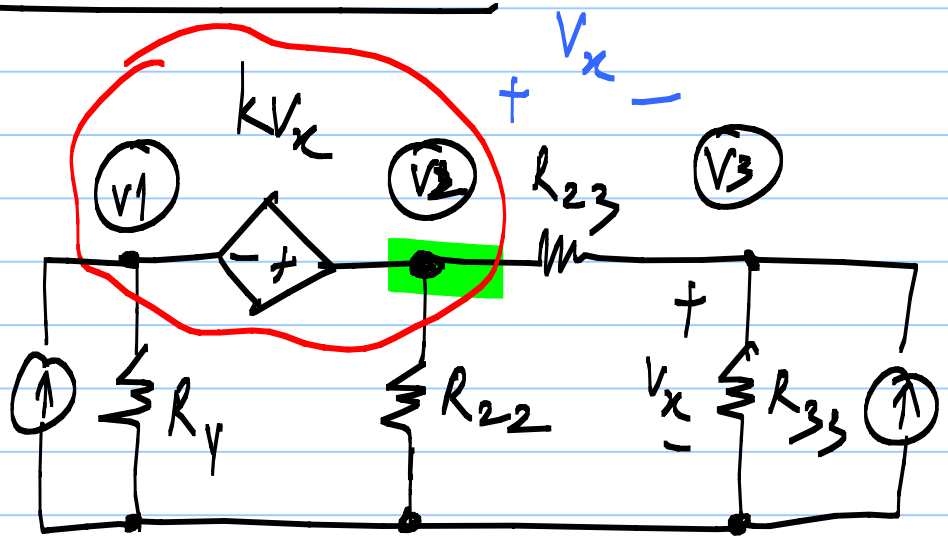
EC 1010: Lecture 7



$k \cdot V_x$

Eq. for the vcvs

$$V_2 - V_1 - kV_3 = 0$$

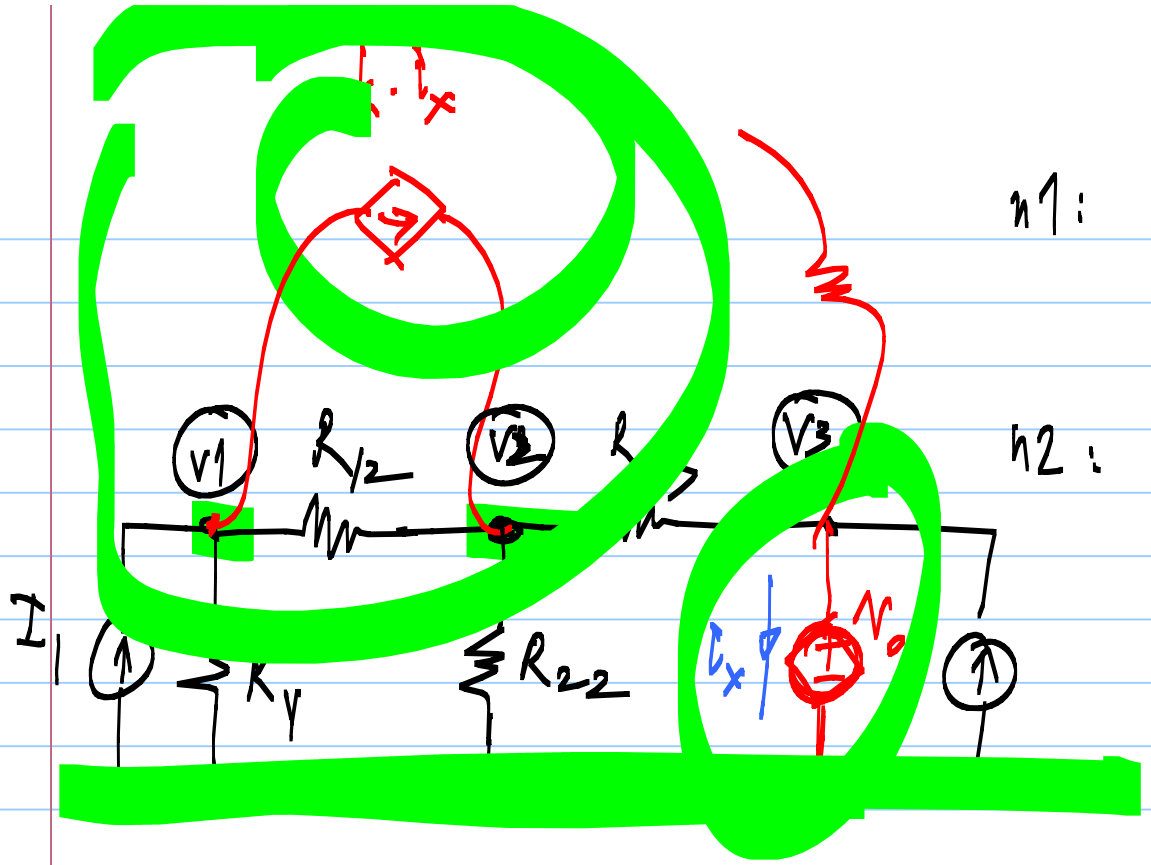
$$V_2 - V_1 - kV_2 + kV_3 = 0$$

- ✓ Resistors & ind. I sources
- ✓ Ind. V sources
- ✓ vccs

---

vcvs

* Combine nodes to which v. source is connected into a supernode



n1: $V_1 \cdot (G_{11} + G_{12}) + k \cdot i_x - V_2 \cdot G_{12} = I_1$

$\left(\frac{V_3}{R_{33}}\right)$

n2: ... $- k i_x$ ...

$$V_2 - V_1 - R_x i_x = 0$$

$$V_2 - V_1 - R_x \cdot V_3 / R_{33} = 0$$

Nodal analysis:

- Node voltages (wrt ref. node): variable vector
- KCL eq. at every node (excl. ref).

$$[G]\,\underline{V} = \underline{I}$$

$[G]$: Conductance matrix

$\underline{V}$: node voltage vector

$\underline{I}$: Source vector

- Circuits w/ resistors & ind. current sources

* Circuits w/ resistors & ind. current sources

Symmetric $[G]$ matrix

- VCCS : $[G]$: asymmetric
- Ind. voltage sources $[G]\,\underline{V} = \underline{I}$
- CCVS / CCCS : Similar form if the controlling current is through a resistor
- Auxiliary variables if the cont. current is through a voltage source

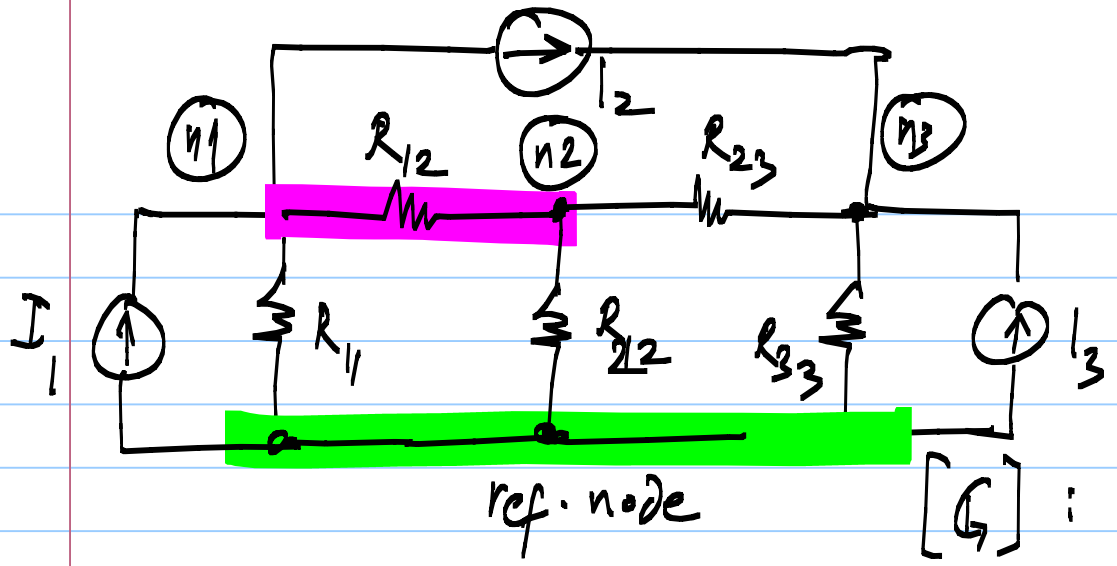
Modified nodal analysis (MNA)

variable vector:

$$\begin{bmatrix} \text{vector of node voltages } \underline{v_n} \\ \hline \text{vector of auxiliary branch currents } \underline{i_b} \end{bmatrix} = \underline{x}$$

$$[G]\,\underline{x} = \underline{I}$$

$[G]$: currents through voltage sources, controlling currents



LHS | RHS

$$n1 \begin{bmatrix} G_{12} & -G_{12} \\ -G_{12} & G_{12} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Big| \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$n2$

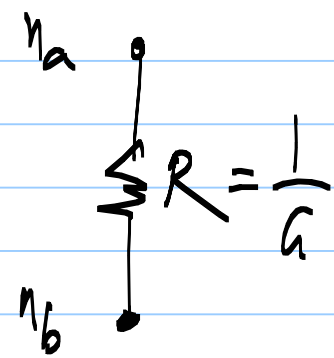
MNA stamp

$[G]$:

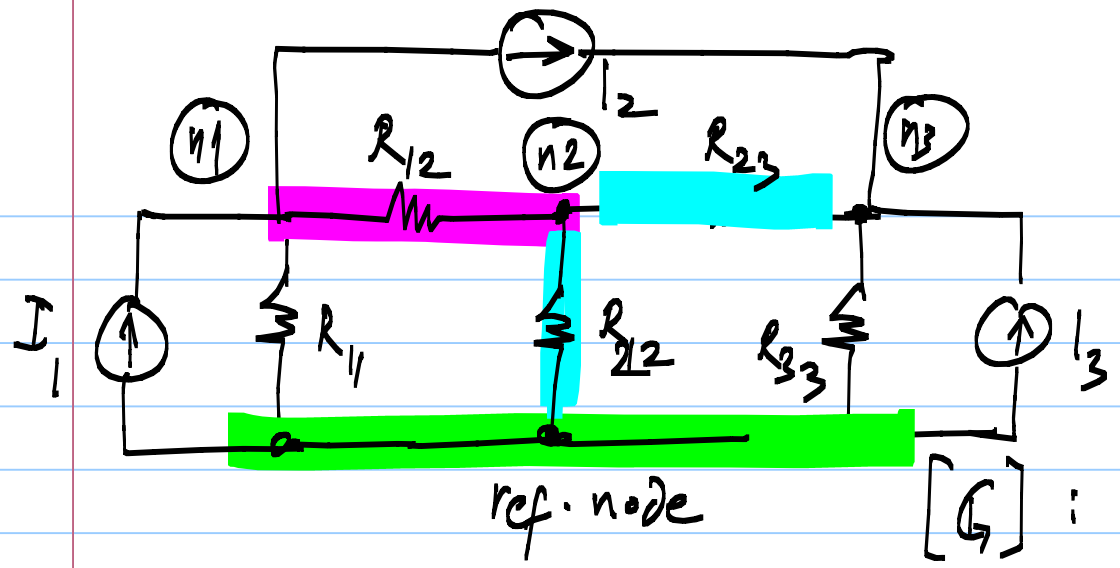
node

$$\begin{matrix} n1 \\ n2 \\ n3 \\ \\ v1 \end{matrix} \begin{bmatrix} G_{11}+G_{12} & -G_{12} & 0 \\ -G_{12} & G_{12}+G_{22}+G_{23} & -G_{23} \\ 0 & -G_{23} & +G_{33}+G_{23} \\ & v2 & v3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ 0 \\ I_2 + I_3 \end{bmatrix}$$

MNA stamp for a resistor:



$$\begin{array}{c} n_a \\ n_b \end{array} \begin{bmatrix} G & -G \\ -G & G \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} \quad \Bigg| \quad \overset{\text{RHS}}{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}$$



LHS | RHS

$$\begin{array}{c} \\ n2 \\ n3 \end{array} \begin{bmatrix} G_{23} & -G_{23} \\ -G_{23} & G_{23} \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 = 0 \end{bmatrix} \Big| \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

MNA stamp

$[G]$:

$$\begin{array}{c} \text{node} \\ n1 \\ n2 \\ n3 \\ \\ v1 \end{array} \begin{bmatrix} G_{11}+G_{12} & -G_{12} & 0 \\ -G_{12} & G_{12}+G_{22}+G_{23} & -G_{23} \\ 0 & -G_{23} & +G_{33}+G_{23} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ 0 \\ I_2 + I_3 \end{bmatrix} \quad \begin{array}{l} \\ \\ \\ \\ v2 \quad v3 \end{array}$$

	v_1	v_2	v_3
n_1	G_{12}	$-G_{12}$	
n_2	$-G_{12}$	$G_{12} + G_{22} + G_{23}$	$-G_{23}$
n_3		$-G_{23}$	$+G_{23}$