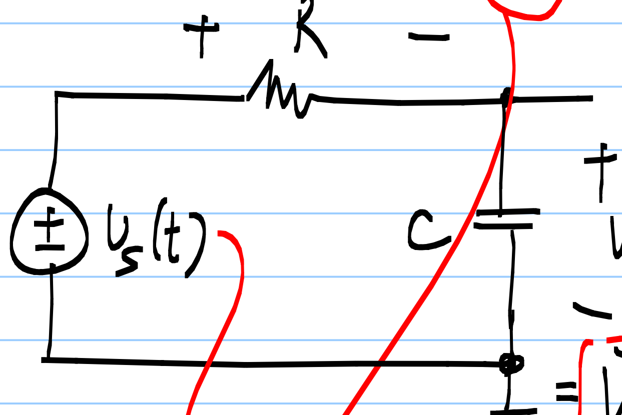
# Lecture 28




$v_R(t) = \bar{V}_R \cdot \exp(j\omega t)$

$v_c(t) = \bar{V}_c \cdot \exp(j\omega t)$

$V_p \cos(\omega t + \phi)$

$V_p \exp(j\phi) \cdot \exp(j\omega t)$

Constants

Can be written in terms of phasors

* KVL, KCL equations

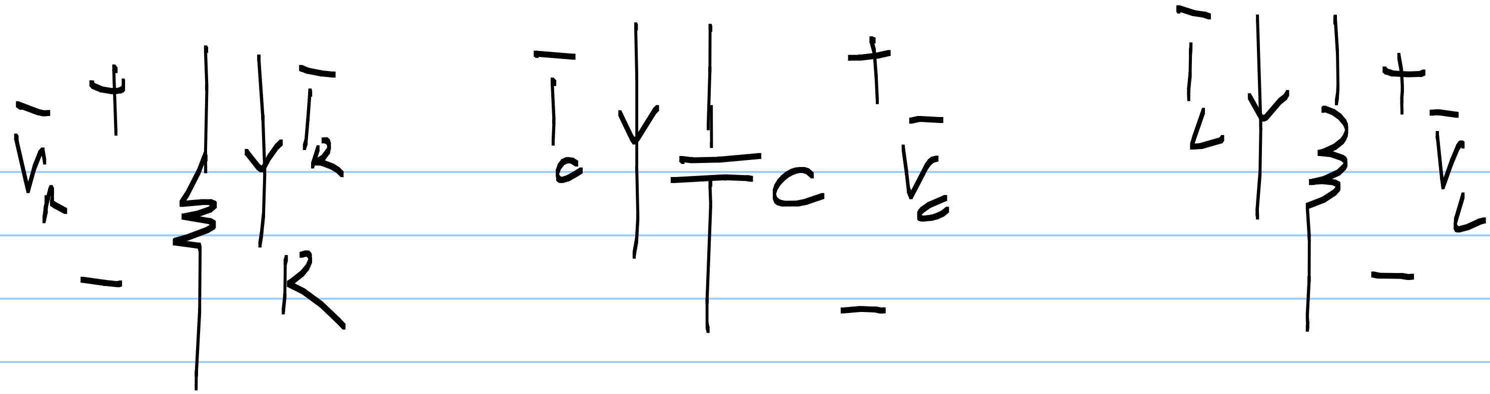
* Response to $V_p \cos(\omega t + \phi)$

$= \text{Real}\{\text{Response to } V_p \exp(j\phi) \exp(j\omega t)\}$

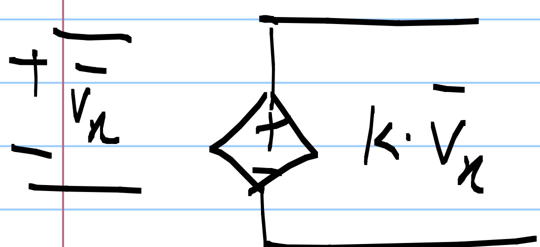
* Steady-state response of a linear circuit to $\exp(j\omega t)$

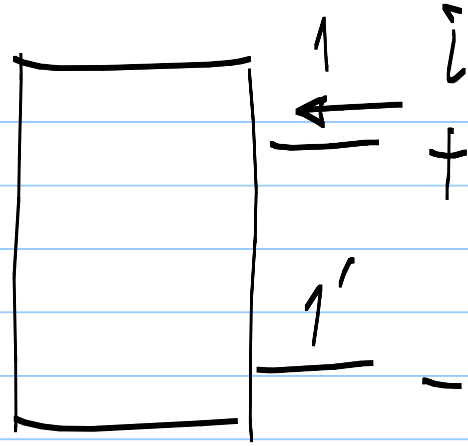
$= (\quad) \cdot \exp(j\omega t)$

Complex number




$$\underbrace{\bar{V}_R = R \cdot \bar{I}_R \qquad \bar{V}_c = \frac{1}{j\omega C} \cdot \bar{I}_c \qquad \bar{V}_L = j\omega L \cdot \bar{I}_L}_{\text{Element relationships } \sim \text{ Ohm's law}}$$






$$i = \bar{I} \cdot \exp(j\omega t)$$

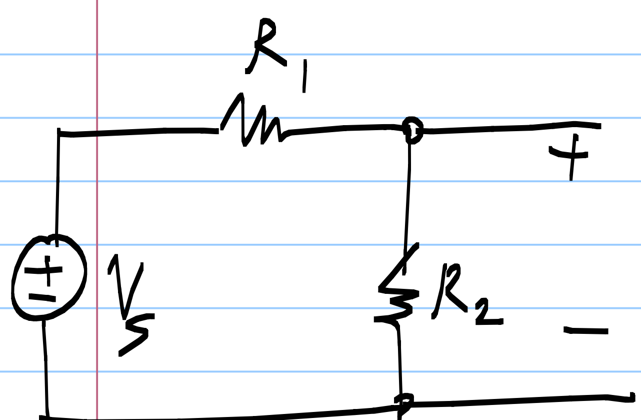
$$v = \bar{V} \cdot \exp(j\omega t)$$

$$\frac{\bar{V}}{\bar{I}} = \bar{Z} = R + jX$$

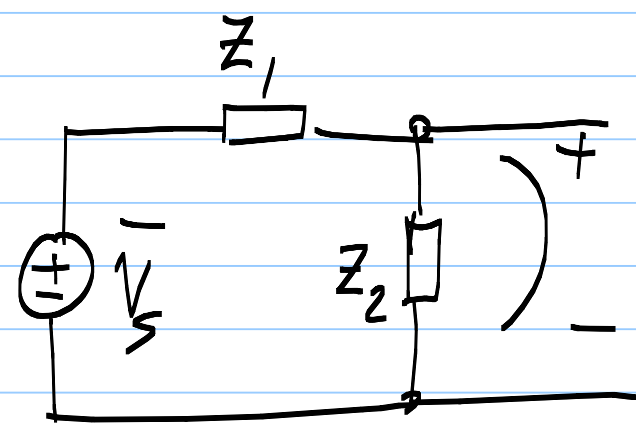
impedance

$$\frac{\bar{I}}{\bar{V}} = \bar{Y} = G + jB$$

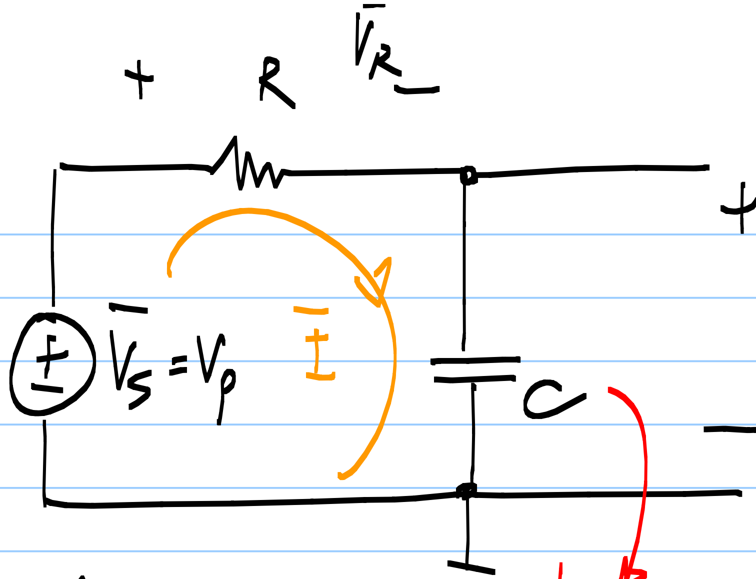
admittance




$$V_s \cdot \frac{R_2}{R_1 + R_2}$$




$$\bar{V}_s \cdot \frac{Z_2}{Z_1 + Z_2}$$




$V_p \cos \omega t$

$V_p \sin \omega t$

$\frac{1}{j\omega C}$

$$\bar{V}_s \cdot \frac{1/j\omega C}{R + 1/j\omega C}$$

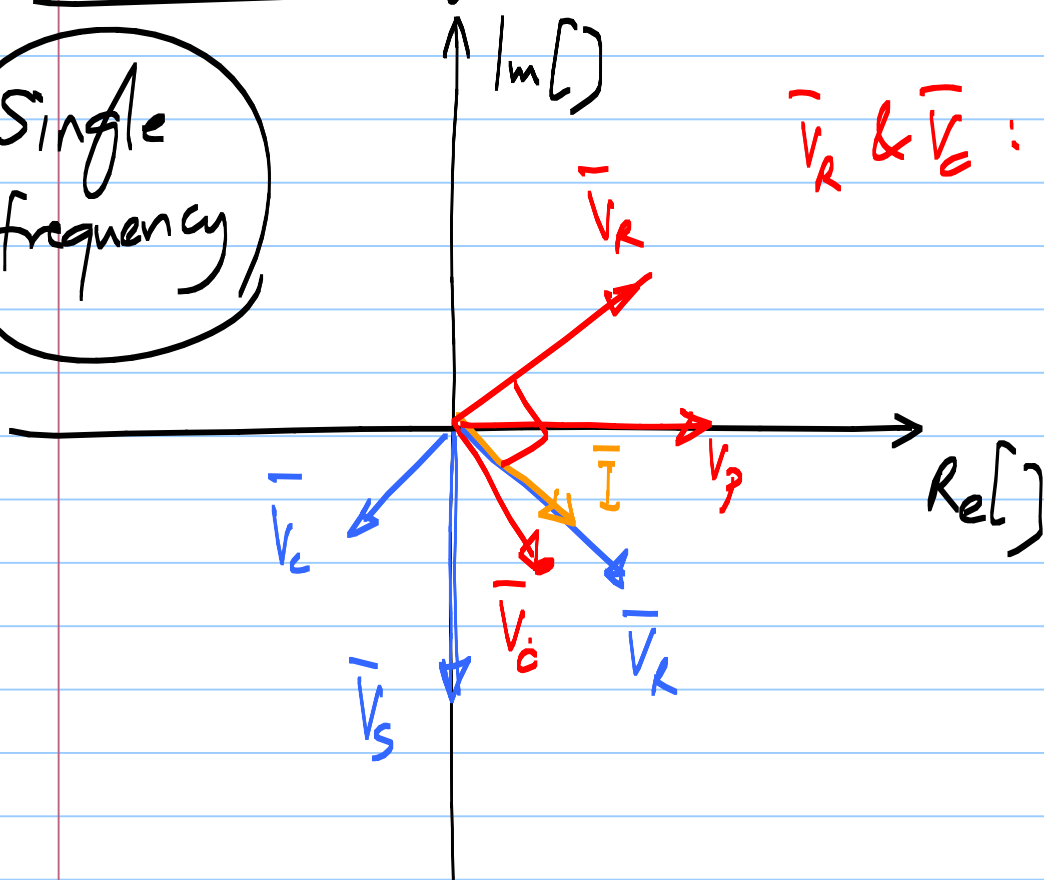
$$= \bar{V}_s \underbrace{\left( \frac{1}{1 + j\omega CR} \right)}$$

Phasor of the output

$$\bar{V}_R = \bar{V}_s \cdot \frac{j\omega CR}{1 + j\omega CR}$$

## Phasor diagram: phasors on the complex plane

Single frequency




$\bar{V}_R$ & $\bar{V}_c$ : 90° wrt each other

cos(ωt)

R: $\bar{V}_R$, $\bar{I}_R$ coincident

L: $\bar{I}_L$ lags $\bar{V}_L$ by 90°

C: $\bar{I}_c$ leads $\bar{V}_c$ by 90°

$$\frac{\bar{V}_c}{\bar{V}_s} = \frac{1}{1 + j\omega CR}$$

Transfer function

Magnitude $= \dfrac{1}{\sqrt{1+(\omega CR)^2}}$

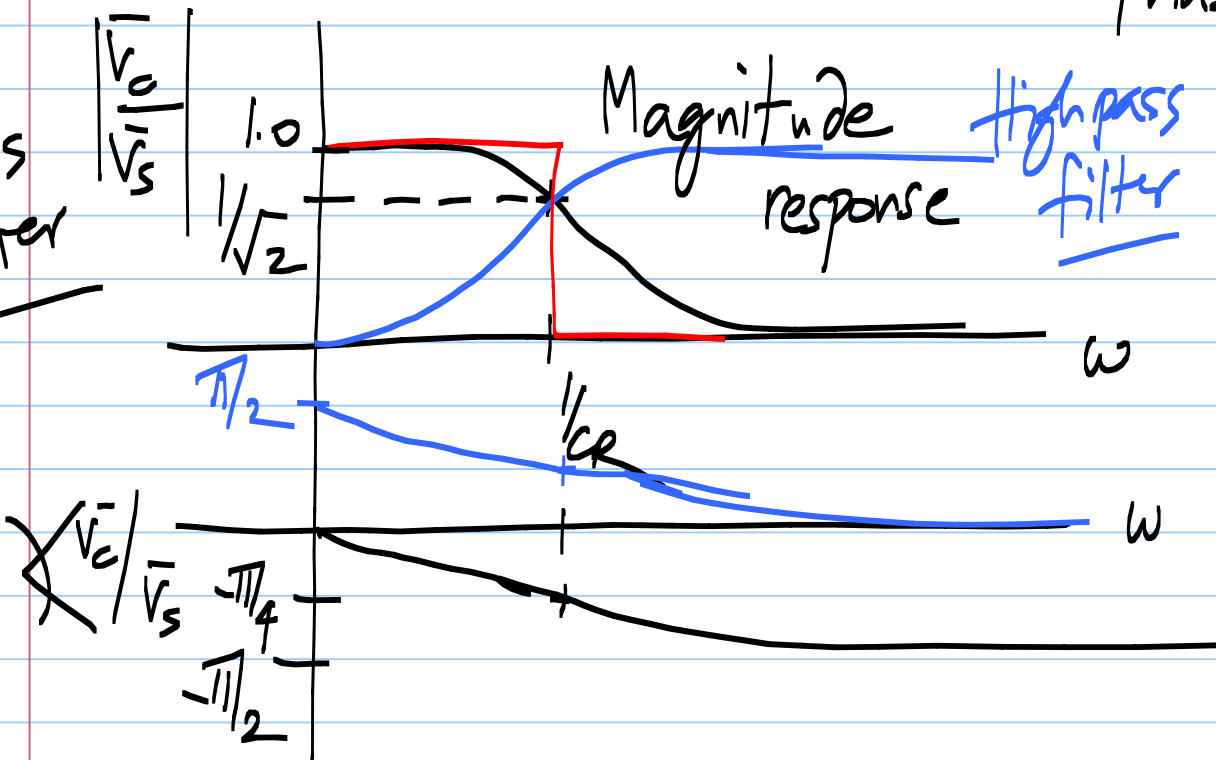
phase $= -\tan^{-1}(\omega CR)$

$\frac{\pi}{2} - \tan^{-1}(\omega CR)$

$\simeq 1, \quad \omega \ll \frac{1}{CR}$

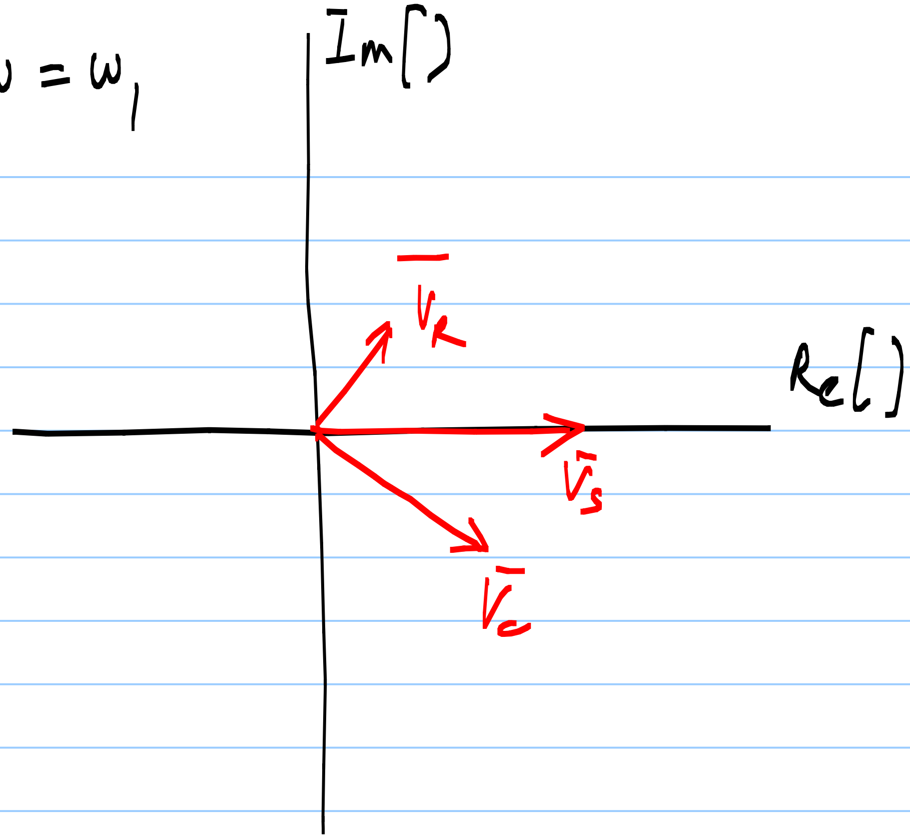
$\simeq \frac{1}{\omega CR} \quad \omega \gg \frac{1}{CR}$

Low pass filter

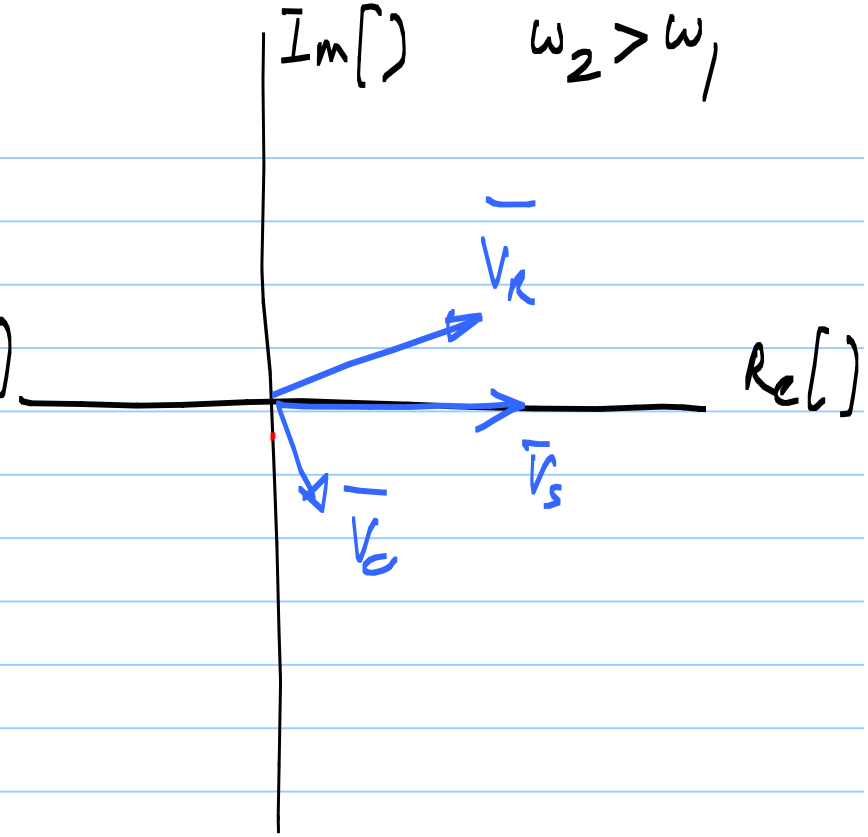


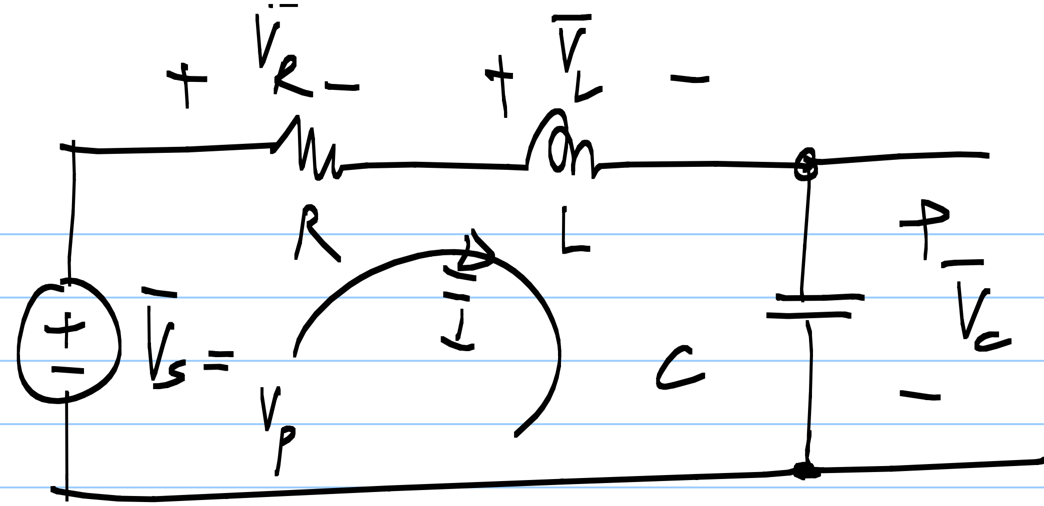

$$\frac{\bar{V}_R}{\bar{V}_s} = \frac{j\omega CR}{1 + j\omega CR}$$

$\omega = \omega_1$




$\omega_2 > \omega_1$






$$\bar{V}_s = V_p + j0$$
$$= V_p \angle 0$$

* Draw phasor diagram with $\bar{V}_s, \bar{V}_R, \bar{V}_L, \bar{V}_c, \bar{I}$
* Draw $\left|\frac{\bar{V}_c}{\bar{V}_s}\right|$ & $\measuredangle \frac{\bar{V}_c}{\bar{V}_s}$ versus $\omega$