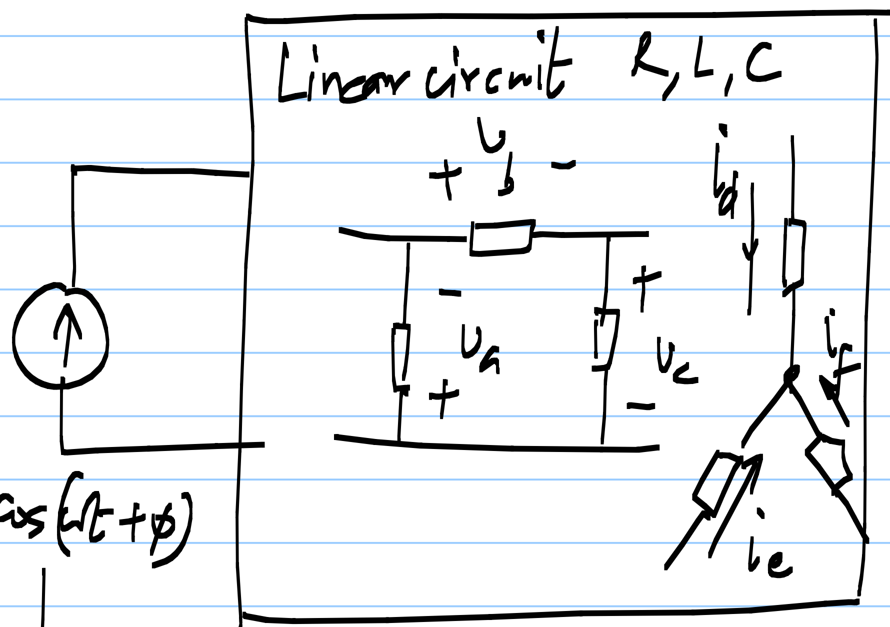
EC1010: Lecture 30.

Phasors

Sinusoidal steady state analysis at a frequency ω



$I_p \cos(\omega t + \phi)$

$\downarrow$

$[I_p \exp(j\phi)] \exp(j\omega t)$

$$\left.\begin{aligned} v_a &= \bar{V}_a \cdot \exp(j\omega t) \\ v_b &= \bar{V}_b \cdot \exp(j\omega t) \\ v_c &= \bar{V}_c \cdot \exp(j\omega t) \end{aligned}\right\}$$

KVL

$v_a + v_b + v_c = 0$

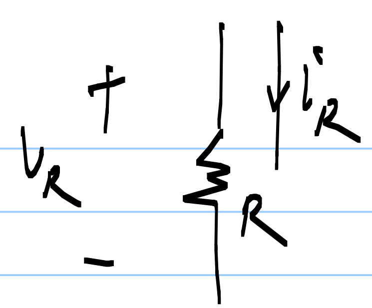
$\bar{V}_a + \bar{V}_b + \bar{V}_c = 0$

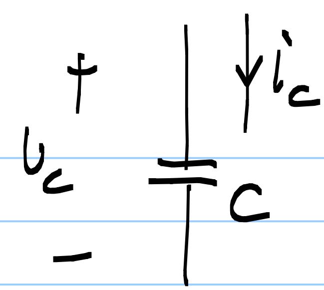
$$\left.\begin{aligned} i_d &= \bar{I}_d \cdot \exp(j\omega t) \\ i_e &= \bar{I}_e \cdot \exp(j\omega t) \\ i_f &= \bar{I}_f \cdot \exp(j\omega t) \end{aligned}\right\}$$

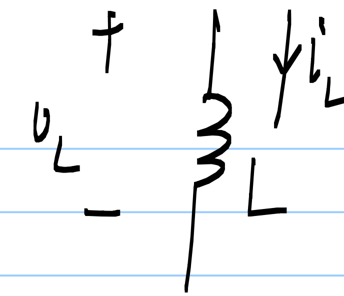
KCL

$i_d + i_e + i_f = 0$

$\bar{I}_d + \bar{I}_e + \bar{I}_f = 0$







voltage $v_R = \bar{V}_R \cdot \exp(j\omega t)$

current $i_R = \bar{I}_R \cdot \exp(j\omega t)$

$$\frac{v_R}{i_R} = R$$

$$\bar{V}_R / \bar{I}_R = R$$

$v_c = \bar{V}_c \cdot \exp(j\omega t)$

$i_c = \bar{I}_c \cdot \exp(j\omega t)$

$$i_c = C \cdot \frac{dv_c}{dt}$$

$$\bar{I}_c \cdot \exp(j\omega t) = j\omega C \cdot \bar{V}_c \cdot \exp(j\omega t)$$

$$\bar{V}_c / \bar{I}_c = 1 / j\omega C$$

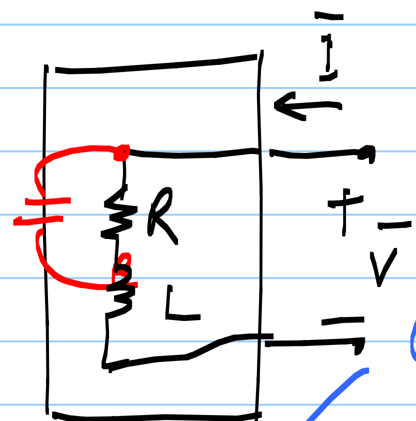
$v_L = \bar{V}_L \cdot \exp(j\omega t)$

$i_L = \bar{I}_L \cdot \exp(j\omega t)$

$$v_L = L \cdot \frac{di_L}{dt}$$

$$\frac{\bar{V}_L}{\bar{I}_L} = j\omega L$$

	Resistor	Capacitor	Inductor
Impedance $Z = \frac{\bar{V}}{\bar{I}}$:	R	$\frac{1}{j\omega C} = \frac{-j}{\omega C}$	$j\omega L$
Admittance $Y = \frac{\bar{I}}{\bar{V}}$:	$G = \frac{1}{R}$	$j\omega C$	$\frac{1}{j\omega L} = \frac{-j}{\omega C}$



$$Z = \frac{\bar{V}}{\bar{I}} = \overset{R + jX}{R + \underbrace{j\omega L}}$$

resistance $Re[Z]$

reactance $Im[Z]$

G = Conductance $= Re[Y]$

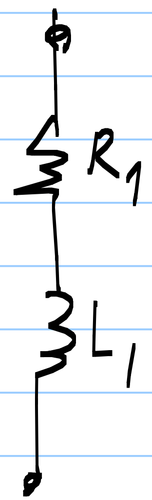
$$\bar{Y} = \frac{\bar{I}}{\bar{V}} = \frac{R}{R^2 + \omega^2 L^2} - \frac{j\omega L}{R^2 + \omega^2 L^2} = G + jB$$

$Im[Y]$: Susceptance

$$Z = \underbrace{R}_{\mathrm{Re}[Z]} + j\underbrace{X}_{\mathrm{Im}[Z]} = R_1 + j\omega L_1$$

$$R = R_1$$

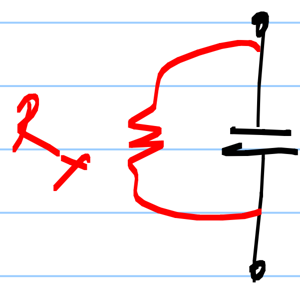
$$X = \omega L_1$$



$$Y = G + jB$$

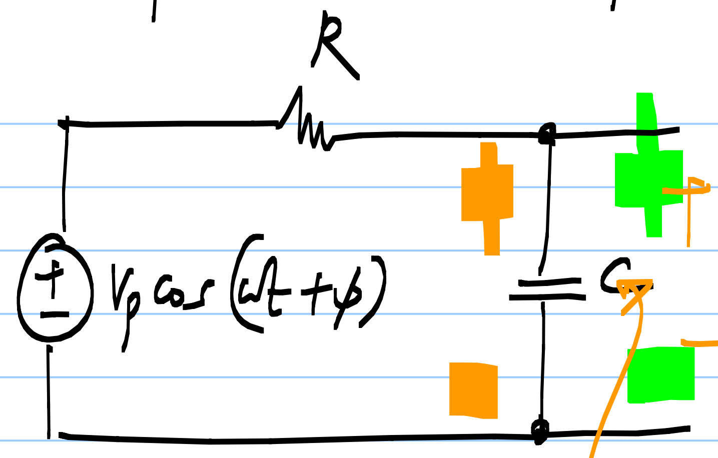
$$G = \frac{R_1}{R_1^2 + \omega^2 L_1^2}$$

$$B = \frac{-\omega L_1}{R_1^2 + \omega^2 L_1^2}$$

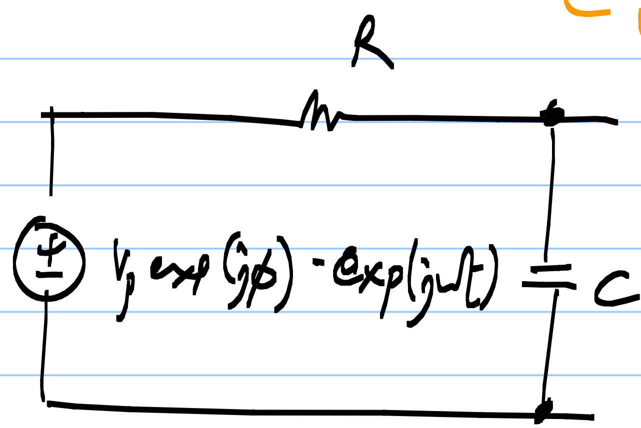


$$Y = \frac{1}{R_x} + j\omega C$$

$V_p \sin \omega t \longrightarrow V_p \angle -\pi/2$

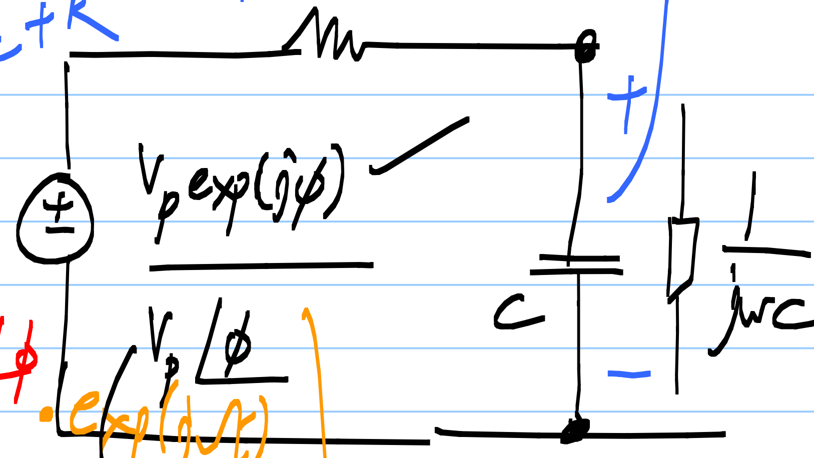


$Re\left[\frac{1}{1+j\omega CR} \cdot V_p\angle\phi \cdot \exp(j\omega t)\right]$

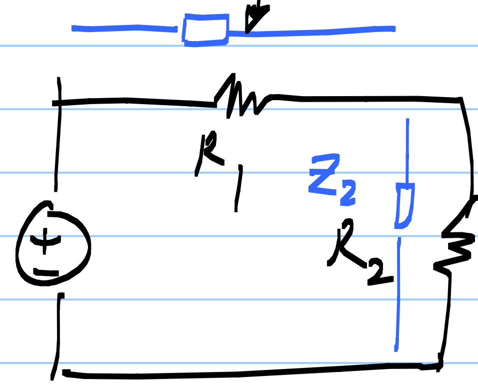


$V_p\angle\phi$

$\frac{1/j\omega C}{1/j\omega C + R} \cdot V_p\angle\phi$



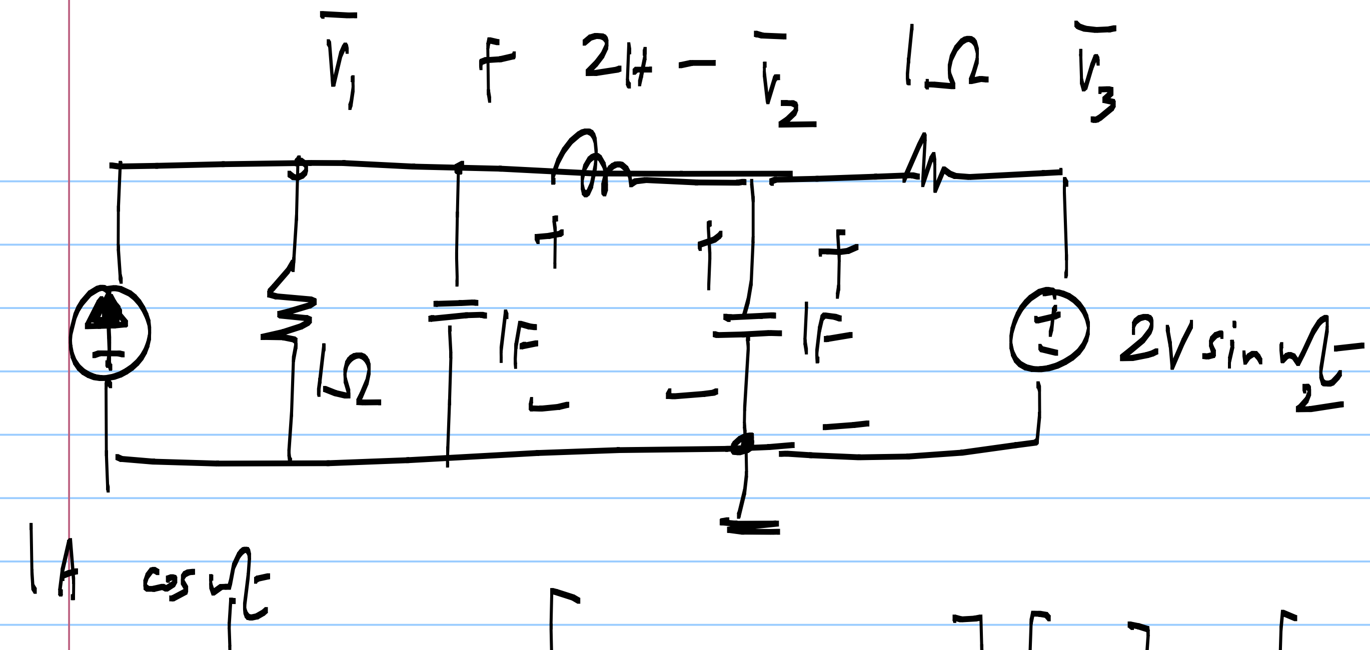
Z_1



$+ \quad \frac{R_2}{R_1+R_2} \cdot V_p\angle\phi$

$-$

$\frac{Z_2}{Z_1+Z_2} V_p\angle\phi$



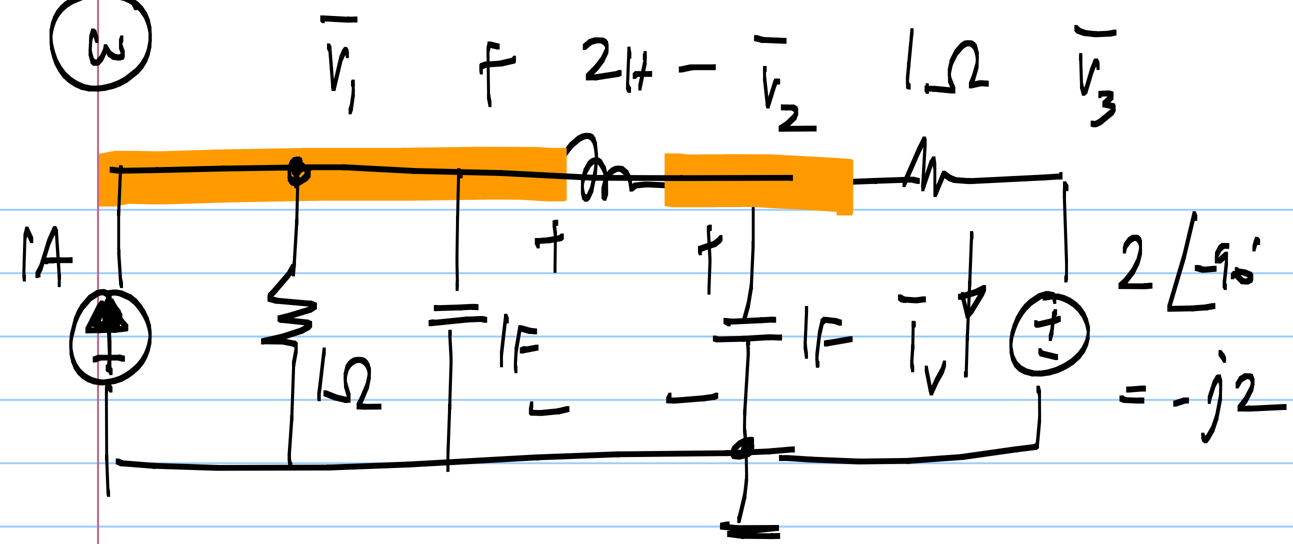
Set up MNA eq.

#1 $Re[\bar{V} \cdot \exp(j\omega_1 t)]$

#2 $Re[\bar{V} \exp(j\omega_2 t)]$

$$\begin{bmatrix} & & \\ & Y & \\ & & \end{bmatrix} \begin{bmatrix} \\ \bar{\underline{V}} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \bar{\underline{I}} \\ \\ \end{bmatrix}$$

ω

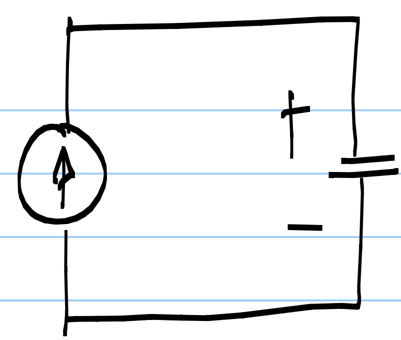


$$\begin{bmatrix} 1+j1\omega+\frac{1}{j2\omega} & -\frac{1}{j2\omega} & 0 & 0 \\ -\frac{1}{j2\omega} & 1+j\omega+\frac{1}{j2\omega} & -1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{I}_V \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -j2 \end{bmatrix}$$

$\omega = 10^3$ $1\,mA \cos(10^3 t)$: $v_c = \sin(10^3 t)\,V$ $i = C\frac{dv}{dt}$

Solution for $\underline{t > 0}$

1 mA



$1\,\mu F$ $v_c = 0$ @ $\underline{\underline{t = 0}}$

$$\frac{1}{j\omega C} = -j\,10^3\,\Omega$$

$1\,mA \sin(10^3 t)$: $v_c = \left[1 - \cos(10^3 t)\right] V$

$1\,mA \cos(10^3 t)$ $\sin(10^3 t)$

$1\,mA \angle 0 \times (-j 10^3 \Omega) = -j\,V$

$1\,mA \sin(10^3 t)$ V

$-j\,1\,mA \times (-j 10^3 \Omega) = -1\,V$ $\rightarrow$ $-\cos(10^3 t)\,V$