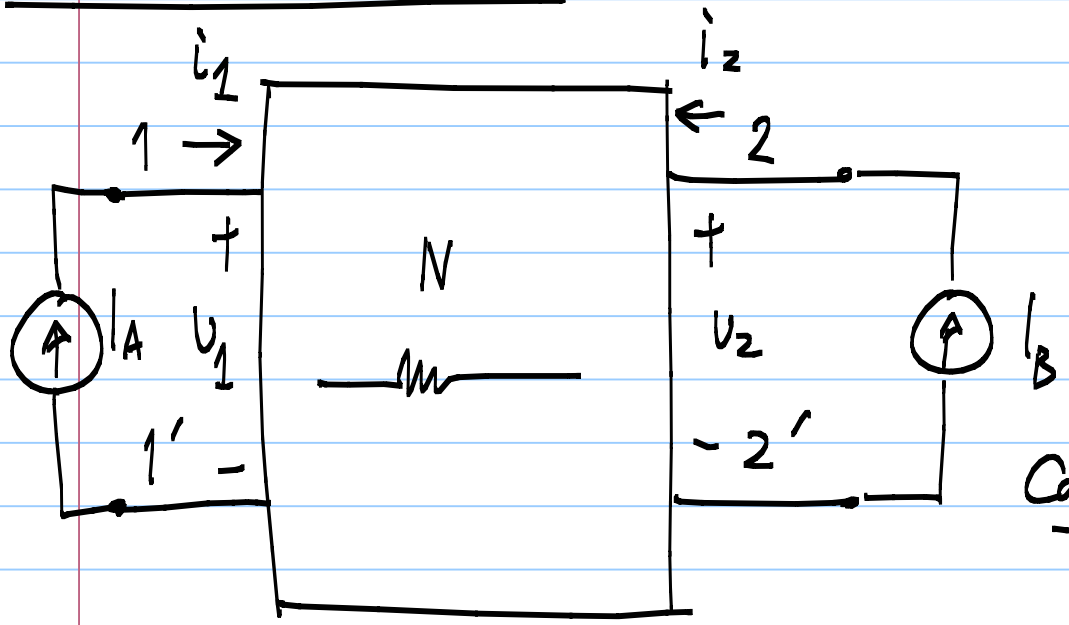
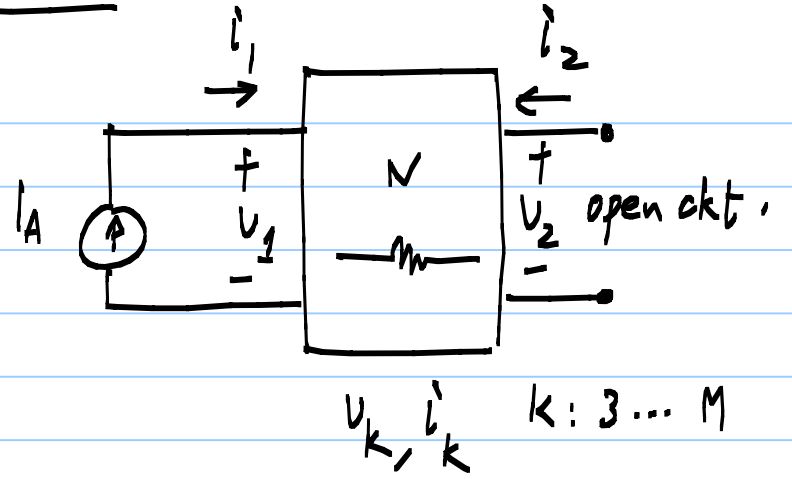
EC1010: Lecture 14

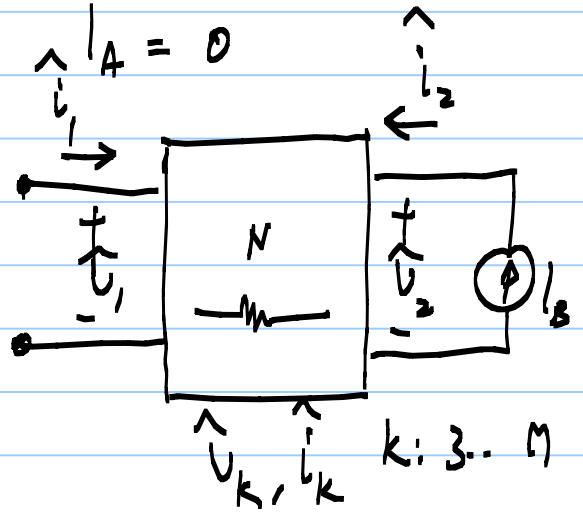


Two port network

Case 1: $I_B = 0$



Case 2: $I_A = 0$



Tellegen's theorem: $\{v_k, i_k\}$

$$v_1 \cdot 0 + v_2(-I_B) + \sum_{3}^{M} v_k \cdot i_k = 0$$

Resistive network $\Rightarrow$ $v_k = R_k \cdot i_k$

$$\sum_{3}^{M} R_k \, i_k \, i_k$$

=

Tellegen's theorem: $\{\hat{v}_k, \hat{i}_k\}$

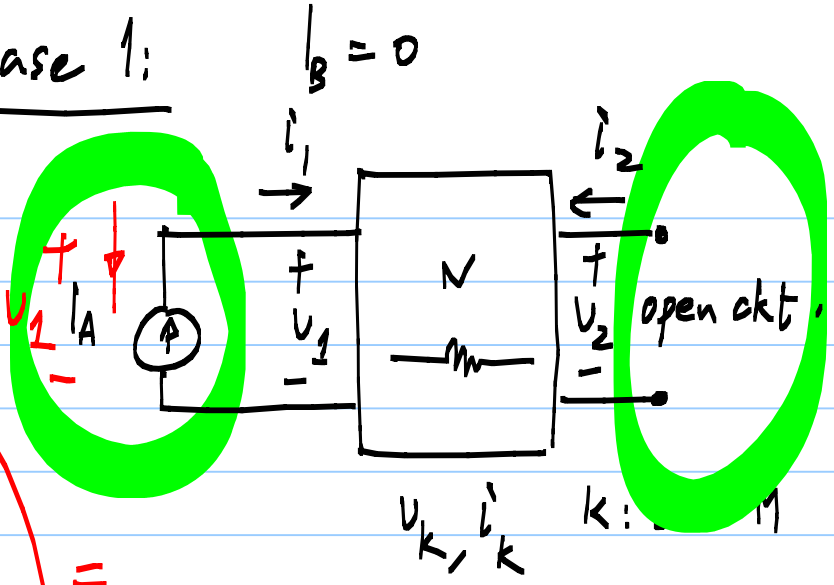
$$\hat{v}_1(-I_A) + \hat{v}_2(0) + \sum_{3}^{M} \hat{v}_k \, i_k = 0$$

$\hat{v}_k = R_k \cdot \hat{i}_k$

$$\sum_{3}^{M} R_k \, \hat{i}_k \, i_k$$

=

Case 1: $I_B = 0$



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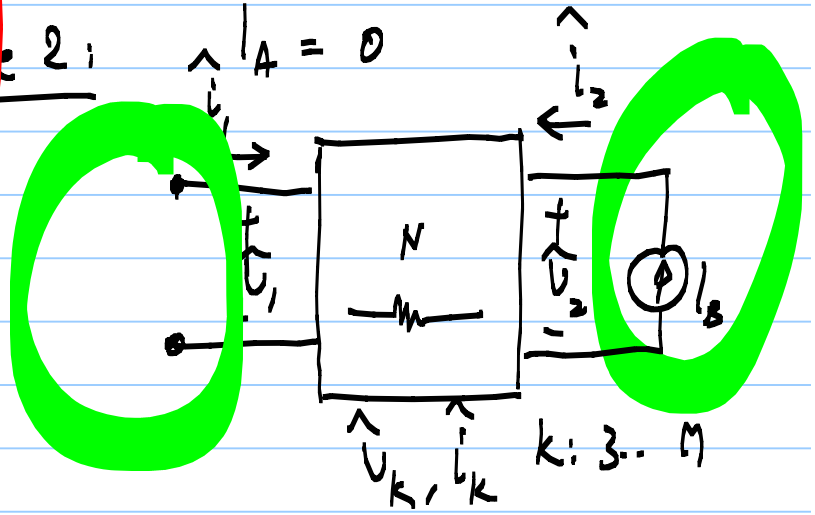
Case 2: $I_A = 0$

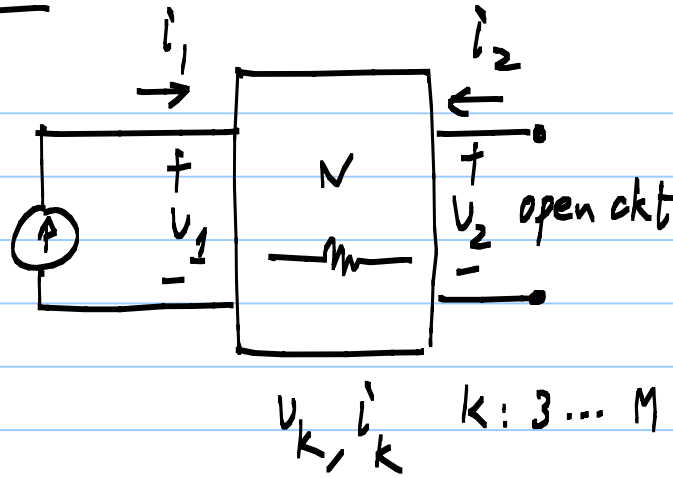


$$\hat{V}_1(-I_A) = V_2(-I_B)$$

$$\frac{V_2}{I_A} = \frac{\hat{V}_1}{I_B}$$

ratio of "output" (@ port #2) to "input" (@ port #1)

Case 1: $I_B = 0$

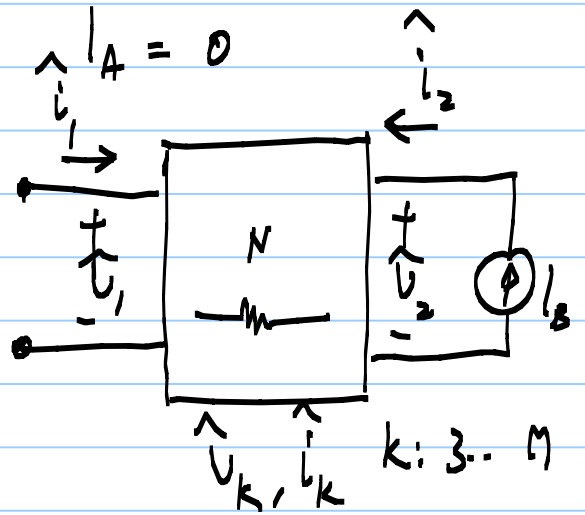


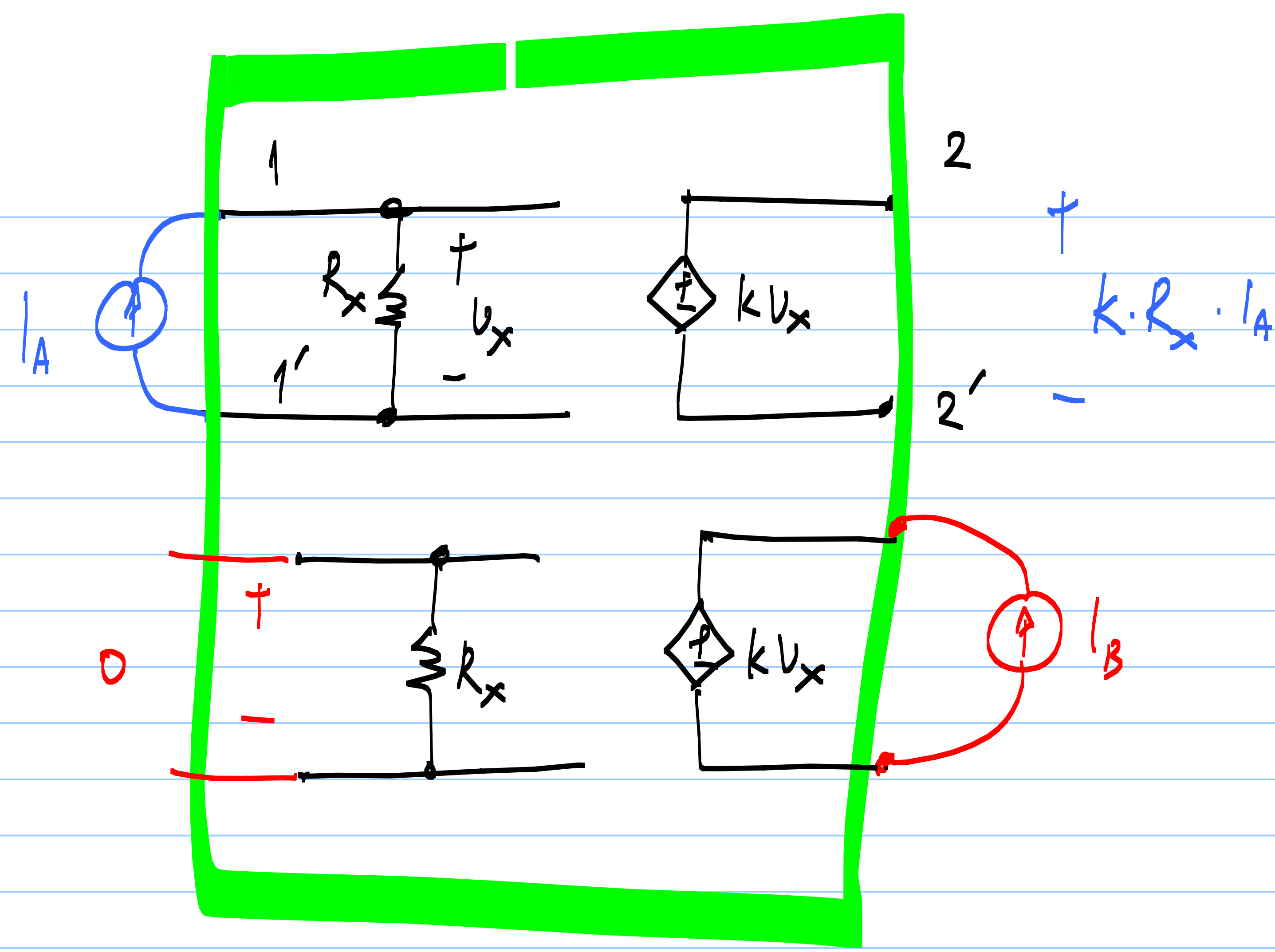
Reciprocity theorem

Reciprocal networks

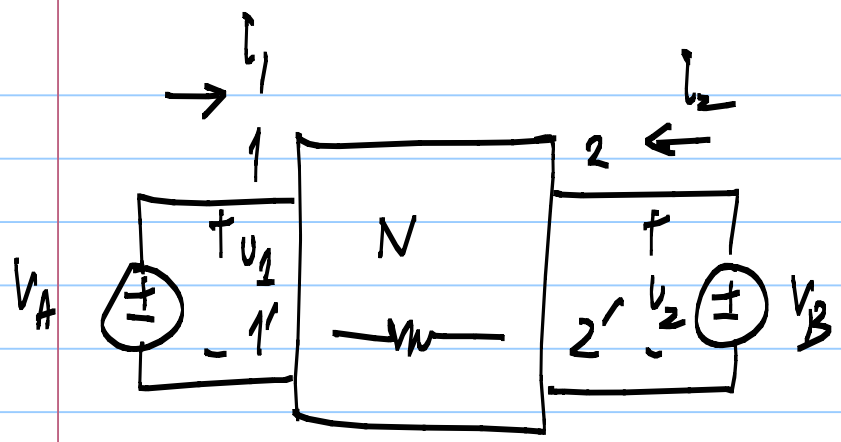
Case 2: $\hat{I}_A = 0$

ratio of "output" (@ port #1) to "input" (@ port # 2)

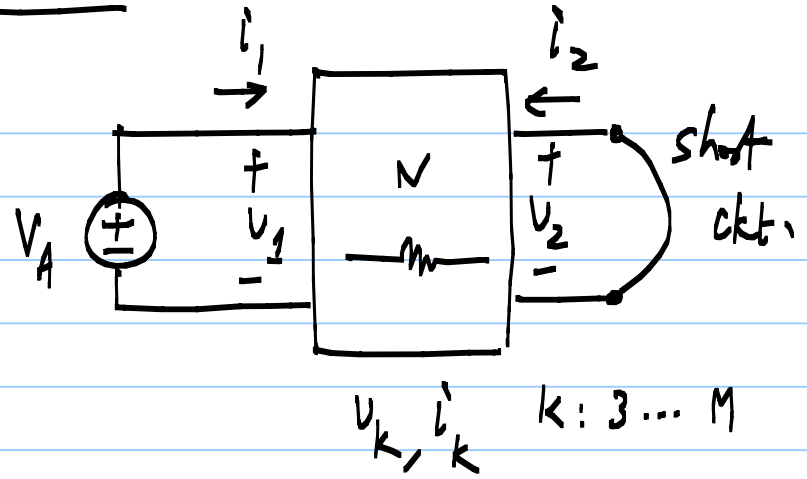




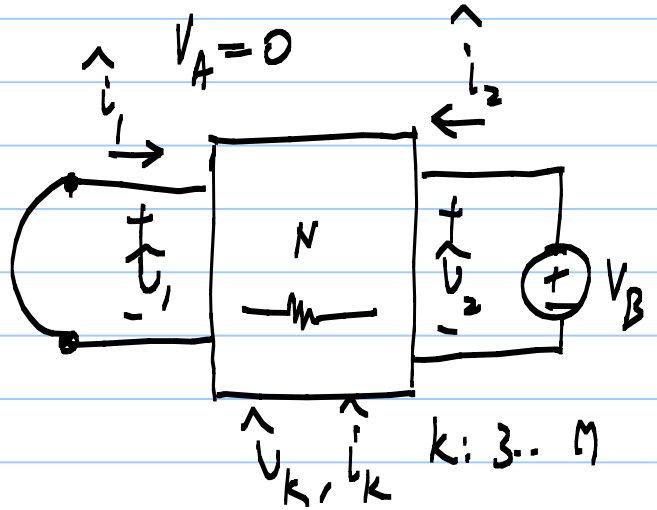
1
2
I_A
R_x
+
U_x
−
$1'$
$2'$
kU_x
+
$k \cdot R_x \cdot I_A$
−
0
R_x
kU_x
I_B



Case 1: $V_B = 0$



Case 2: $V_A = 0$



Tellegen's theorem: $\{\hat{v}_k, \hat{i}_k\}$

$$V_A\left(-\hat{i}_1\right) + 0\left(-\hat{i}_2\right) + \sum_{3}^{M} v_k \hat{i}_k = 0$$

$$= \quad \{\hat{v}_k, i_k\} \; R_k i_k \hat{i}_k$$

$$0\left(-\hat{i}_1\right) + V_B\left(-\hat{i}_2\right) + \sum_{3}^{M} \hat{v}_k i_k = 0$$

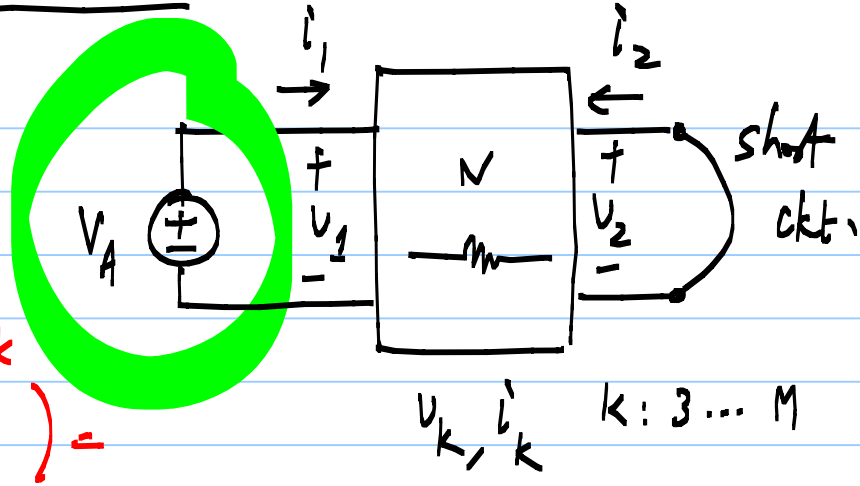
$$R_k \hat{i}_k i_k$$

$$V_A\left(-\hat{i}_1\right) = V_B\left(-\hat{i}_2\right)$$

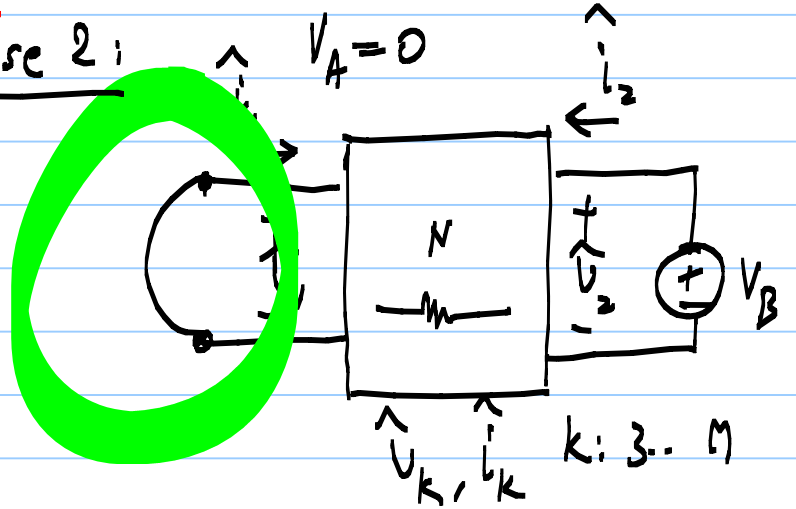
$$\frac{\hat{i}_2}{V_A} = \frac{\hat{i}_1}{V_B}$$

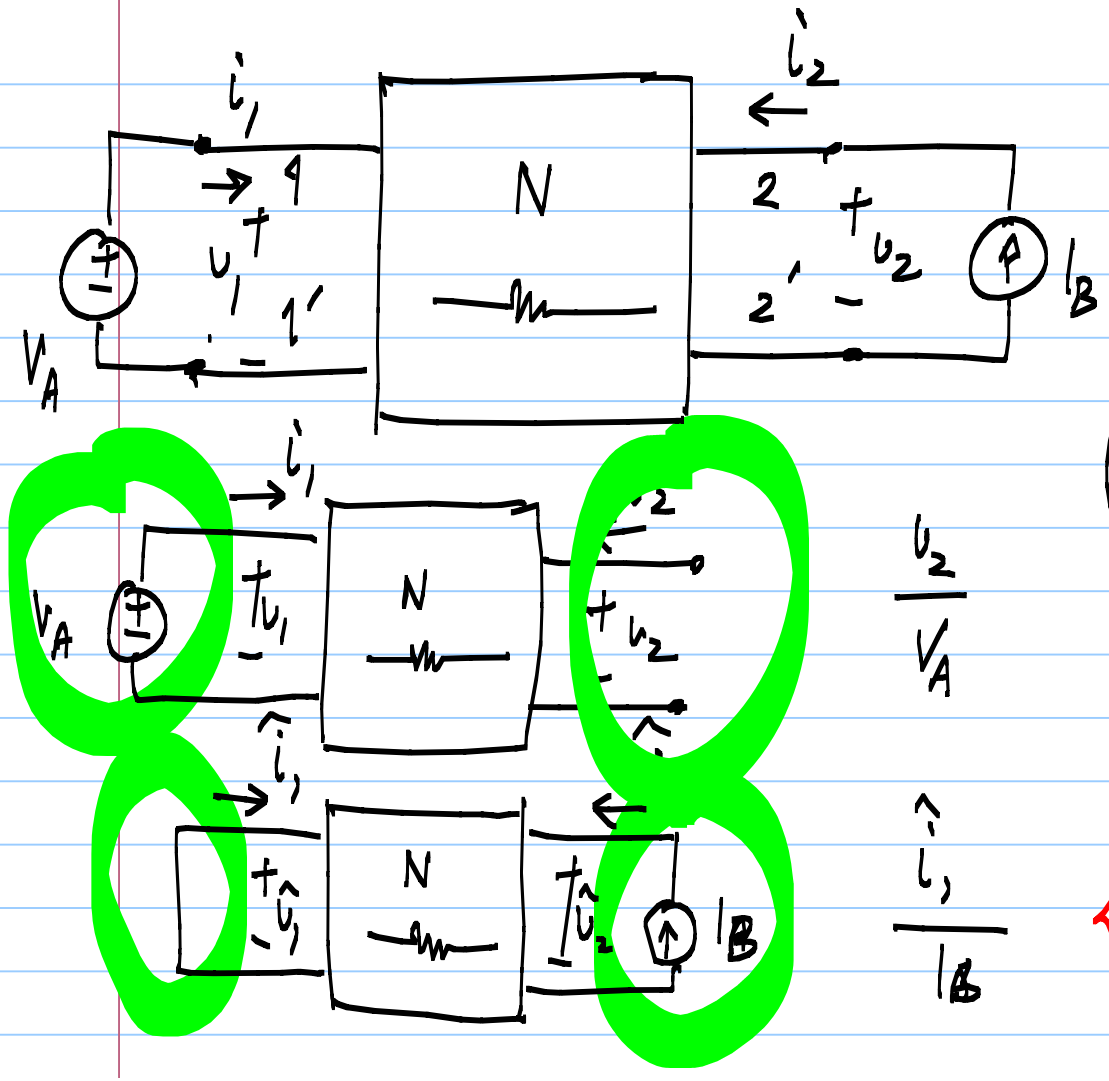
Reciprocity theorem

Case 1: $V_B = 0$



Case 2: $V_A = 0$





$$\frac{v_2}{V_A}$$

$$\frac{\hat{i}_1}{I_B}$$

Case#1 : $I_B = 0$, v_k, i_k'

Case#2 : $V_A = 0$, $\hat{v}_k, \hat{i}_k'$

Tellegen's theorem $\{v_k, \hat{i}_k'\}$

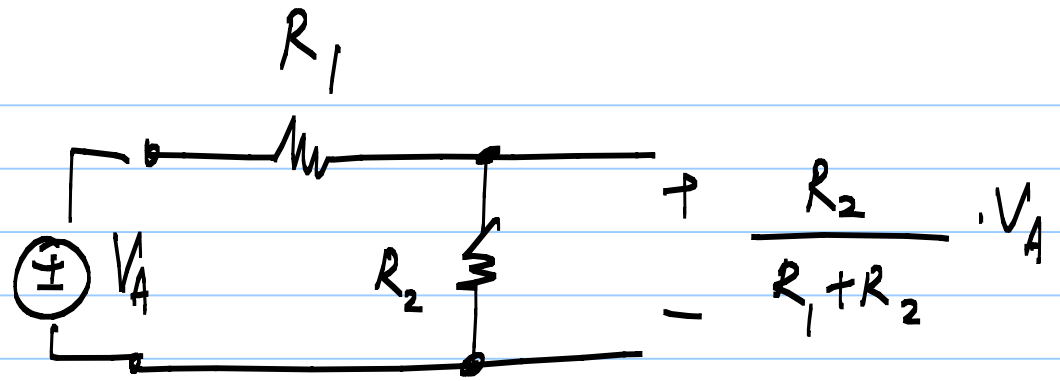
$$v_A(-\hat{i}_1') + v_2(-I_B) + \sum_{3}^{M} v_k \hat{i}_k' = 0$$

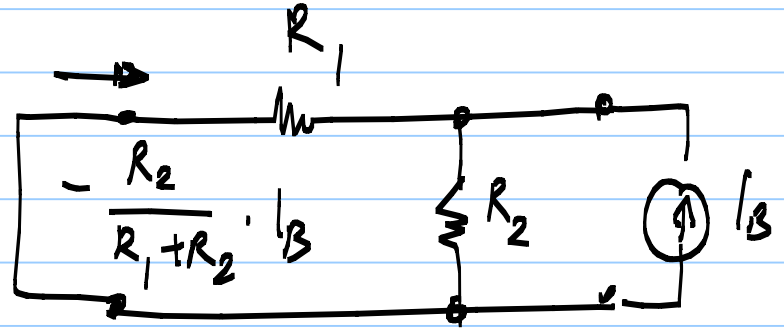
$= R_k i_k' \hat{i}_k'$

$\{\hat{v}_k, i_k'\}$:

$$0(-\hat{i}_1') + \hat{v}_2(0) + \sum_{3}^{M} \hat{v}_k i_k' = 0$$

$$\left\{ \frac{v_2}{V_A} = -\frac{\hat{i}_1}{I_B} \right\}$$


R1
R2
VA
+
R2 / (R1+R2) · VA
−


R1
R2
IB
− R2 / (R1+R2) · IB

Two port networks: