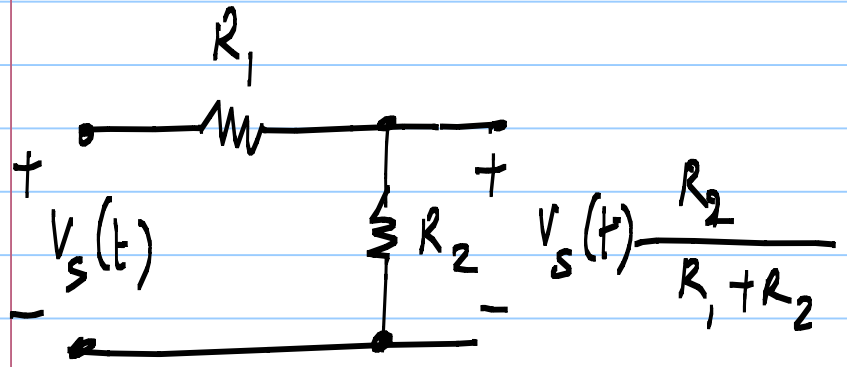
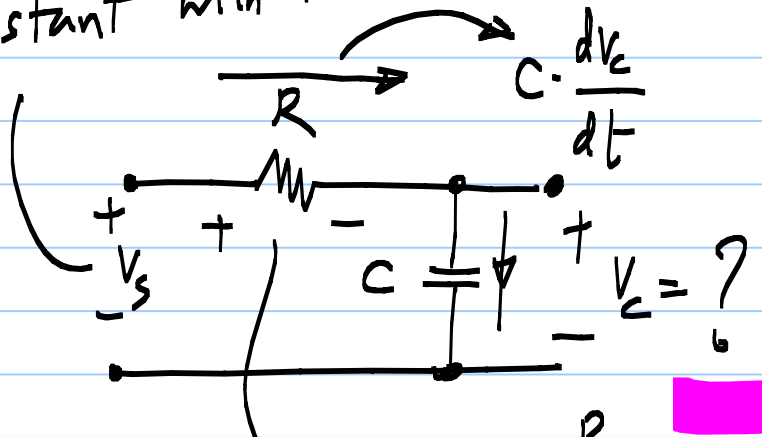
EC 1010 : Lecture 23

direct current (dc)



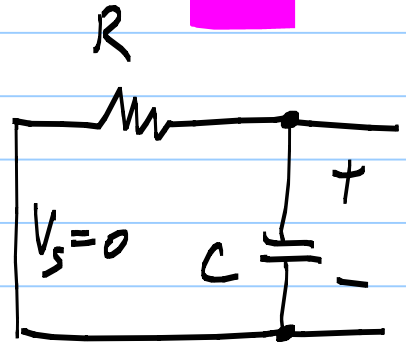
$$V_s(t)\,\frac{R_2}{R_1+R_2}$$

Constant with time



$$RC\cdot\frac{dV_c}{dt} + V_c = V_s$$

$$RC\cdot\frac{dV_c}{dt}$$



$$V_s = 0$$

$$RC\,\frac{dV_c}{dt} + V_c = 0$$

First order diff. equation

linear

$$RC \cdot \frac{dV_c}{dt} + V_c = V_s$$

$V_s = 0$

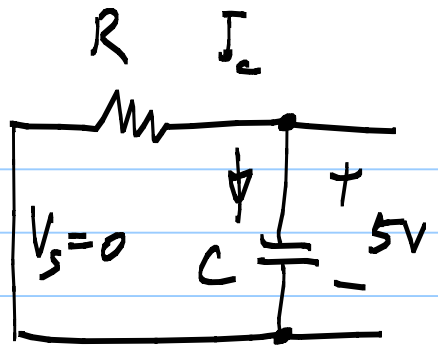
$$RC \frac{dV_c}{dt} + V_c = 0$$

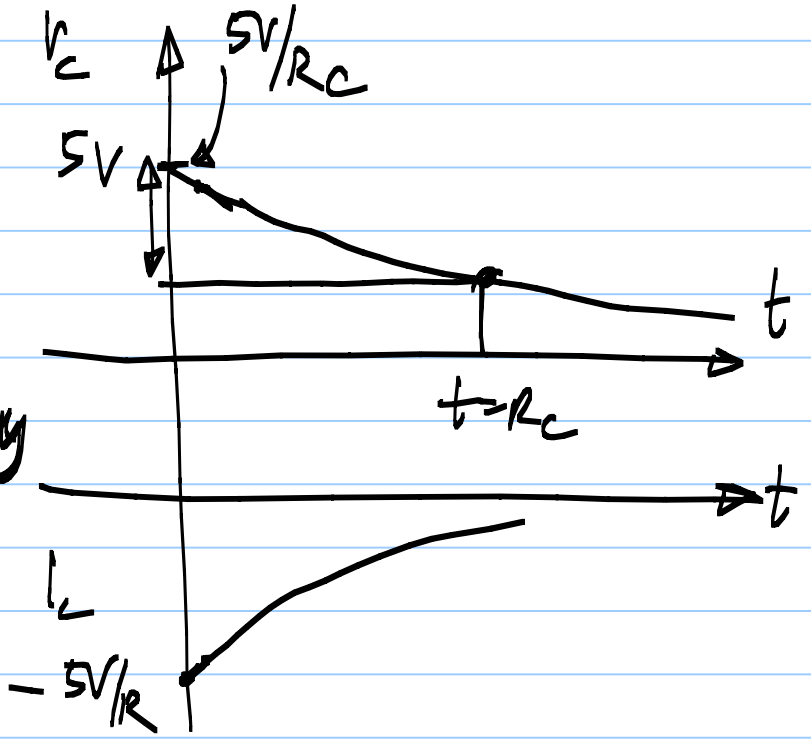
homogeneous eq.

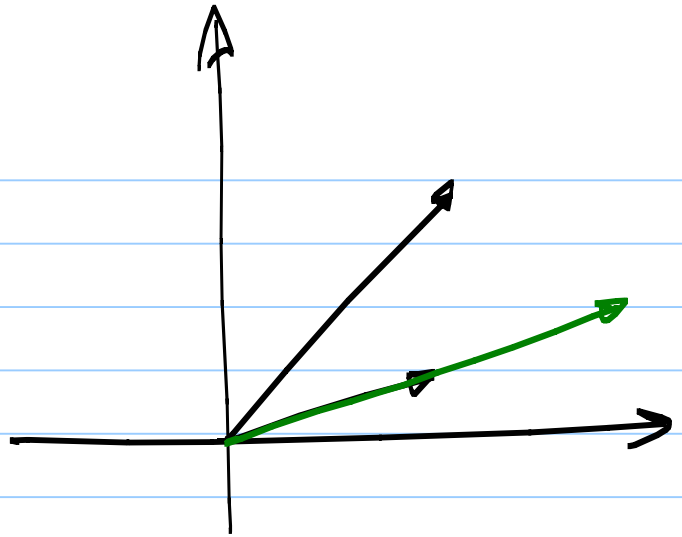
$$\frac{dV_c}{dt} = -\left(\frac{1}{RC}\right) \cdot V_c$$

$\exp(\alpha x)$

$$\frac{dy}{dx} = \alpha y$$





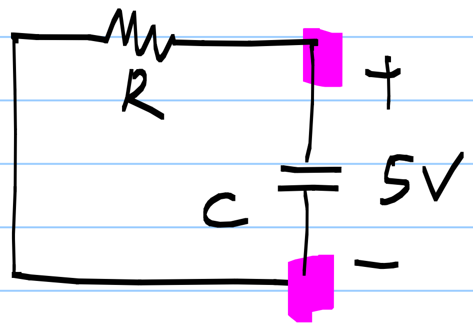


$$\begin{bmatrix} A \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

2×2

$$\propto \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

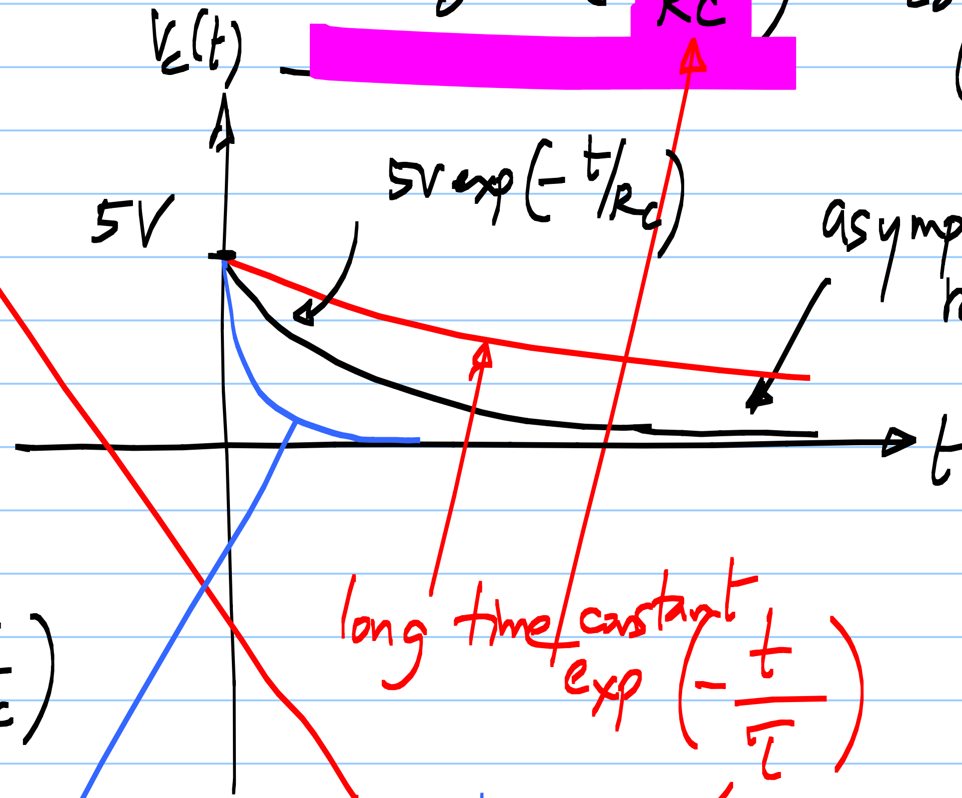
$$\frac{dV_c}{dt} = -\frac{1}{RC} \cdot V_c$$



$$\frac{dV_c}{dt} = -\frac{V_{c0}}{RC} \exp\left(-\frac{t}{RC}\right)$$

$$V_c = V_{c0} \exp\left(-\frac{t}{RC}\right)$$?

$V_{c0} = 5V$ $(t=0)$



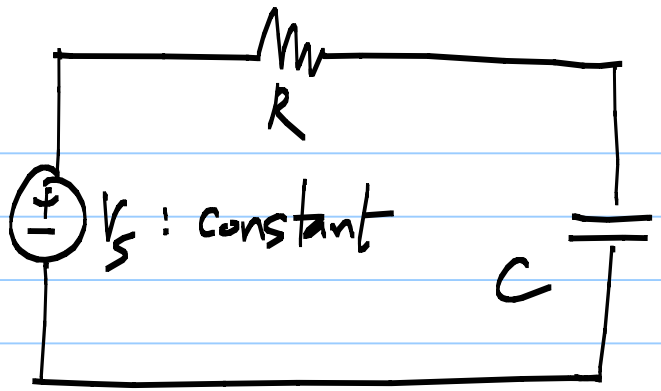
$t = 0$

long time constant

$\exp\left(-\frac{t}{\tau}\right)$

short time constant

time constant τ RC



$$RC\frac{dV_c}{dt} + V_c = V_s$$

$$\frac{dV_c}{dt} + \frac{V_c}{RC} = \frac{V_s}{RC}$$

$$RC\frac{dV_c}{dt} + (V_c - V_s) = 0$$

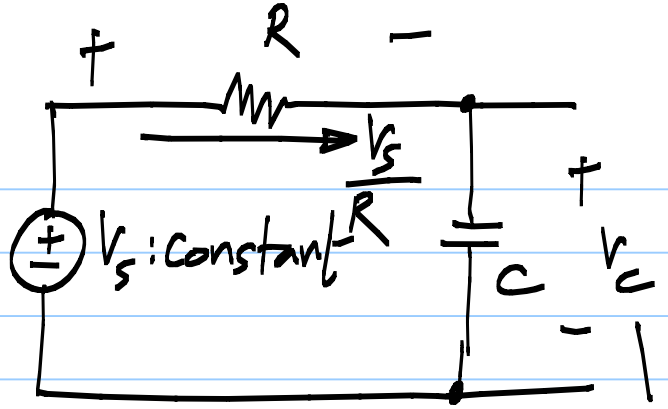
$$V_{c_1} = V_c - V_s$$

$$\frac{dV_{c_1}}{dt} = \frac{dV_c}{dt}$$

$$RC\frac{dV_{c_1}}{dt} + V_{c_1} = 0$$

$$V_{c_1} = V_{c_{1_0}} \cdot \exp\left(-\frac{t}{RC}\right)$$

$$V_c = V_s + V_{c_{1_0}} \exp\left(-\frac{t}{RC}\right)$$



$$RC\frac{dV_c}{dt} + V_c = V_s$$

$$V_c(t) = V_s + V_{c_{10}} \cdot \exp\left(-\frac{t}{RC}\right)$$

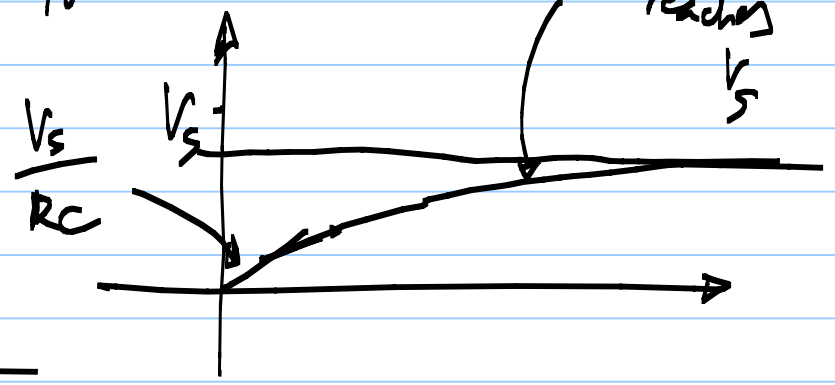
$$V_c(t) = V_s + \left(V_c(0) - V_s\right)\exp\left(-\frac{t}{RC}\right) \qquad V_c(0) = V_s + V_{c_{10}}$$

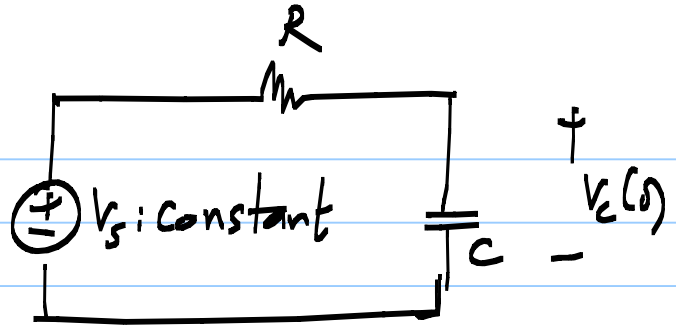
$$V_{c_{10}} = V_c(0) - V_s$$

$V_c(0) = 0$:

$$V_c(t) = V_s - V_s \exp\left(-\frac{t}{RC}\right)$$

$$= V_s\left(1 - \exp\left(-\frac{t}{RC}\right)\right)$$

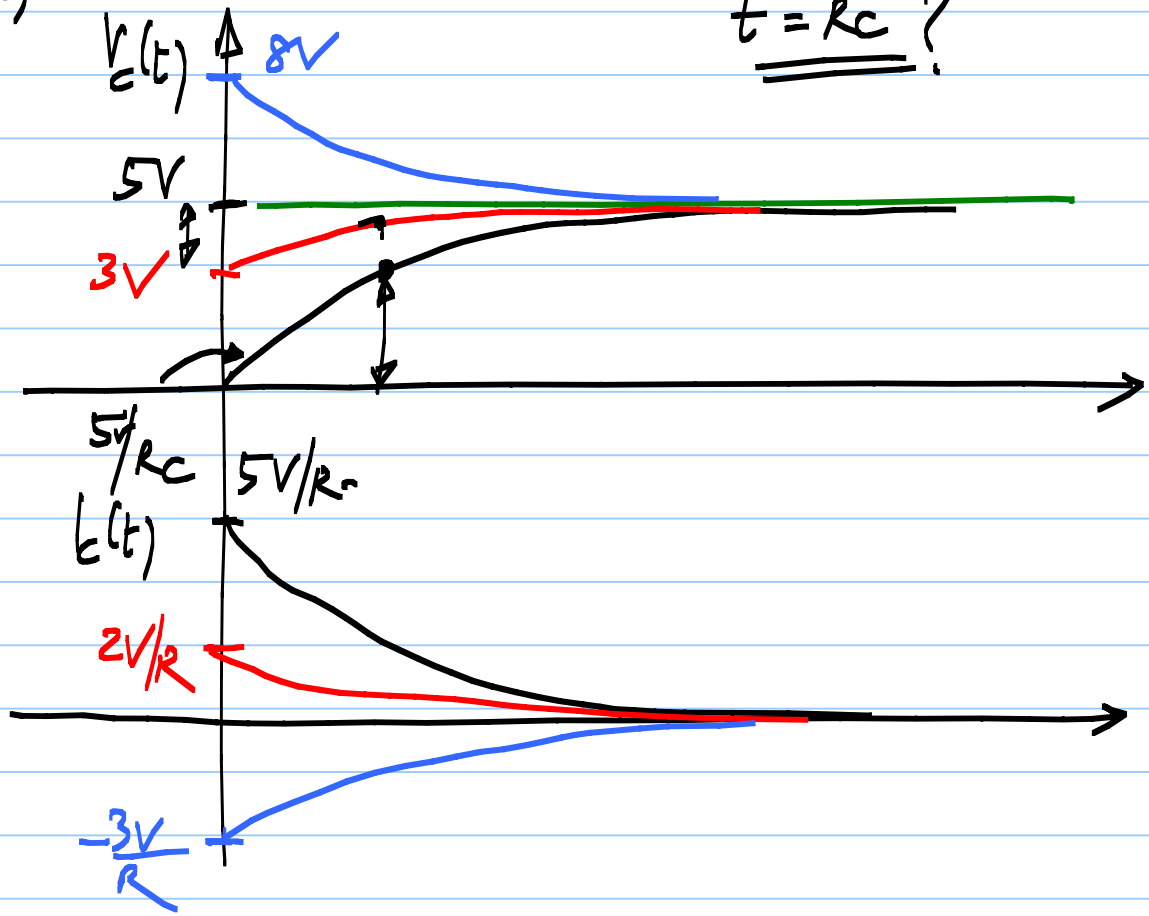




$$V_c(t) = V_s + (V_c(0) - V_s)\exp\left(-\frac{t}{R_c}\right)$$

$t = Rc$?

$$V_s\left(1 - \frac{1}{e}\right) \simeq 0.63$$



$$RC\frac{dv_c}{dt} + v_c = V_s \quad : \quad v_c(t) = V_s + \left(v_c(0) - V_s\right)\exp\left(-\frac{t}{RC}\right)$$

Steady state response + Transient response

Forced response + Natural response

Particular solution + Solution to homogeneous equation

$$RC\frac{dv_c}{dt} + v_c = 0 \quad : \quad v_c(t) = v_c(0)\exp\left(-\frac{t}{RC}\right)$$

Forcing function

$$V_s\left(1 - \exp\left(-\frac{t}{RC}\right)\right) + v_c(0)\exp\left(-\frac{t}{RC}\right)$$

Zero state response + Zero input response