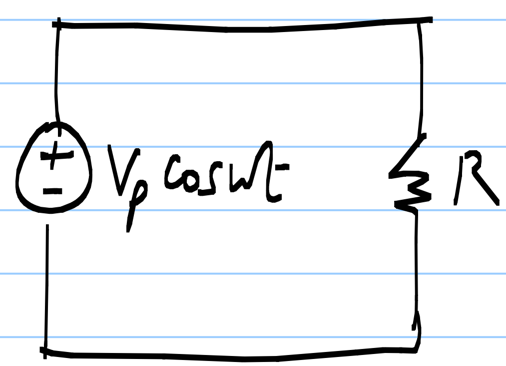
Lecture 31

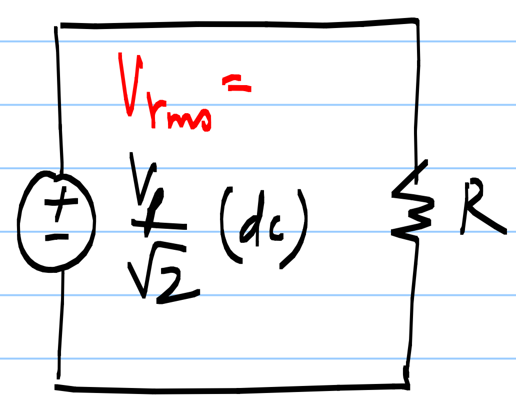


$$P_R = \frac{V_p^2}{2R}$$

mean squared voltage $= \frac{1}{T}\int_0^T (V_p \cos\omega t)^2 \, dt$

period

root mean squared voltage $= \sqrt{\frac{1}{T}\int_0^T (V_p \cos\omega t)^2 \, dt}$



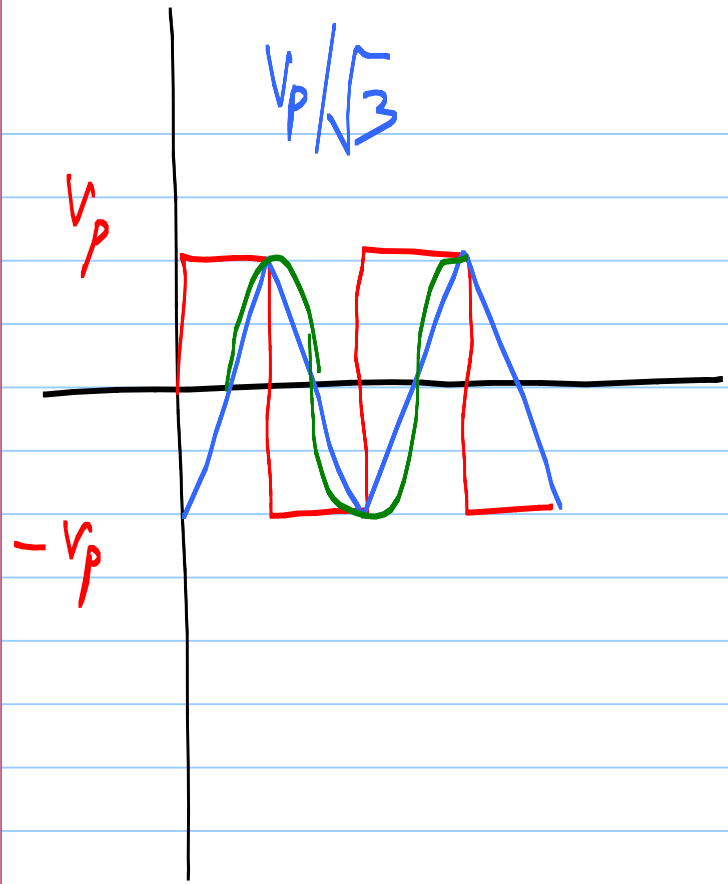
$$P_R = \frac{V_p^2}{2R}$$

$230 V_{rms}$

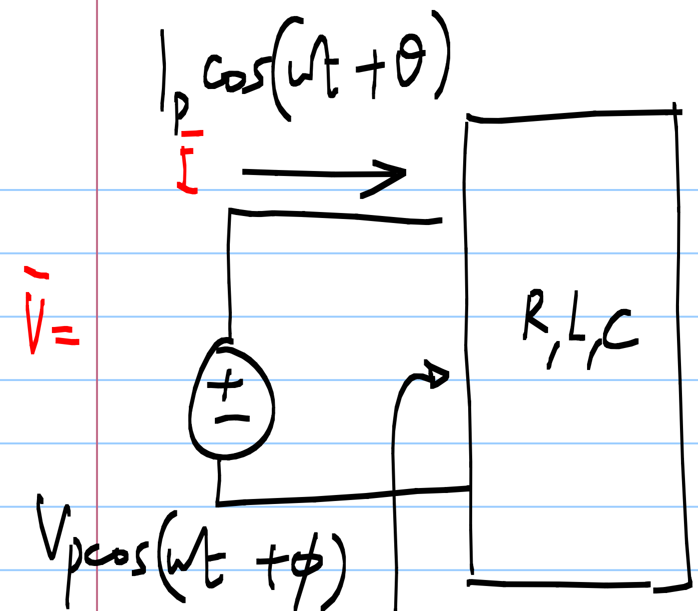
@ 50Hz

100π rad/s

$230\sqrt{2}$ V



$V_p/\sqrt{3}$
V_p
$-V_p$



$\bar{V} = V_p \exp(j\phi)$

$\bar{I} = I_p \exp(j\theta)$

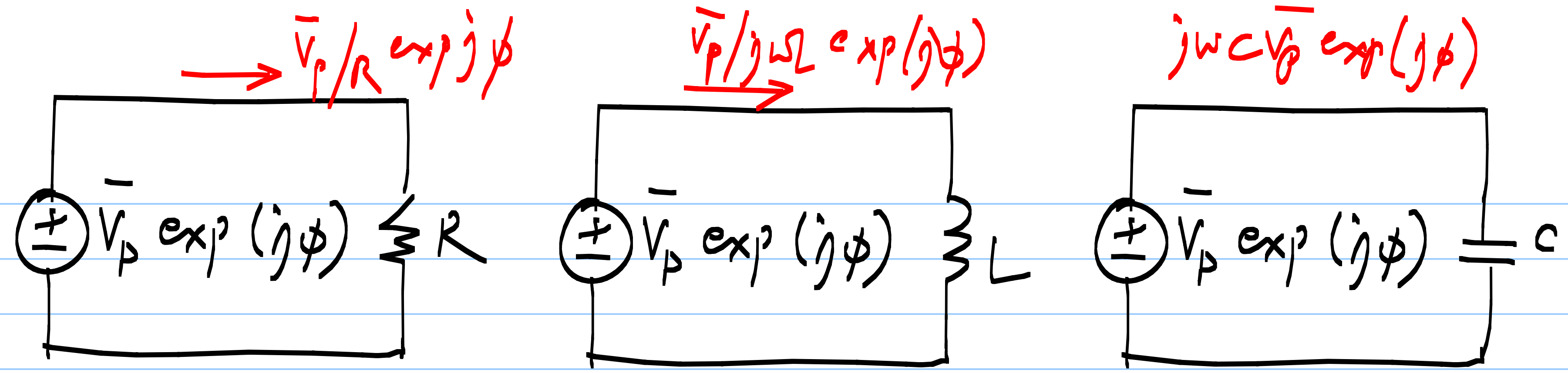
$$\frac{\bar{V}\cdot\bar{I}^*}{2} = \frac{V_p I_p}{2} \exp(j(\phi-\theta))$$

$$\text{Average Power} = \frac{V_p I_p}{2}\cos(\phi-\theta)$$

$\bar{P} = \dfrac{\bar{V}\cdot \bar{I}^*}{2}$ complex power

$\mathrm{Re}[\bar{P}] = \mathrm{Re}\left[\dfrac{\bar{V}\,\bar{I}^*}{2}\right]$ = Avg. power

$P_r + jP_i$



$$\bar{P} = \frac{V_p^2}{2R} \qquad j\frac{V_p^2}{2\cdot\omega L} \qquad -j\,\omega C\cdot\frac{V_p^2}{2}$$

+ve for an Inductor

−ve for a Capacitor

$$j\frac{V_p^2}{2X_L} \qquad +j\frac{V_p^2}{2X_C}$$

$$\bar{P} = P_r + jP_i$$

P_i: Reactive power V·A

P_r: Real power, (~ work) W

$$\bar{V} = V_p \exp(j\phi), \quad \bar{I} = I_p \exp(j\theta)$$

$$\bar{P} = \frac{V_p I_p}{2} \exp(j(\phi-\theta)) = \underbrace{\frac{V_p I_p}{2}} \left[\cos(\phi-\theta) + j\sin(\phi-\theta)\right]$$

$$|\bar{P}| = \frac{V_p I_p}{2} \quad (\text{V.A}, \text{ apparent power})$$

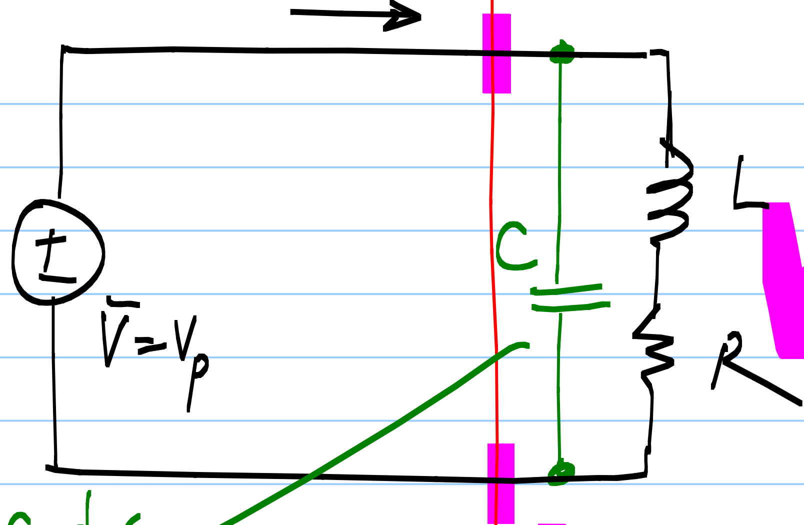
$$\mathrm{Re}[\bar{P}] = \frac{V_p I_p}{2} \cos(\phi-\theta) \quad (\text{W}, \text{ real power})$$

$$\mathrm{Im}[\bar{P}] = \frac{V_p I_p}{2} \sin(\phi-\theta) \quad (\text{VA}, \text{ reactive power})$$

$$\frac{\text{Real power}}{\text{Apparent power}} = \frac{Re[\bar{p}]}{|\bar{p}|} = \underset{PF}{\text{Power factor}}$$

$$0 \leq PF \leq 1$$

$\bar{i} = V_p / R + j\omega L \;\; + V_p j\omega C$



$$\frac{\bar{V}\cdot\bar{I}^*}{2} = \frac{V_p^2}{2(R - j\omega L)}$$

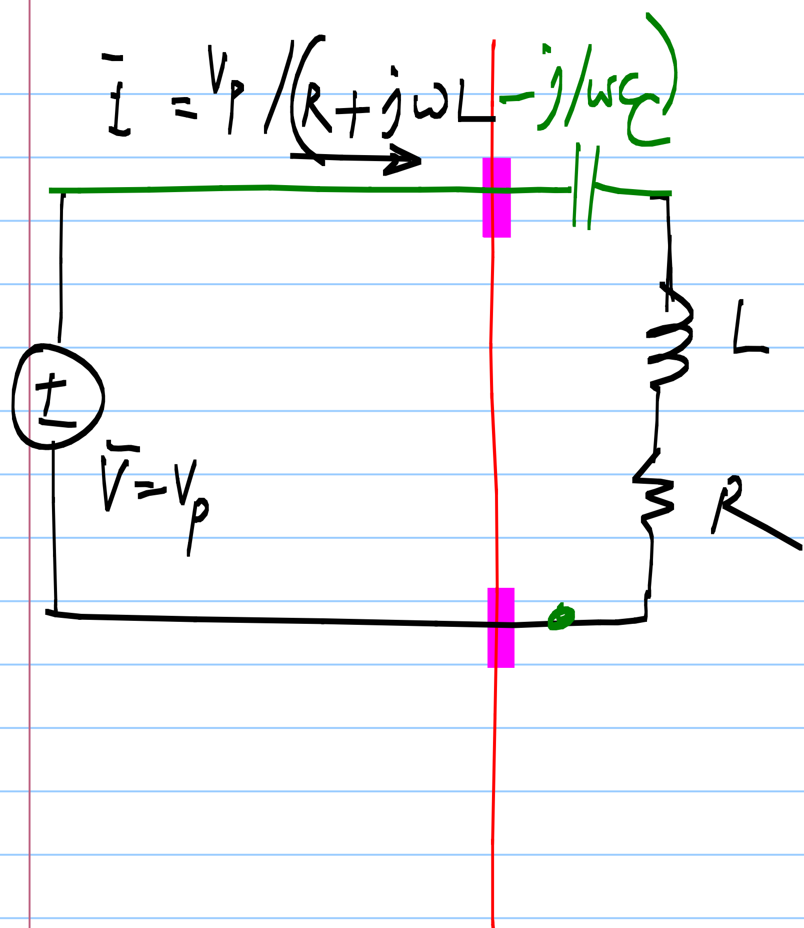
$$= \frac{V_p^2}{2}\left[\frac{R}{R^2+\omega^2L^2} + \frac{j\omega L}{(R^2+\omega^2L^2)}\right]$$

Power factor correction capacitor

$$\bar{I} = V_p\left[\frac{R}{R^2+\omega^2L^2} - \frac{j\omega L}{R^2+\omega^2L^2} + j\omega C\right]$$

Parallel:

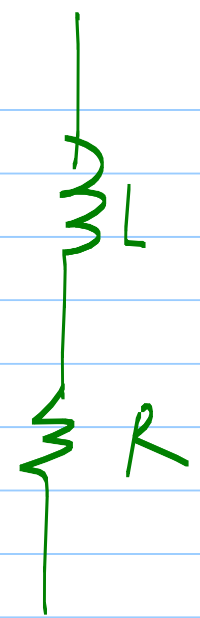
$$C = \frac{L}{R^2+\omega^2L^2}$$



$$\bar{I} = \frac{V_p}{R + \underline{\underline{j\omega L}} - \underline{\underline{j/\omega C}}}$$

$$C = \frac{1}{\omega^2 L}$$

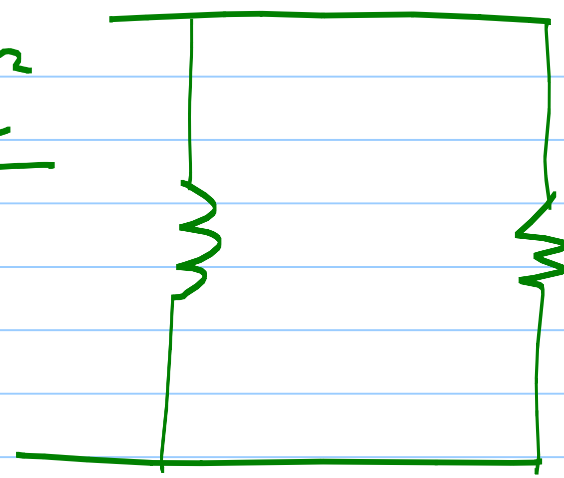
$Z \rightarrow$



L, R

$= R + j\omega L$

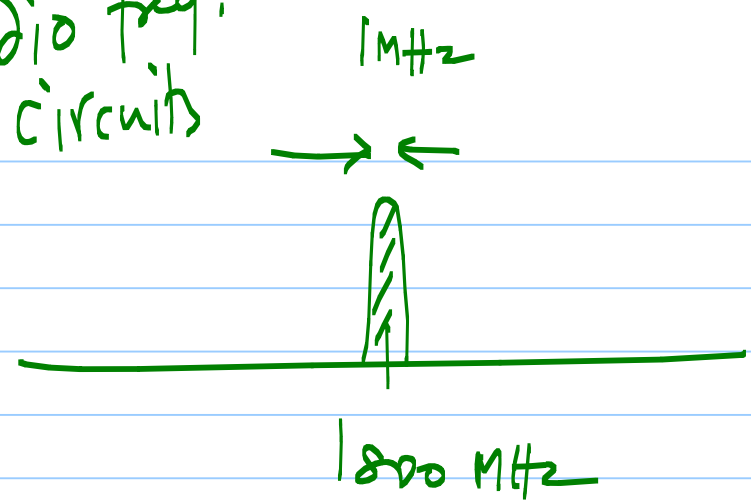
$\xrightarrow[\text{frequency } \underline{\underline{\omega}}]{@\, a}$



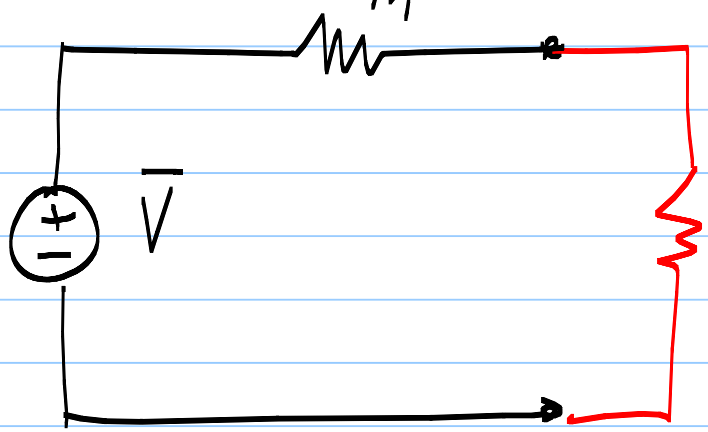
$\frac{R^2+\omega^2L^2}{\omega^2 L}$ $\qquad$ $\frac{R^2+\omega^2L^2}{R}$

$$Y = \frac{1}{Z} = \frac{1}{R+j\omega L} = \frac{R}{R^2+\omega^2L^2} - \underbrace{\frac{j\omega L}{R^2+\omega^2L^2}}$$

Radio freq. circuits



$$\bar{Z}_{th} = R_{th} + jX_{th}$$



$$\bar{Z}_L = \underbrace{R_L + jX_L}$$

$$Re\left[\frac{1}{2}\cdot\bar{V}\cdot\underbrace{\frac{Z_L}{Z_L+Z_{th}}\cdot\frac{\bar{V}^*}{Z_L^*+Z_{th}^*}}\right] = \frac{|\bar{V}|^2\cdot Re[Z_L]}{2\cdot|Z_L+Z_{th}|^2}$$

$$P_L = \frac{|\tilde{V}|^2}{2R_{th}} \cdot \frac{R_L R_{th}}{(R_{th}+R_L)^2 + (X_{th}+X_L)^2}$$

adjust R_L, X_L to maximize P_L

$$= \frac{|\tilde{V}|^2}{2R_{th}} \cdot \frac{1}{\underbrace{\left(\sqrt{\frac{R_{th}}{R_L}} + \sqrt{\frac{R_L}{R_{th}}}\right)^2}_{R_L = R_{th}} + \underbrace{\frac{(X_{th}+X_L)^2}{R_{th}R_L}}_{X_L = -X_{th}}} = \frac{|\bar{V}|^2/2}{4R_{th}}$$

$P_{L,max}$

Available power

$$Z_L = Z_{th}^*$$

Conjugate matching

$+X$