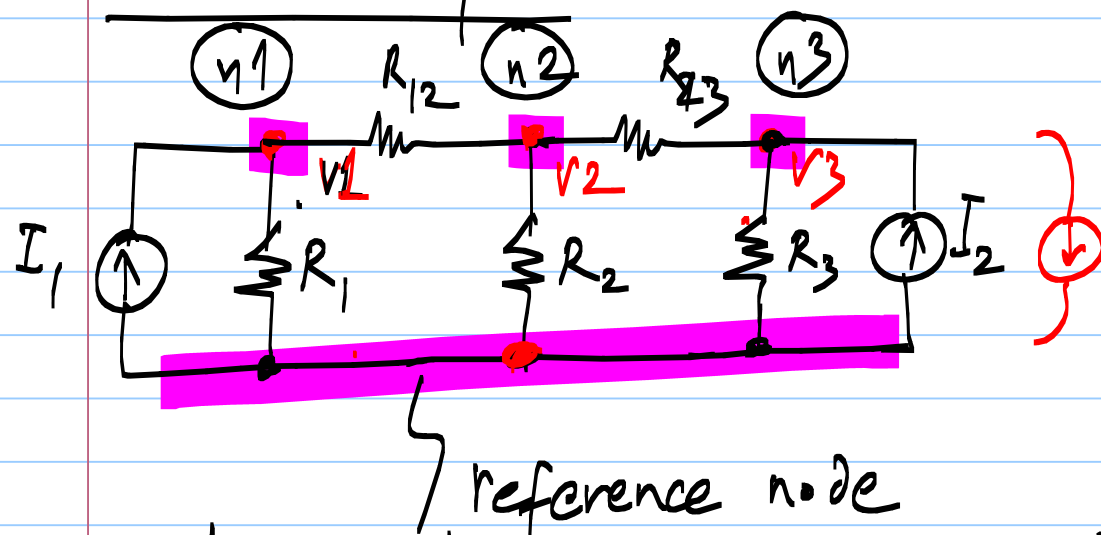
Nodal analysis: KCL at N-1 nodes



$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_{12}} = I_1$$

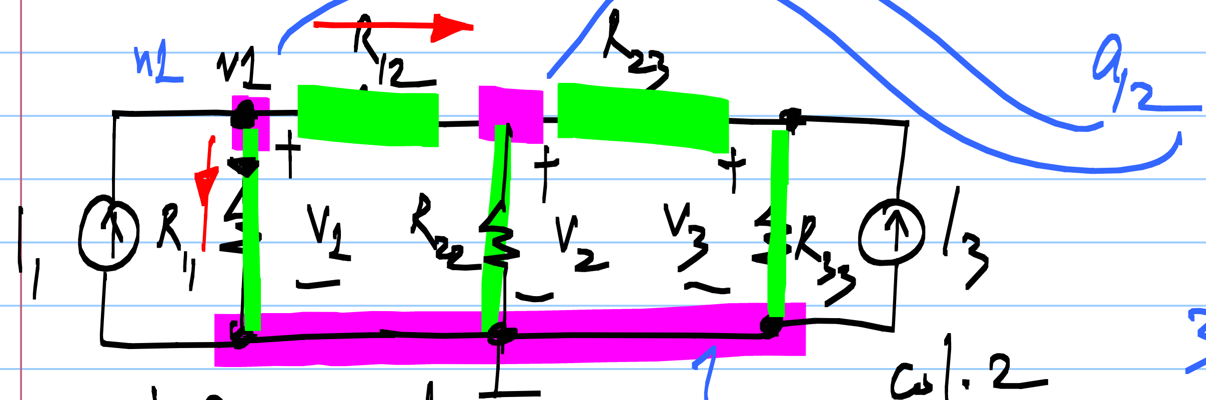
$$\frac{V_2 - V_1}{R_{12}} + \frac{V_2}{R_2} + \frac{V_2 - V_3}{R_{23}} = 0$$

$$\frac{V_3 - V_2}{R_2} + \frac{V_3}{R_3} = I_2$$

$$\begin{array}{c} n1 \\ n2 \\ n3 \end{array}
\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_{12}} & -\frac{1}{R_{12}} & 0 \\ -\frac{1}{R_{12}} & \frac{1}{R_{12}}+\frac{1}{R_2}+\frac{1}{R_{23}} & -\frac{1}{R_{23}} \\ 0 & -\frac{1}{R_{23}} & \frac{1}{R_3}+\frac{1}{R_{23}} \end{bmatrix}
\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ I_2 \end{bmatrix} \quad -I_2$$

EC1010: Lecture 6

Resistances & ind. current sources



$\underline{V}$: vector of (unknown) node voltages

row 1: (node 1)

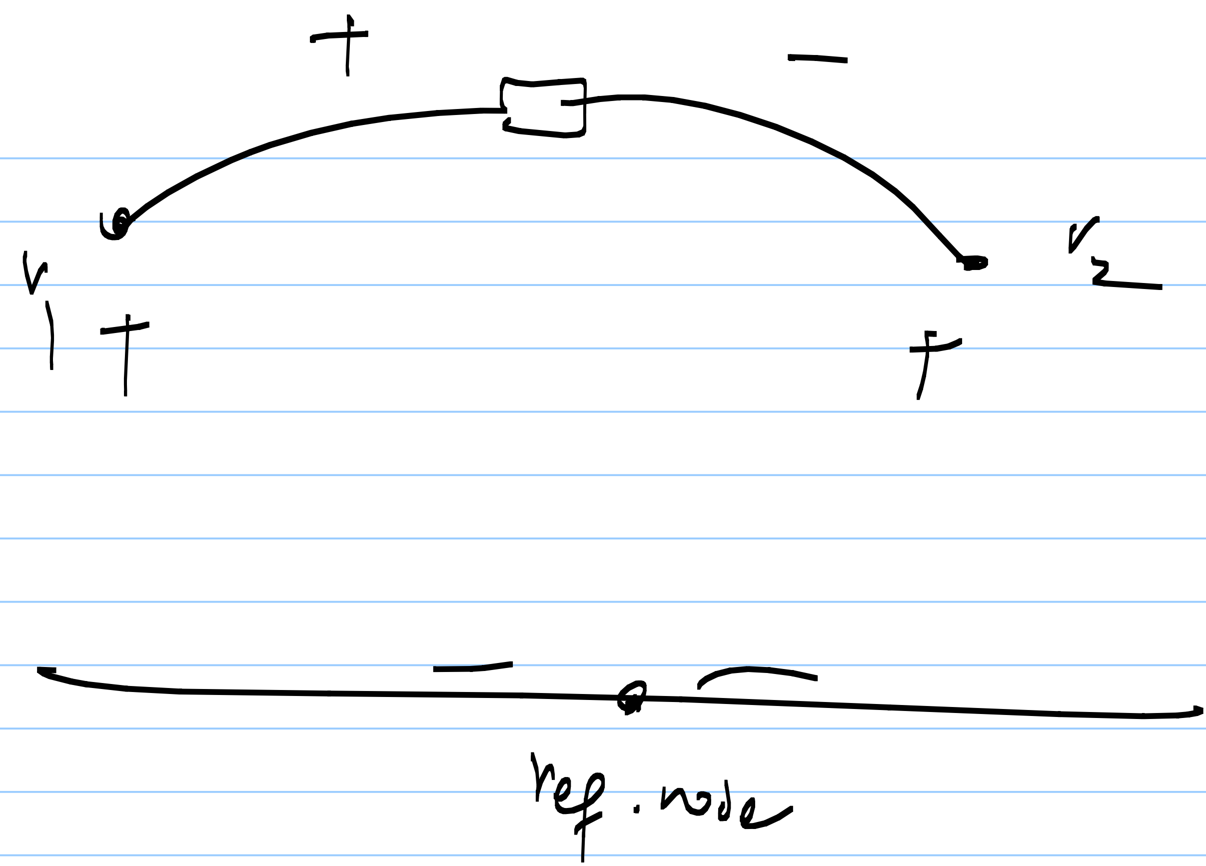
col. 2

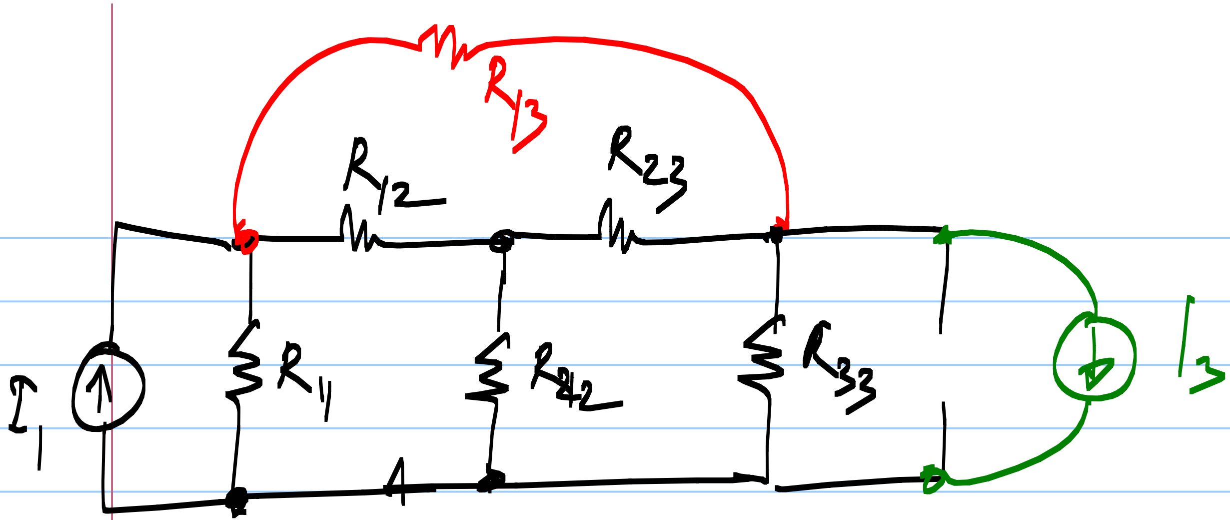
$$\begin{bmatrix} G_{11}+G_{12} & -G_{12} & 0 \\ -G_{12} & G_{12}+G_{22}+G_{23} & -G_{23} \\ 0 & -G_{23} & G_{23}+G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ I_3 \end{bmatrix}$$

source vector

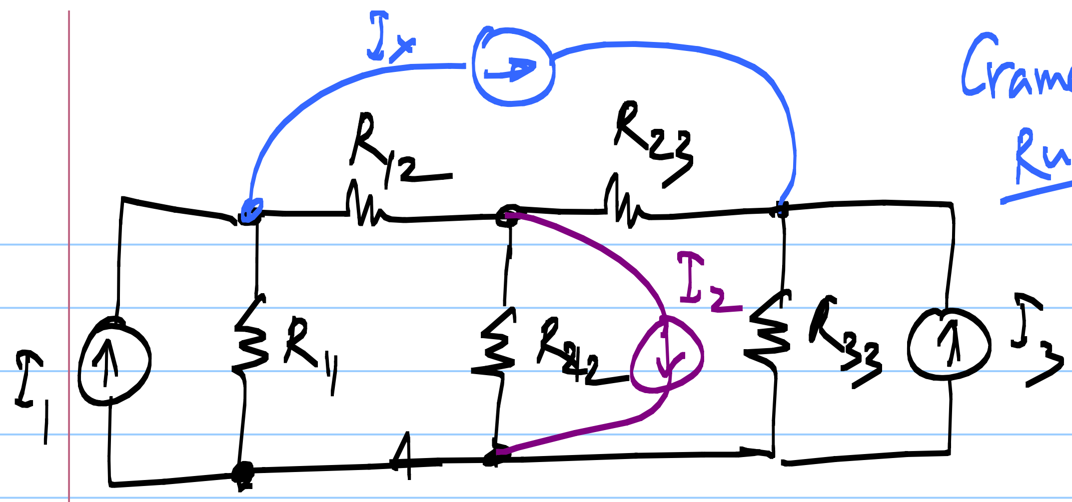
$$[G]\,\underline{V} = \underline{I}$$

$$\underline{V} = [G]^{-1} \cdot \underline{I}$$


+
−
v_1
+
+
v_2
−
−
ref. node



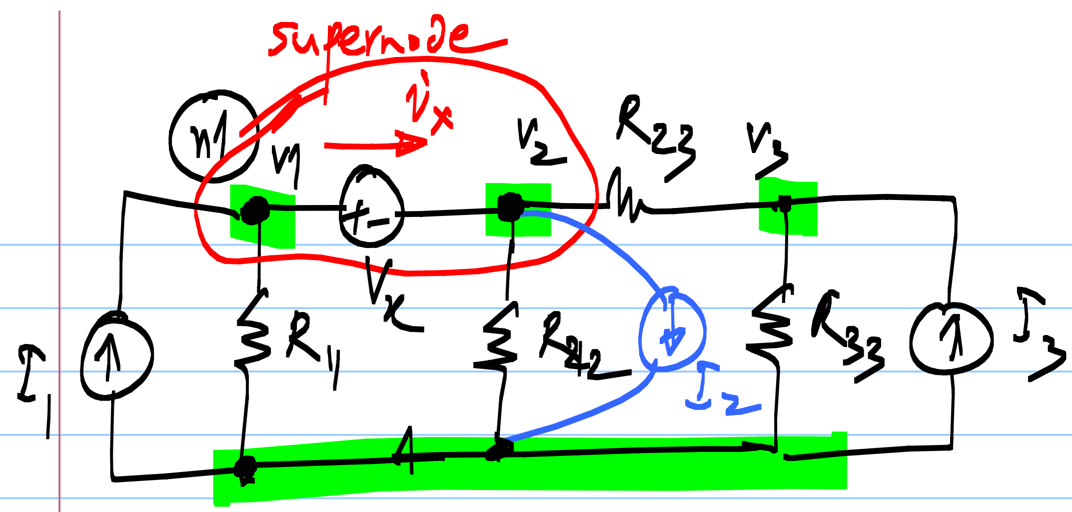
$$\begin{bmatrix} G_{11}+G_{12}+G_{13} & -G_{12} & \overset{-G_{13}}{0} \\ -G_{12} & G_{12}+G_{22}+G_{23} & -G_{23} \\ \underset{-G_{13}}{0} & -G_{23} & G_{13}+G_{23}+G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ I_3 \end{bmatrix} \begin{matrix} \\ \\ -I_3 \end{matrix}$$



Cramer's Rule

$$V_1 = \frac{\left|\begin{bmatrix} I_1 & -G_{12} & 0 \\ 0 & G_{12}+G_{22}+G_{23} & -G_{23} \\ I_3 & -G_{23} & G_{23}+G_{33} \end{bmatrix}\right|}{\left|\begin{bmatrix} & & \\ & G & \\ & & \end{bmatrix}\right|}$$

$$\begin{bmatrix} G_{11}+G_{12} & -G_{12} & 0 \\ -G_{12} & G_{12}+G_{22}+G_{23} & -G_{23} \\ 0 & -G_{23} & G_{23}+G_{33} \end{bmatrix} \begin{bmatrix} \underline{\underline{V_1}} \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ I_3 \end{bmatrix} \begin{matrix} -I_x \\ -I_2 \\ +I_x \end{matrix}$$



$$\boxed{v_1 - v_2 = V_x}$$

node 1 $\quad v_1 \cdot G_{11} + i_x = I_1$

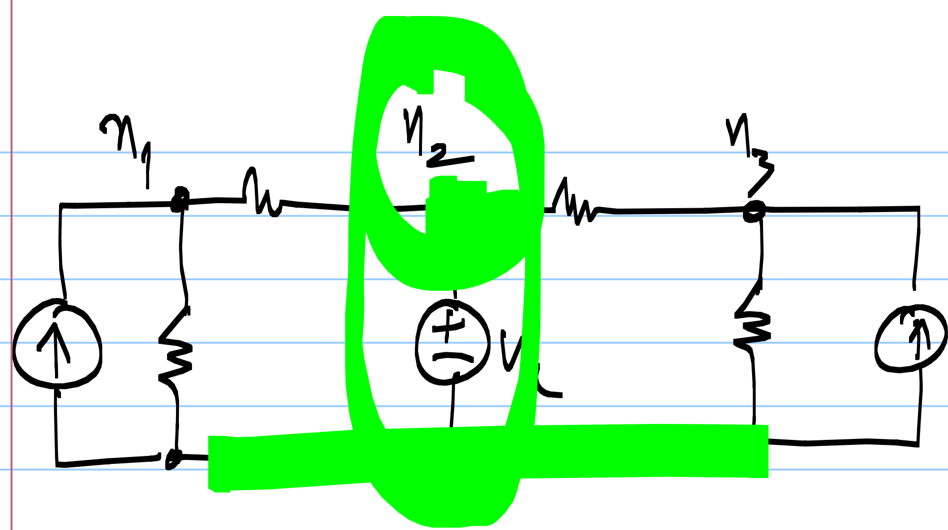
node 2 $\quad v_2 (G_{22} + G_{23}) - v_3 \cdot G_{23} - i_x = 0$

Supernode (1,2) $\quad \boxed{v_1 G_{11} + v_2 (G_{22} + G_{23}) - v_3 G_{23} = I_1} - I_2$

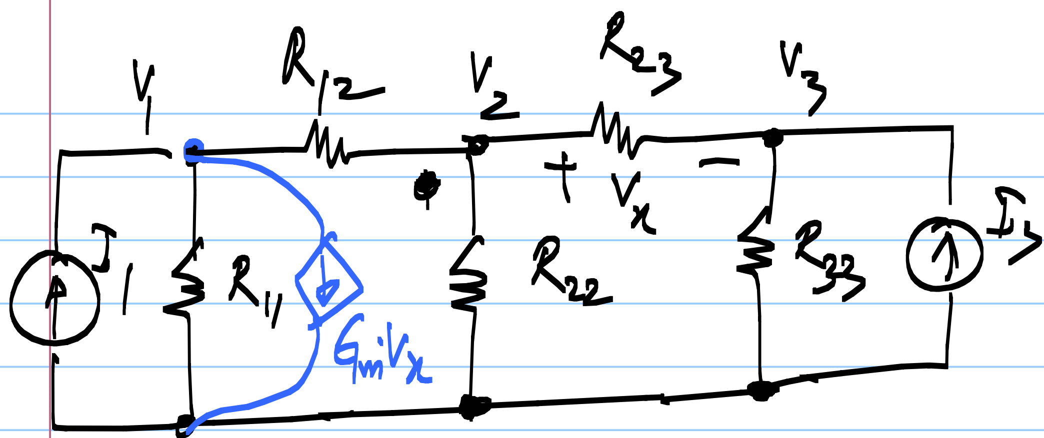
Nodal analysis:

* Circuits w/ conductances & ind. current sources
  symmetric $[G]$ matrix.

* Circuits w/ conductances, ind. current & ind. voltage sources
  - Combine nodes connected to voltage source into a supernode – single KCL eq. for it
  - Voltage source constraint



$V_2 = V_x$



$$\begin{bmatrix} G_{11}+G_{12} & -G_{12}+G_m & -G_m \\ -G_{12} & G_{12}+G_{22}+G_{23} & -G_{23} \\ 0 & -G_{23} & G_{23}+G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ I_3 \end{bmatrix}$$