EC1010: Lecture 27

Forced response to $V_p \cos(\omega t + \phi)$

$V_p \cos(\omega t + \phi)$ → Lin. system w/ real coeff. → Forced response to $V_p \cos(\omega t + \phi)$

$V_p \sin(\omega t + \phi)$ → Lin. system w/ real coeff. → Forced response to $V_p \sin(\omega t + \phi)$

$j = \sqrt{-1}$

j

1

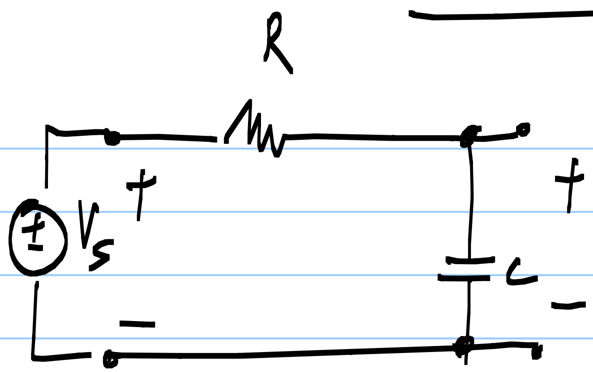
1

$V_p \exp(j(\omega t + \phi))$ → Lin. System w/ real coeff. → Forced response to $V_p \exp(j(\omega t + \phi))$

real part

Easy to calculate!

Sinusoidal steady state response

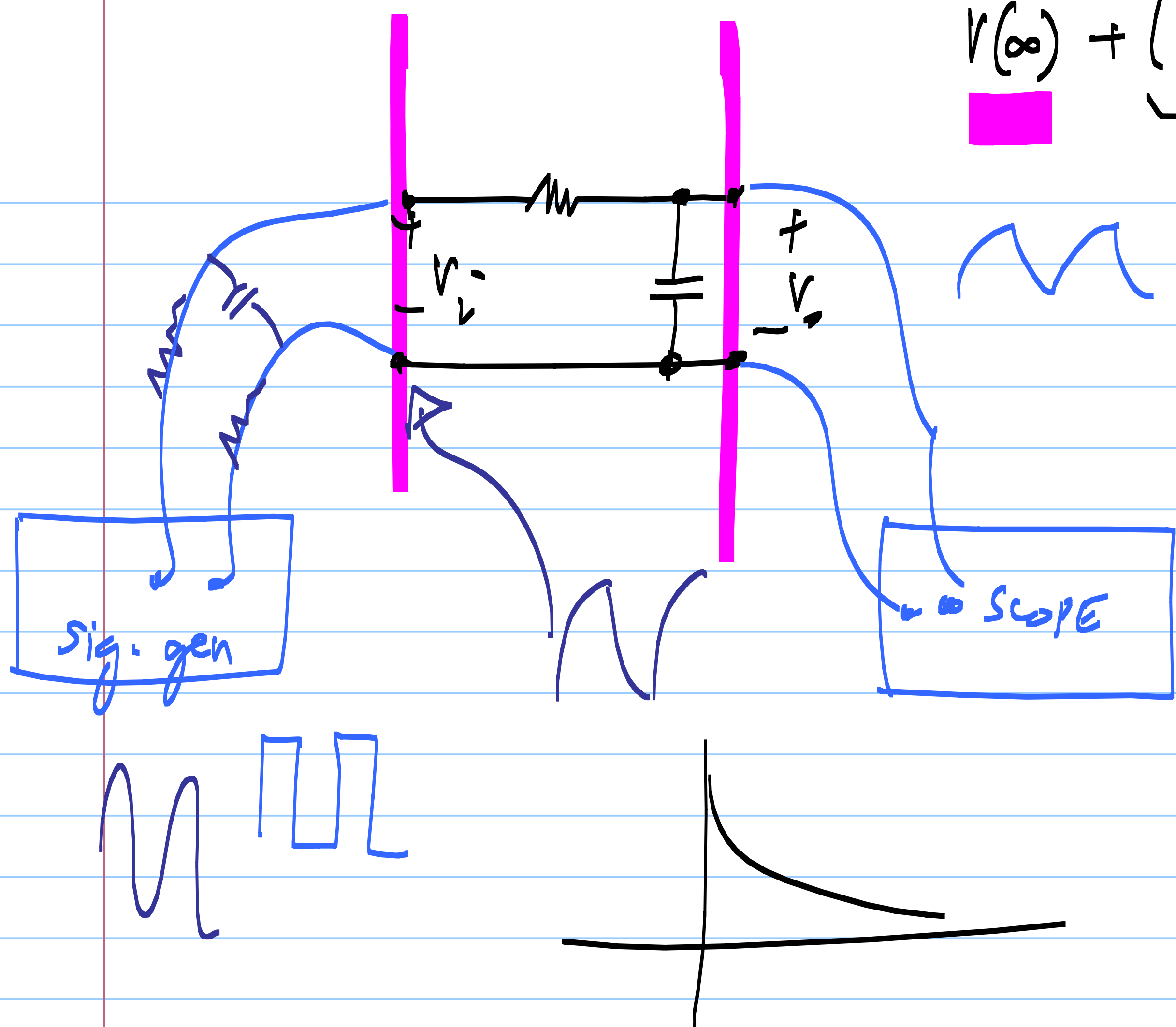


V_s	Forced resp.
$V_p \exp(st)$	$\dfrac{V_p}{1+sCR} \exp(st)$
$V_p \exp\left(j(\omega t+\phi)\right)$	
$= \underbrace{V_p \exp(j\phi)} \cdot \exp(j\omega t)$	$\dfrac{V_p \exp(j\phi)}{1+j\omega CR} \exp(j\omega t)$
	$= \dfrac{V_p \exp\left(j(\omega t+\phi)\right)}{(1+j\omega CR)}$

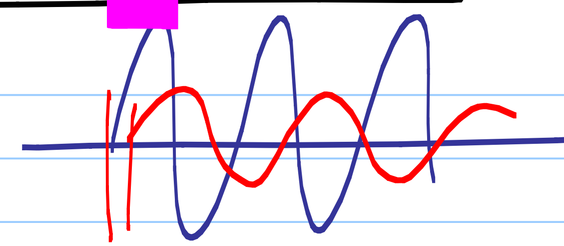
$$\underbrace{x+jy} = \underbrace{A \exp(j\theta)}$$

$$\frac{1}{1+j\omega CR} = \frac{1}{\sqrt{1+(\omega CR)^2}} \cdot \exp\left(-j\tan^{-1}(\omega CR)\right)$$

$$\left[\frac{V_p}{\sqrt{1+(\omega CR)^2}}\right] \cdot \cos\left(\omega t+\phi-\tan^{-1}(\omega CR)\right) \quad V_p \cos(\omega t+\phi) \qquad \downarrow \; \mathrm{Re}[\;]$$



$$V(\infty) + \underbrace{\left(V(0) - V(\infty)\right)\exp(-t/RC)}$$

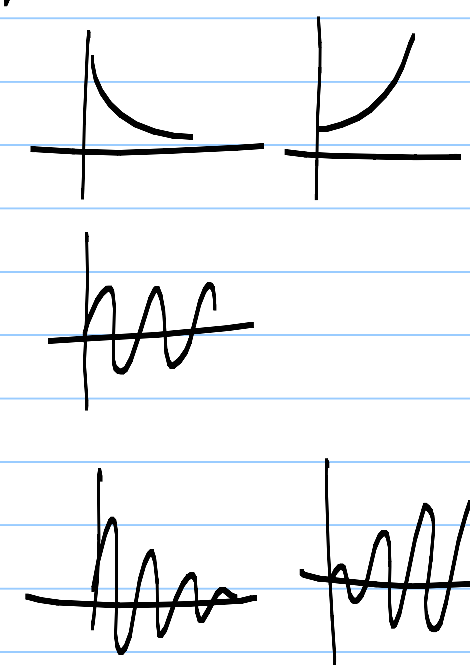


$\exp(st)$

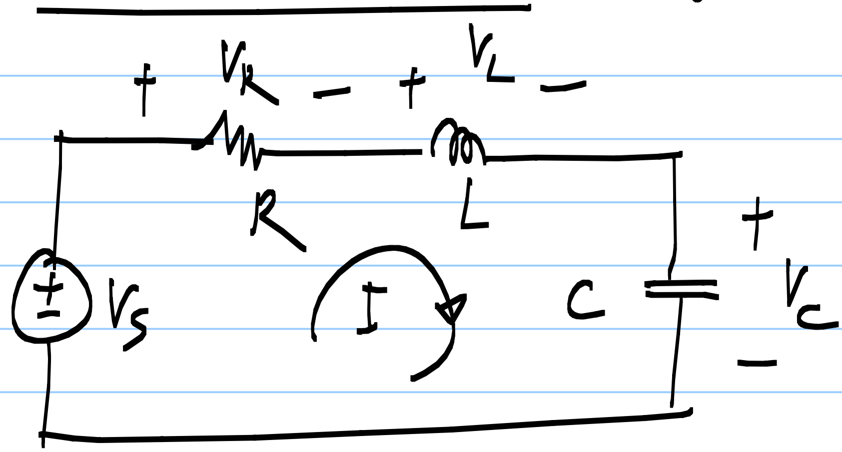
s: real

s: purely imag.

s: complex



Second order circuits: e.g: RLC



Diff. equation in terms of V_c

$$\boxed{LC\frac{d^2V_c}{dt^2} + RC\frac{dV_c}{dt} + V_c = V_s}$$

Natural response: $V_s = 0$

$$LC\frac{d^2V_c}{dt^2} + RC\frac{dV_c}{dt} + V_c = 0$$

$V_c = V_0 \exp(pt)$

$$\underbrace{(LC\cdot p^2 + RC\cdot p + 1)}_{\text{Characteristic eq. of the D.E}} V_0 \exp(pt) = 0$$

$$RC\frac{dV_c}{dt} + V_c = 0 \qquad p = -\frac{1}{RC}$$

First order circuit

$$V_c = V_0 \exp(pt) \qquad \underbrace{(RC \cdot p + 1)}_{\neq 0} \cdot V_0 \exp(pt) = 0$$

$$\overbrace{(LC \cdot p^2 + RC \cdot p + 1)}^{\neq 0} V_0 \exp(pt) = 0$$

Characteristic eq. of the D.E

Second order circuit

$$p = \frac{-RC \pm \sqrt{(RC)^2 - 4LC}}{2LC} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Two possible solutions: p_1, p_2 $\exp(p_1 t)$ & $\exp(p_2 t)$ are possible solutions.

In general: Solution (Natural response) $= A_1 \exp(p_1 t) + A_2 \exp(p_2 t)$

$$LC p^2 + RC \cdot p + 1 = 0$$

$$p_{1,2} = \frac{-RC \pm \sqrt{(RC)^2 - 4LC}}{2LC} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$p_{1,2}$: real & distinct : $\boxed{A_1 \exp(p_1 t) + A_2 \exp(p_2 t)}$

$(RC)^2 > 4LC$ $R^2 > \frac{4L}{C}$

$p_{1,2}$: real & identical $\quad (A_1 + A_2 t)\exp(p_1 t)$

$R^2 = \dfrac{4L}{C}$

Forced response to $\exp(st)$ of an RC circuit

$$sCR + 1 = 0$$

$$\left(v_c(0) - \frac{V_p}{1+sCR}\right) \qquad v_c(0)$$

$$\frac{V_p \exp(st)}{1+sCR} + V_0 \exp\left(-t/_{RC}\right) = v_c(t)$$

Limit: $1+sCR \to 0$

$\delta = 1+sCR$

$$\frac{V_p}{(1+sCR)}\left[\exp(st) - \exp\left(-t/_{RC}\right)\right] + v_c(0)\exp\left(-t/_{RC}\right) = v_c$$

$$\frac{V_p}{\delta}\left[\exp\left(\frac{\delta-1}{RC}\cdot t\right) - \exp\left(-\frac{t}{RC}\right)\right] + V_c(0)\exp\left(-\frac{t}{RC}\right)$$

$$\lim_{\delta \to 0} \frac{V_p}{\delta}\underbrace{\left[\exp\left(\frac{\delta t}{RC}\right) - 1\right]}\exp\left(-\frac{t}{RC}\right) + V_c(0)\exp\left(-\frac{t}{RC}\right)$$

$$\frac{V_p}{\cancel{\delta}}\left[\cancel{1} + \frac{\delta t}{RC} + \left(\frac{\delta t}{RC}\right)^2\cdot\frac{1}{2} + \ldots \cancel{-1}\right]$$

$$V_p\cdot\frac{t}{RC}\cdot\exp\left(-\frac{t}{RC}\right) + V_c(0)\exp\left(-\frac{t}{RC}\right)$$

$$(k_1 + k_2 t)\exp\left(-t/RC\right)$$

$p_{1,2}$: complex conjugates of each other

$R^2 < \frac{4L}{C}$

$$p_{1,2} = -\frac{R}{2L} \pm j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = p_r \pm j p_i$$

$$A_1 \exp(p_1 t) + A_2 \exp(p_2 t) \qquad A_2 = A_1^*$$

$$v_c(t) = A_1 \exp(p_1 t) + A_1^* \exp(p_2 t)$$

$$A_1 \exp(p_r t) \exp(j p_i t) + A_1^* \exp(p_r t) \exp(-j p_i \cdot t)$$

$$(\quad) \cos(\quad) \cdot \exp(\quad)$$