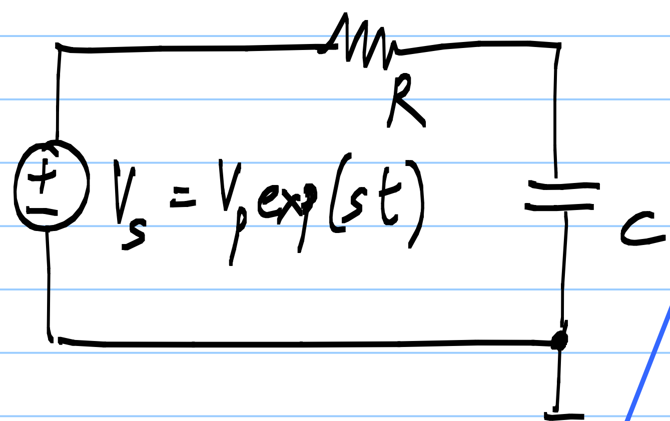
EC1010: Lecture 26



$$RC\frac{dV_{c_2}}{dt} + V_{c_2} = 0 \qquad V_{c_2} = V_{c_{20}} \exp\left(\frac{-t}{RC}\right)$$

$$RC\frac{dV_c}{dt} + V_c = V_p \exp(st)$$

$$V_{c_1} = V_c - V_p \exp(st)$$

$$RC\frac{dV_{c_1}}{dt} + V_{c_1} = -sCR \cdot V_p \cdot \exp(st)$$

$\times sCR$

$$RC\frac{d}{dt}\Big(\underbrace{sCR \cdot V_c + V_{c_1}}_{V_{c_2}}\Big) + \Big(\underbrace{sCR\, V_c + V_{c_1}}_{V_{c_2}}\Big) = 0$$

$V_p \exp(-t/RC)$

$$V_{c_2} = V_{c_{20}} \exp\left(-\frac{t}{RC}\right) = sCR\, V_c + V_{c_1}$$

$$= sCR\, V + V_c - V_p \exp(st)$$

$sCR + 1 = 0$

$$V_c(t) = \frac{V_p \exp(st) + V_{c_{20}} \exp\left(-\frac{t}{RC}\right)}{sCR + 1}$$

Found from initial conditions

$$\lim_{1+sCR \to 0} = \frac{V_p \exp(st)}{sCR+1} + (V_0) \cdot \exp\left(-\frac{t}{RC}\right)$$

$\delta = 1 + sCR$

Steady state / Forced response

Natural response

Steady state response (of any linear system) to exp(st) : scaled version of itself

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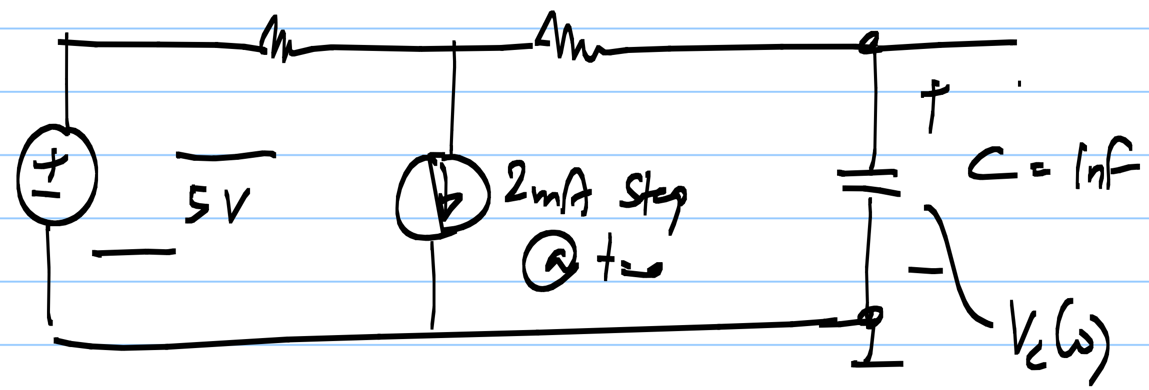
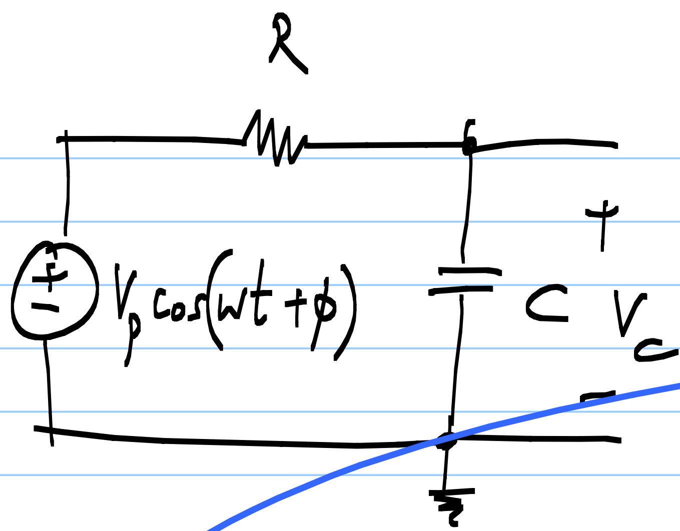
<u>Exercise</u>: ① Find out $V_c(t)$ for $sCR + 1 = 0$ by taking the appropriate limit

②



Try superposing the <u>total</u> responses from <u>the 2 sources</u>



$$RC \frac{dv_c}{dt} + V_c = V_p \cos(\omega t + \phi)$$

$$\gamma \cos(\Omega t + \theta)$$

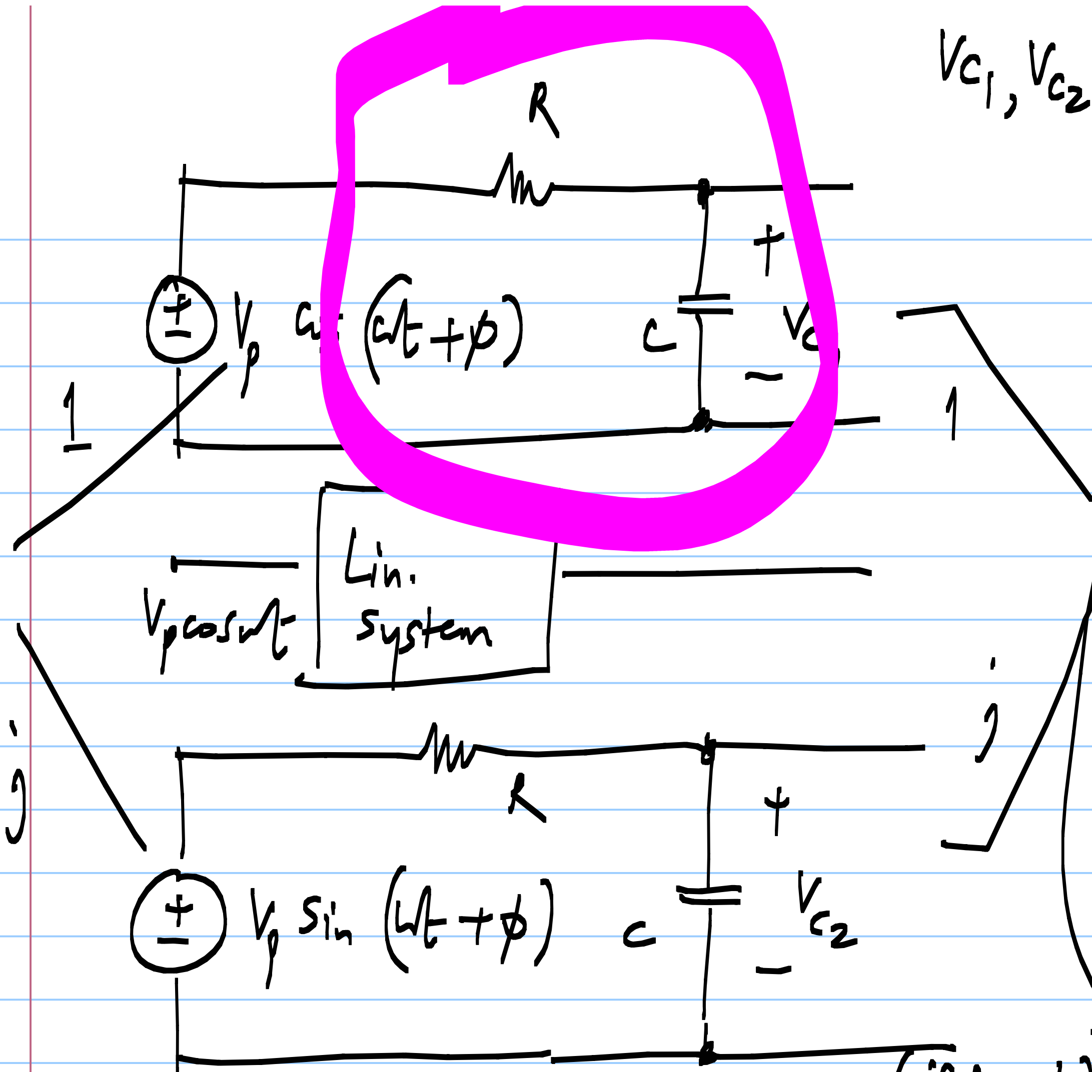
$$V_c(t) = V_p \left[\alpha \cos(\omega t + \phi) + \beta \sin(\omega t + \phi) \right]$$

$$\frac{dV_c}{dt} = V_p \cdot \omega \left[-\alpha \sin(\omega t + \phi) + \beta \cos(\omega t + \phi) \right]$$

$$V_p \cos(\omega t + \phi) = \frac{V_p}{2} \left[\exp(j(\omega t + \phi)) + \exp(-j(\omega t + \phi)) \right]$$

$$\exp(j\phi)\exp(j\omega t)$$

$$\exp(st)$$



V_{c_1}, V_{c_2} : steady state response

$V_p \exp(j(\omega t + \phi))$

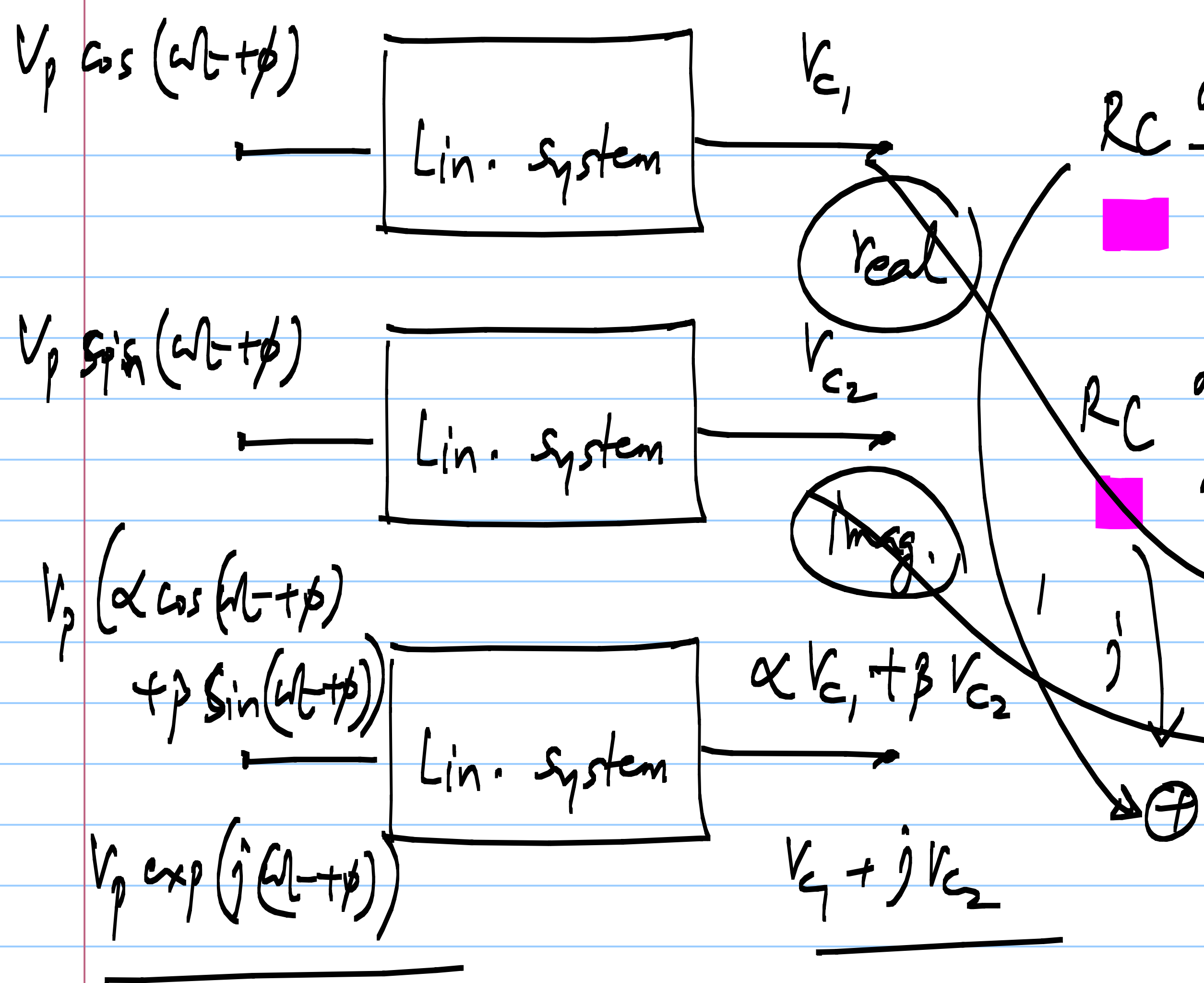
$V_p \exp(j\phi)\exp(j\omega t)$

$V_{c_1} + j V_{c_2}$

$V_p \exp(st) \rightarrow \frac{V_p}{1 + sCR} \exp(st)$

$V_p \frac{\exp(j(\omega t + \phi))}{1 + j\omega CR}$

$\frac{V_p \exp(j\phi)}{1 + j\omega CR} \exp(j\omega t)$



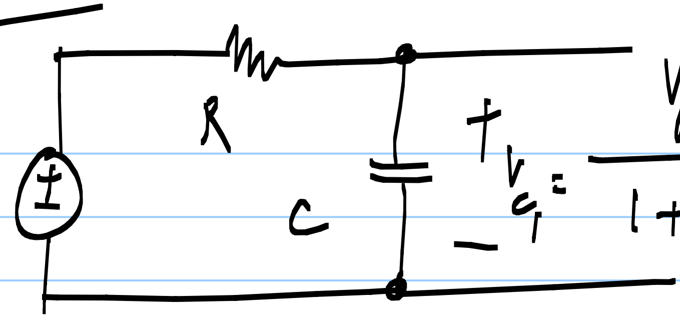
$$RC \frac{dv_{c_1}}{dt} + v_{c_1} = V_p \cos(\omega t + \phi)$$

$$RC \frac{dv_{c_2}}{dt} + V_{c_2} = V_p \sin(\omega t + \phi)$$

$$V_p \exp(j(\omega t + \phi))$$

Forced response:

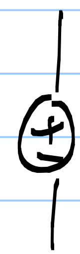
$V_p \exp(j(\omega t + \phi))$



$$V_{c_1} = \frac{V_p}{1 + j\omega CR} \exp(j(\omega t + \phi))$$

real

$V_p \cos(\omega t + \phi)$



$$\left(\frac{V_p}{\sqrt{1 + \omega^2 C^2 R^2}}\right) \cos\left(\omega t + \phi - \tan^{-1}(\omega CR)\right)$$