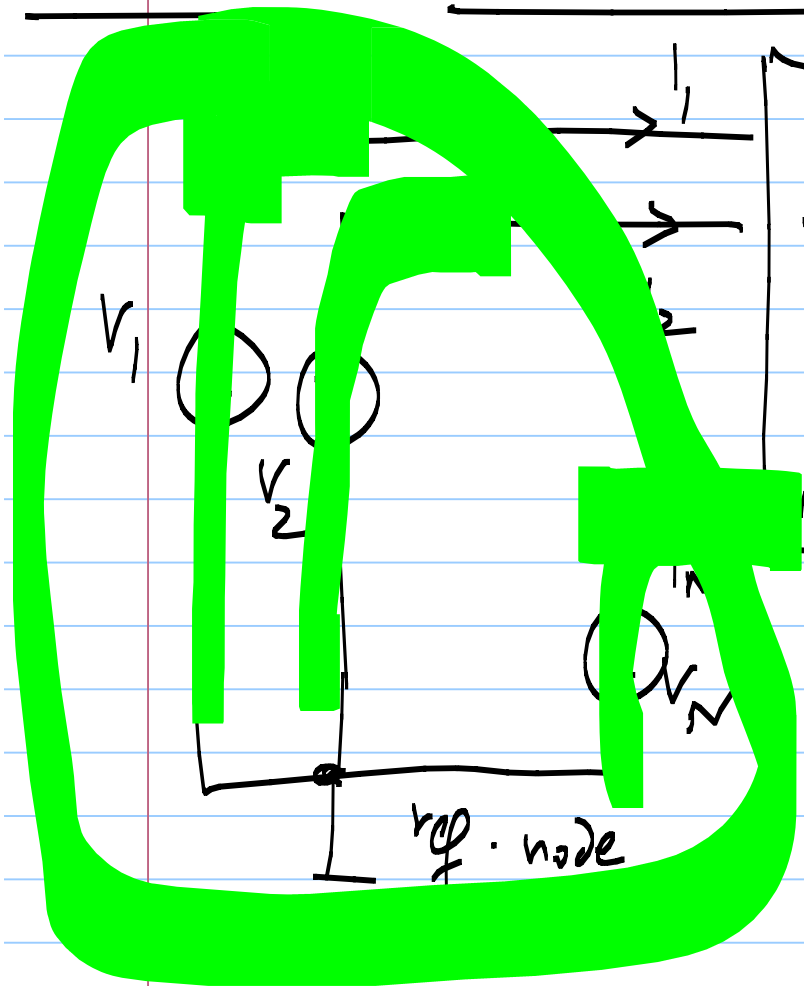
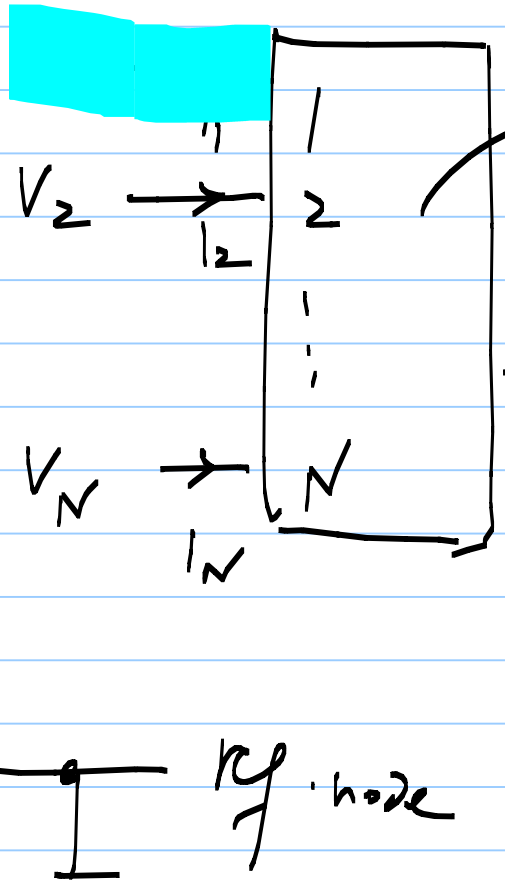
EE2015 Power in a multi-terminal element 9/8/2017



$$\sum_{k=1}^{N} V_k I_k$$



$$I_N = -(I_1 + I_2 + \cdots + I_{N-1})$$

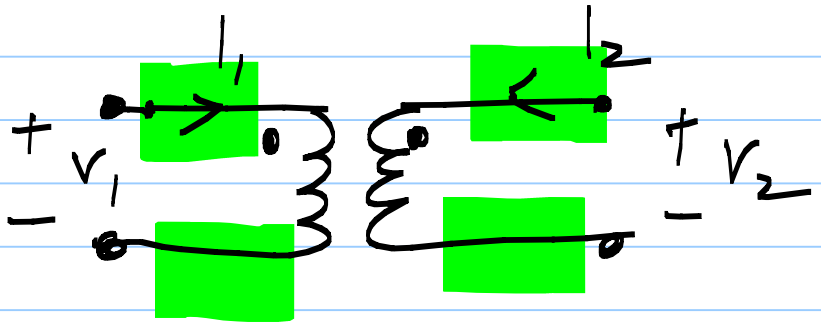
$$V_1 I_1 + V_2 I_2 + \cdots + V_{N-1} I_{N-1}$$

$$- V_N I_2 - \cdots - V_N I_{N-1}$$

$$(V_1 - V_N) I_1 + (V_2 - V_N) I_2 + \cdots (V_{N-1} - V_N) I_{N-1}$$

Power in an n-terminal element $= \sum_{k=1}^{N-1} (V_k - V_N) \cdot I_k$

Mutual inductor :



$$P = V_1 I + V_2 I_2$$

$$I_1 V_1 =$$

$$I_2 V_2 =$$

Calculate the energy stored in the mutual inductor

$$\int_0^t P \cdot dt = \frac{1}{2} L_1 I_1^2 +$$

$$+ M \cdot I_1 I_2$$

$t = 0, \quad I_1 = 0, \quad I_2 = 0$

I_1, I_2

$$\frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2 > 0$$

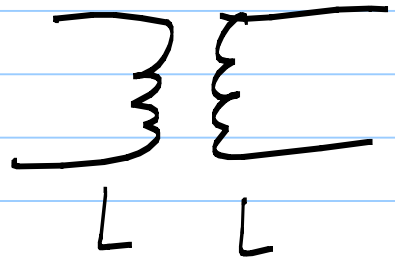
I_1

I_2

$$\left(\sqrt{\frac{L_1}{2}}I_1 + \sqrt{\frac{L_2}{2}}I_2\right)^2 + \left(M - \sqrt{L_1 L_2}\right) I_1 I_2 > 0$$

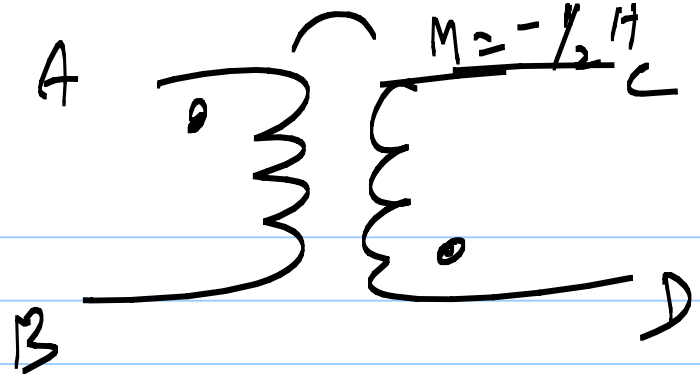
$$|M| \leq \sqrt{L_1 L_2} \qquad M = k \cdot \sqrt{L_1 L_2}$$

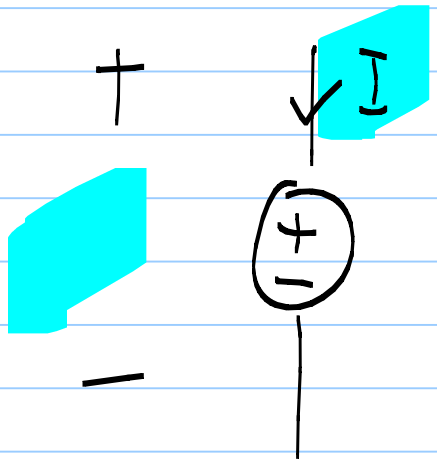
$k \leq 1$

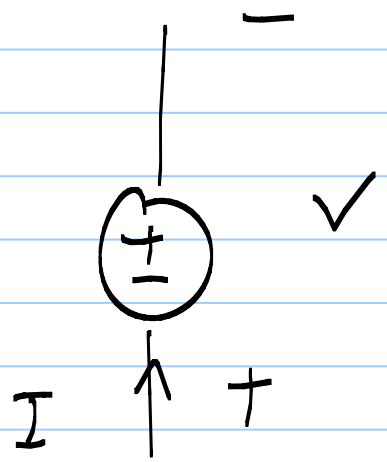


$M < L$


M = 1/2 H
A
C
B
D
1 H
1 H


M = -1/2 H
A
C
B
D


+
I
−


−
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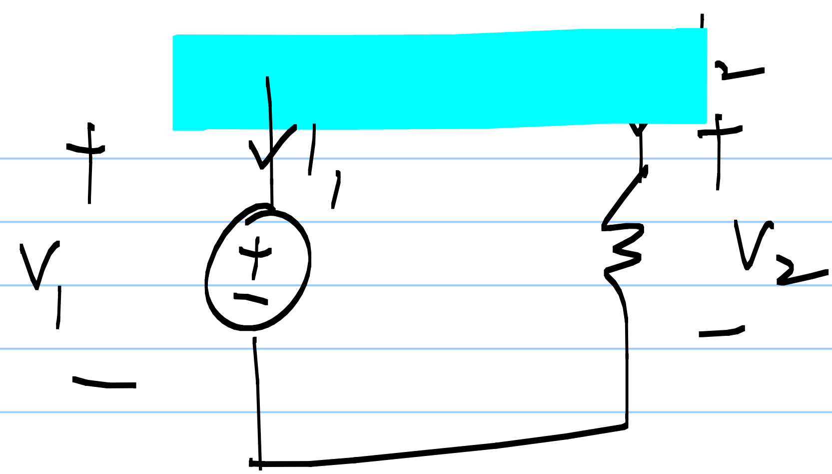
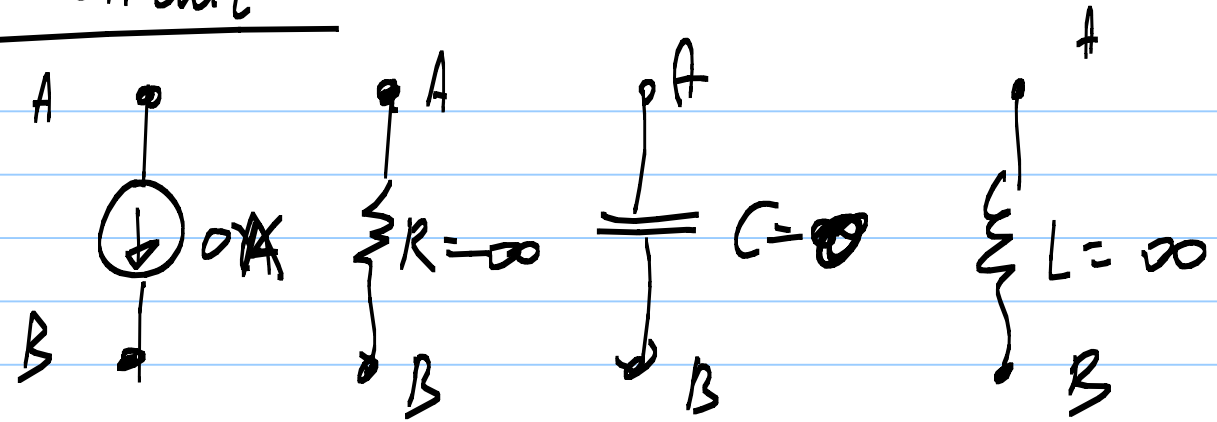


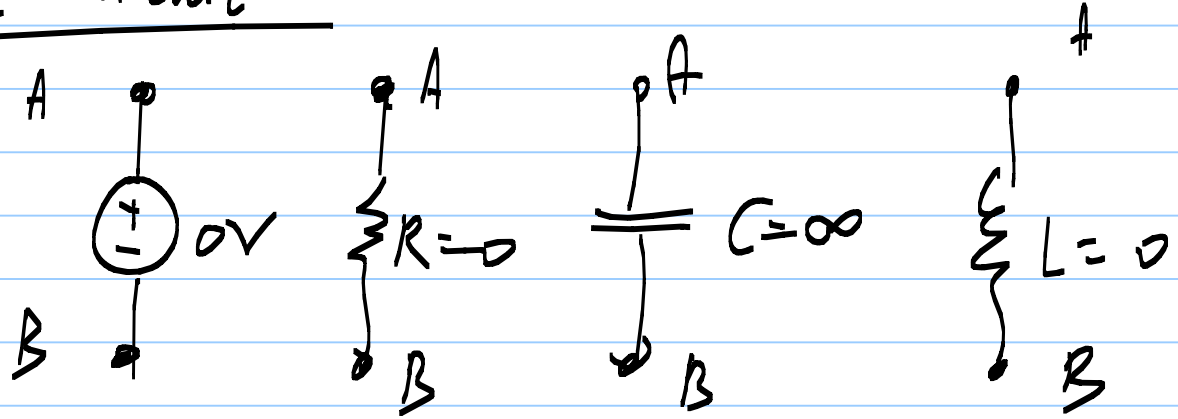
$$I_1 + I_2 = 0$$

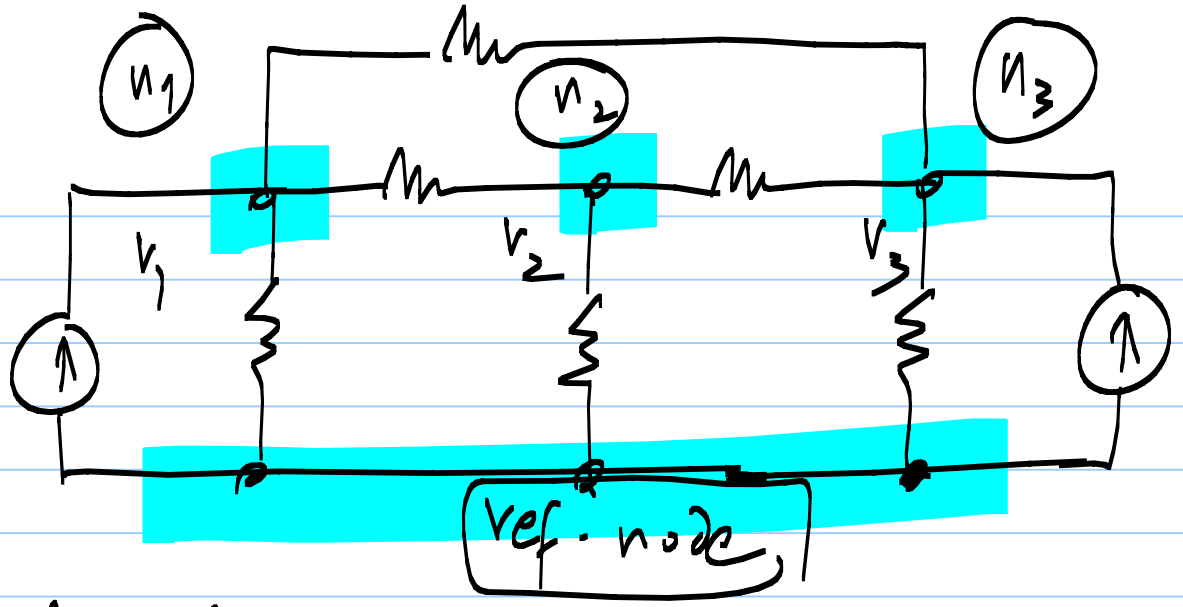
$$V_1 - V_2 = 0$$

open circuit.



Short circuit.





N = 4

B = 8

3 KCL

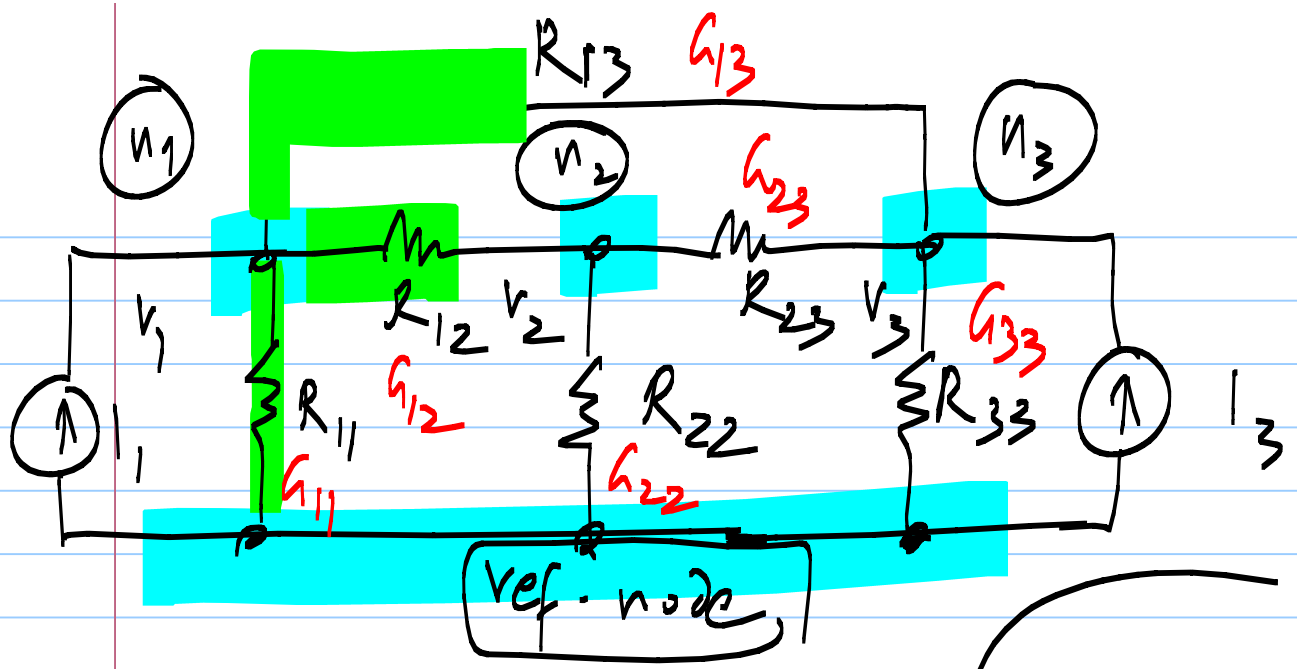
5 KVL

Nodal analysis: start with KCL eq. @ N-1 nodes

Circuits w/ linear elements & ind. sources

resistors

→ Resistors & ind. current sources



KCL @ n_1, n_2, n_3

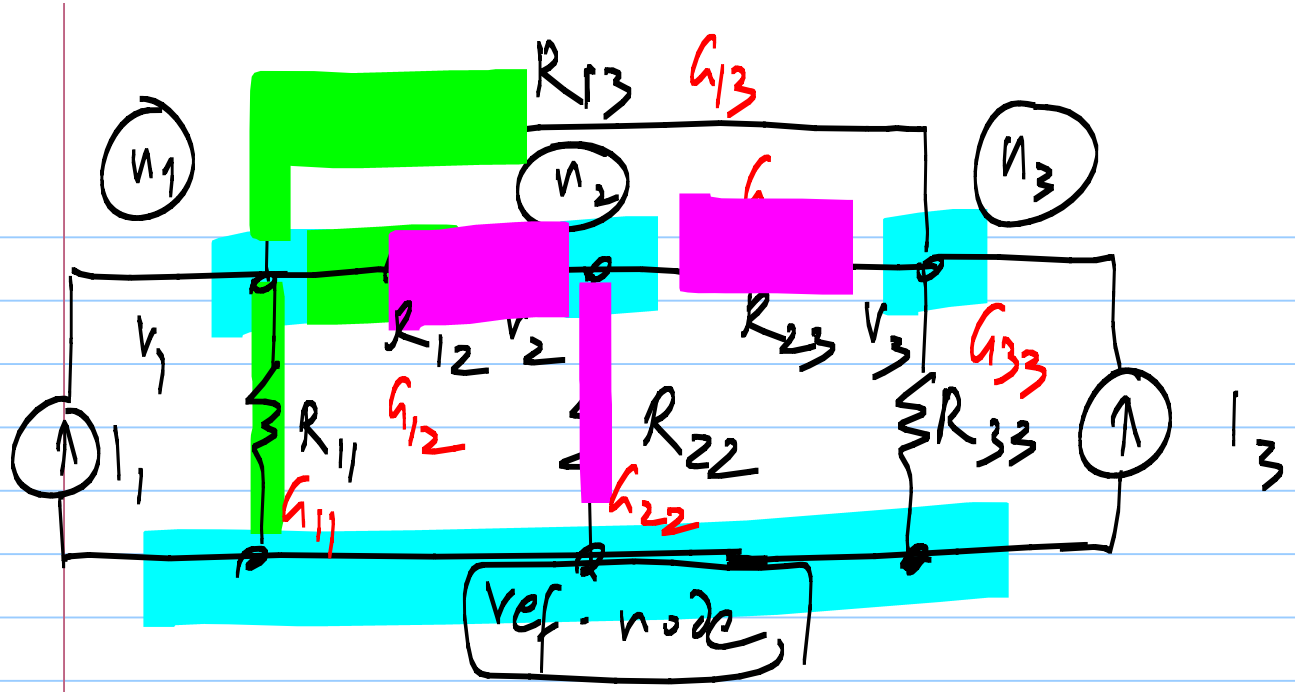
n_1: $\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_{12}} + \frac{V_1 - V_3}{R_{13}} = I_1$

Currents flowing out of n_1

Current flowing into n_1 by an ind. current source

$$V_1 \cdot G_{11} + (V_1 - V_2) G_{12} + (V_1 - V_3) G_{13} = I_1$$

$$V_1 (G_{11} + G_{12} + G_{13}) - V_2 \cdot G_{12} - V_3 G_{13} = I_1$$



n_1, n_2, n_3

n_2:

$$(v_2 - v_1) \cdot G_{12} + v_2 \cdot G_{22} + (v_2 - v_3) G_{23} = 0$$

$$-v_1 \cdot G_{12} + v_2 (G_{12} + G_{22} + G_{23}) - v_3 G_{23} = 0$$