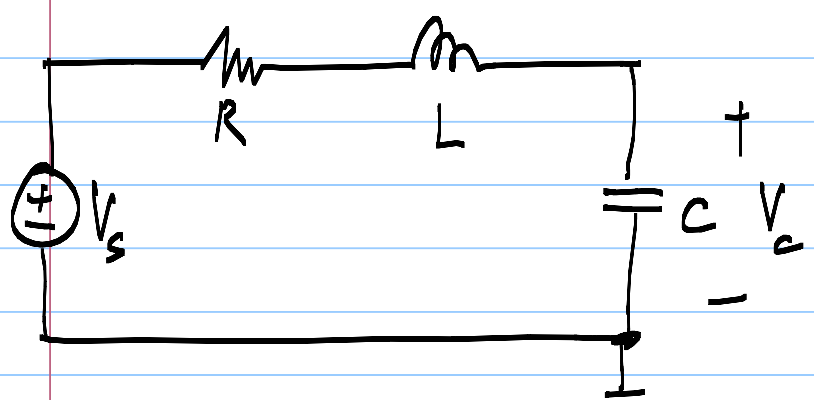
EC 1010: Lecture 28



$v_c = V_0 \exp(pt)$ → homogeneous eq.

$$LC \frac{d^2 v_c}{dt^2} + RC \frac{dv_c}{dt} + v_c = V_s$$

$p_{1,2}$: $LC \cdot p^2 + RC \cdot p + 1 = 0$

Natural response: $\exp(p \cdot t)$

$A_1 \exp(p_1 t) + A_2 \exp(p_2 t)$ ✓

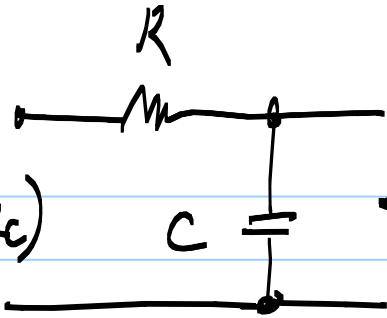
$(A_1 + A_2 t) \exp(pt)$ ✓

$p_{1,2}$: $p_r \pm j p_i$

$A_1 \exp(p_r t) \exp(j p_i t) + A_1^* \exp(p_r t) \exp(-j p_i t)$ ✓

$A_1 = A_0 \exp(j\phi_1)$

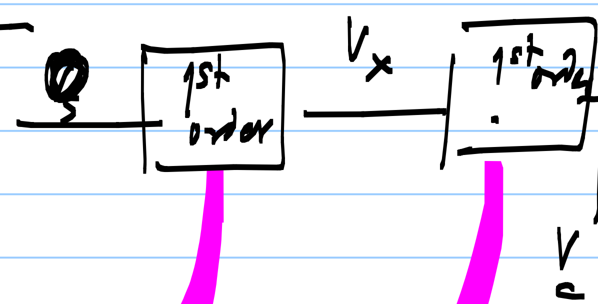
$2A_0 \exp(p_r t) \cdot \cos(p_i t + \phi_1)$ ✓



$\exp(-t/RC)$

$(k_1 + k_2 t)\exp(-t/RC)$

$(\,)\exp(+p_1 t)$



$\left(-\frac{1}{p_1}\right)\left(-\frac{1}{p_2}\right)$ $(b_1 c_1)$ $(b_1 + c_1)$

$$a_2 \frac{d^2 V_c}{dt^2} + a_1 \frac{dV_c}{dt} + V_c = V_s$$

$-\left(\frac{1}{p_1} + \frac{1}{p_2}\right)$

$$-\frac{1}{p_1}\frac{dV_x}{dt} + V_x = V_s$$

$$-\frac{1}{p_2}\frac{dV_c}{dt} + V_c = V_x$$

$-\frac{1}{p_2}$

$\exp(p_2 t)$

$p_{1,2}$:

$$a_2 p^2 + a_1 p + 1 = 0$$

$b_1 c_1$ $(b_1 + c_1)$

Diff. equation:

$\omega_n^2 = \frac{1}{LC}$ $\quad \frac{1}{\sqrt{LC}}$

ω_n: natural frequency

ζ: zeta: damping factor

Q: Quality factor

Q: $\frac{1}{2\zeta}$; $\zeta = \frac{1}{2Q}$

$$LC \cdot \frac{d^2 V_c}{dt^2} + RC \frac{dV_c}{dt} + V_c = V_s$$

p_1, p_2

$$\frac{d^2 V_c}{dt^2} + \frac{R}{L} \frac{dV_c}{dt} + \frac{V_c}{LC} = \frac{V_s}{LC}$$

$$\frac{d^2 V_c}{dt^2} + 2\zeta \omega_n \frac{dV_c}{dt} + \omega_n^2 V_c = \omega_n^2 V_s$$

zeta

$$\frac{d^2 V_c}{dt^2} + \frac{\omega_n}{Q} \frac{dV_c}{dt} + \omega_n^2 V_c = \omega_n^2 V_s$$

Characteristic polynomial:

$$LC \cdot p^2 + RC \cdot p + 1 = 0$$

$$p^2 + 2\zeta \cdot \omega_n \cdot p + \omega_n^2 = 0$$

$$p^2 + \frac{\omega_n}{Q} \cdot p + \omega_n^2 = 0$$

$$p_{1,2} : -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$: -\frac{\omega_n}{2Q} \pm \sqrt{\left(\frac{\omega_n}{2Q}\right)^2 - \omega_n^2}$$

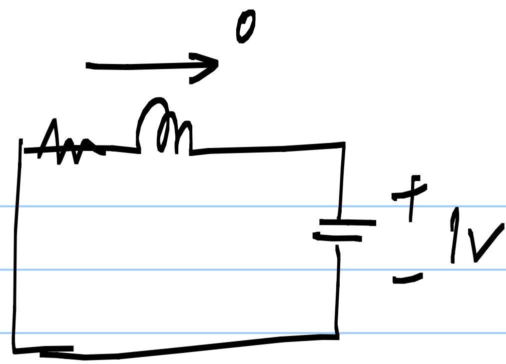
$$\omega_n \left[-\frac{1}{2Q} \pm \sqrt{\left(\frac{1}{2Q}\right)^2 - 1} \right]$$

$$\omega_n \left[-\zeta \pm \sqrt{\zeta^2 - 1} \right]$$

$p_{1,2}$

$$Q : \frac{1}{R}\sqrt{\frac{L}{C}} \quad : \quad \zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

Real & distinct	$Q < 1/2$, $\zeta > 1$	$A_1 \exp(p_1 t) + A_2 \exp(p_2 t)$ overdamped
Real, repeated	$Q = 1/2$, $\zeta = 1$	$(A_1 + A_2 t) \exp(p_1 t)$ critically damped
Complex conj	$Q > 1/2$, $\zeta < 1$	$(2A_0) \exp(p_r t) \cos(p_i t + \phi)$ underdamped.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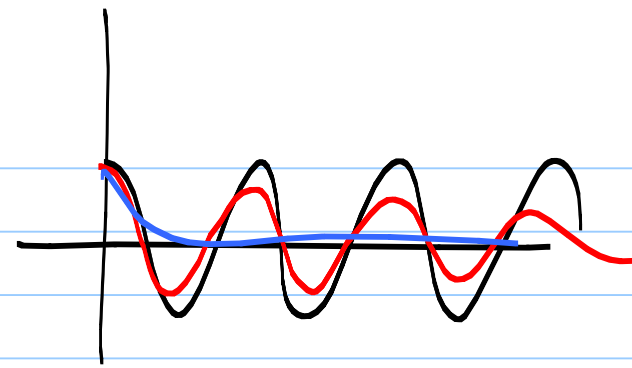
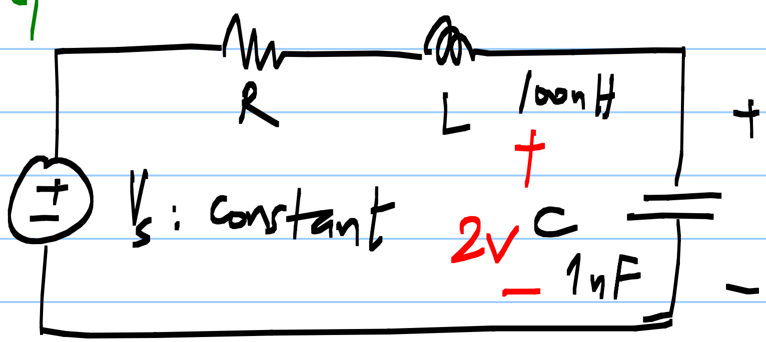


Forced response of a 2^{nd} order system

50Ω, 20Ω 2Ω → $5mA$ I: final value: 0



Final values: open circuit C; short circuit L

v_c = final value $= V_s$

$\zeta = 1/10$

$Q = 5$

Steady state response + natural response

$$v_c(t) = V_s + A_1 \exp(p_r t) \cos(p_i t + \phi)$$

$$v_c(0) = 2V = V_s + A_1 \cdot \cos(\phi)$$

$$C \cdot \frac{dv_c}{dt}\Big|_{t=0} = 5mA$$