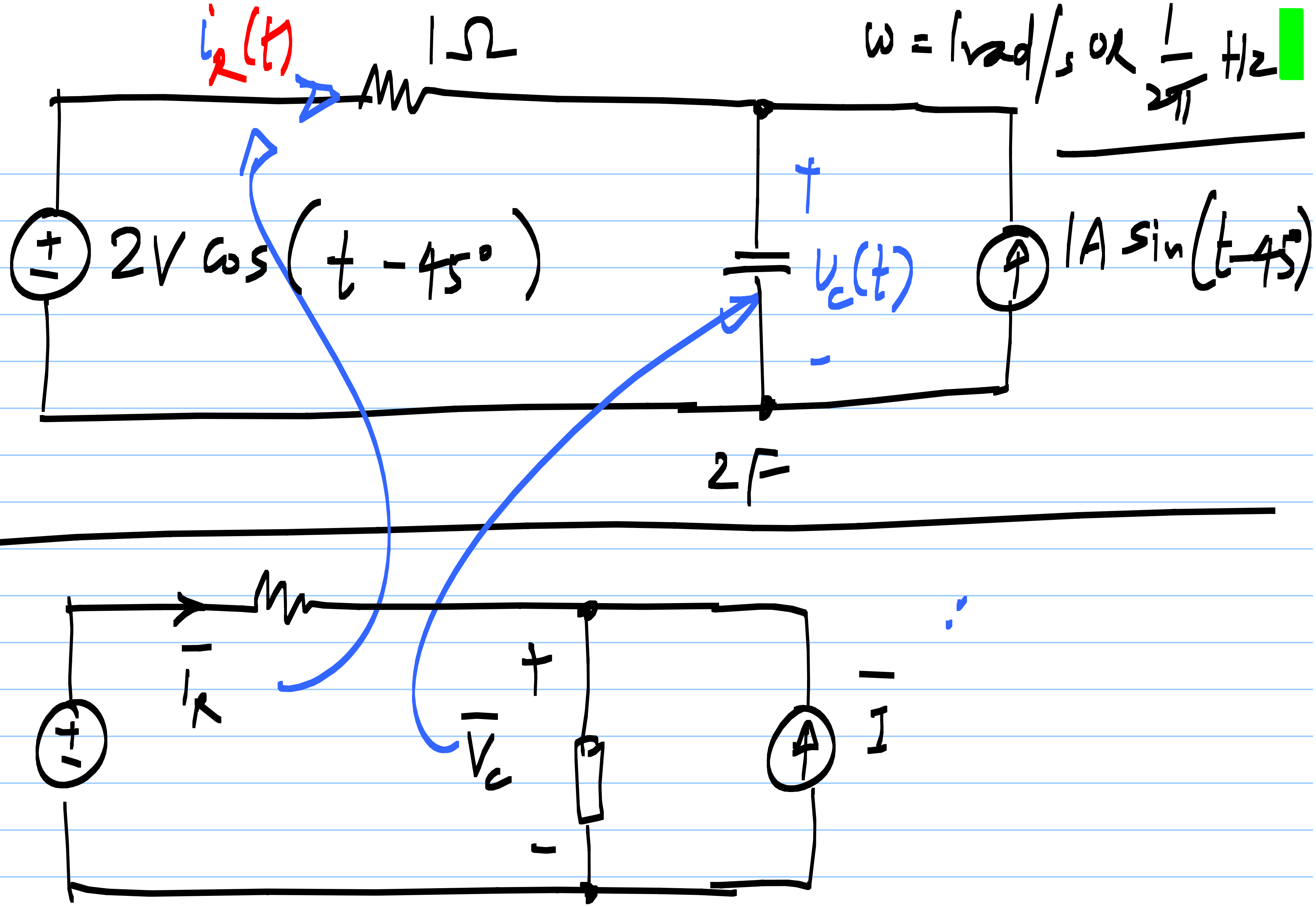
$i(t) = \bar{I} \cdot \exp(j\omega t)$

$v(t)$ phasor $\Rightarrow$

In steady-state w/ $\exp(j\omega t)$ as input

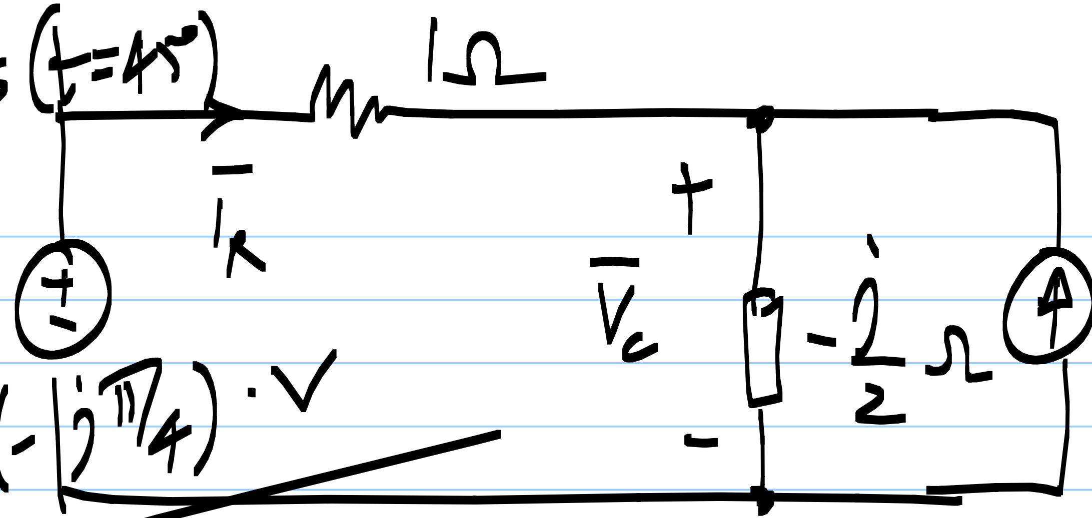
$v(t) = \bar{V} \cdot \exp(j\omega t)$

$\omega = 1\,\mathrm{rad/s}$ or $\frac{1}{2\pi}$ Hz



$2V\cos(t-45°)$

$\bar{V}_s$



$\cancel{2\exp(-j\,\pi/4)\cdot V}$

$i_s(t) = 1A\sin(t-45°)$

$\bar{I}_s$ $1A\cos(t-135°)$

$1A\angle -135°$

$1A\exp\left(-j\frac{3\pi}{4}\right)$

$$\bar{V}_c = \bar{V}_s\,\frac{(-j/2)}{1-j/2} + \bar{I}_s\left[\frac{1\cdot(-j/2)}{1-j/2}\,\Omega\right]$$

$$\bar{V}_c = \left[2\exp\left(-j\frac{\pi}{4}\right)\cdot\frac{(-j/2)}{(1-j/2)} + 1\cdot\exp\left(-j\frac{3\pi}{4}\right)\cdot\frac{1(-j/2)}{(1-j/2)}\right]V$$

$$\bar{V}_c = 1\exp\left(-j\frac{3\pi}{4}\right) \qquad v_c(t) = A\cdot\cos(t-\theta)\cdot V$$

$$\frac{\left(\sqrt{2}-j\sqrt{2}\right)\left(-j/2\right) - \left(\frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)\cdot\left(-j/2\right)}{1 - j/2}$$

$A\exp(j\theta)$

$1\exp(-j3\pi/4)$

$$= \frac{\left(\sqrt{2}+1/\sqrt{2}\right) - j\left(\sqrt{2}-1/\sqrt{2}\right)}{1-j/2}\cdot\left(-j/2\right)$$