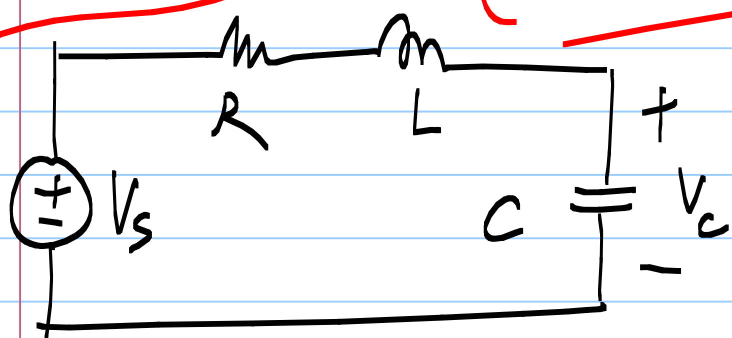
Lecture 27

single loop of series R, L, C (nulled ckt)



Series R, LC circuit

$$LC \frac{d^2 V_c}{dt^2} + RC \cdot \frac{dV_c}{dt} + V_c = V_s$$

Zero input response $(V_s = 0)$

$$a_2 \frac{d^2 V_c}{dt^2} + a_1 \frac{dV_c}{dt} + V_c = 0$$

$$a_2 p^2 + a_1 p + 1 = 0$$

real & distinct: $k_1 \exp(p_1 t) + k_2 \exp(p_2 t)$

real, repeated: $(k_1 + k_2 t) \exp(p_1 t)$

complex conj.: $k_1 \exp(p_r t) \cdot \exp(j p_i t) + k_1^* \exp(p_r t) \exp(-j p_i t)$

real & distinct: $k_1 \exp(p_1 t) + k_2 \exp(p_2 t)$

real, repeated: $(k_1 + k_2 t)\exp(p_1 t)$ $\qquad k_0 = |k_1|$

complex conj.: $2 \cdot k_0 \exp(p_r t) \cdot \cos(p_i t + \phi_k)$ $\qquad \phi_k = \measuredangle k_1$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$a_2 p^2 + a_1 p + 1$$

$$\approx p^2 + 2\zeta \omega_n \cdot p + \omega_n^2$$

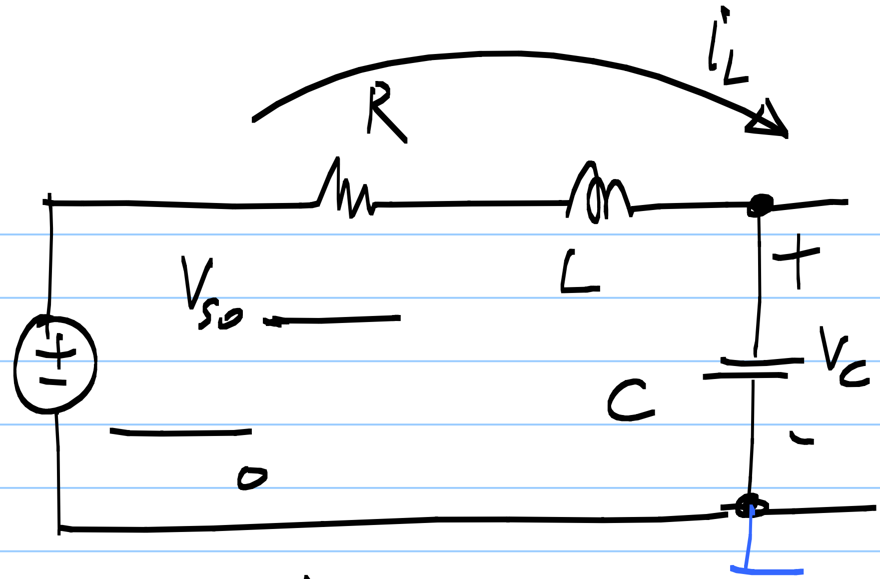
$$p^2 + \frac{\omega_n \cdot p}{Q} + \omega_n^2$$

natural frequency

damping factor

Quality factor

$$Q = \frac{1}{2\zeta}$$

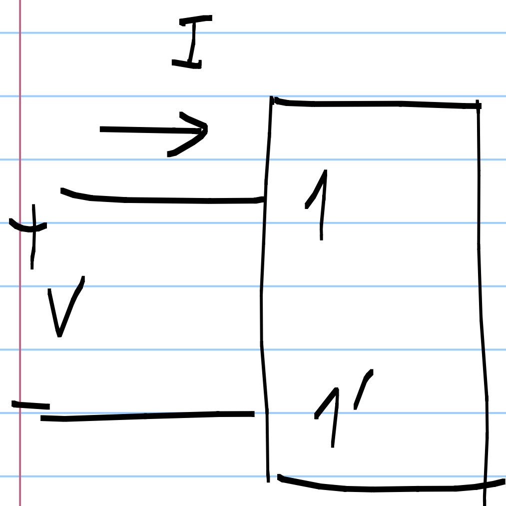


Critically damped:

Sinusoidal steady state response:

G: Conductance

B: Susceptance



$V_p \exp(j\omega t)$

$I_p \exp(j(\omega t + \phi))$

(steady state)

$$\frac{\bar{V}}{\bar{I}} = \bar{Z} = R + jX$$

(Impedance)

Reactance

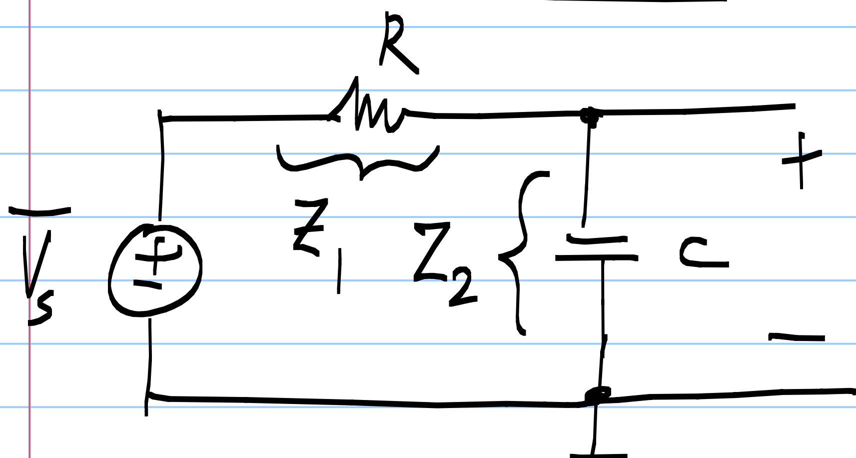
Resistance

$$\frac{\bar{I}}{\bar{V}} = \bar{Y} = G + jB$$

(Admittance)

	$\bar{Z}$	$\bar{Y}$
Resistor R	$R + j0$	$\frac{1}{R} + j \cdot 0$
Inductance L	$0 + j\omega L$	$0 - \frac{j}{\omega L}$
Capacitance C	$0 - \frac{j}{\omega C}$	$0 + j\omega C$

First order (RC)



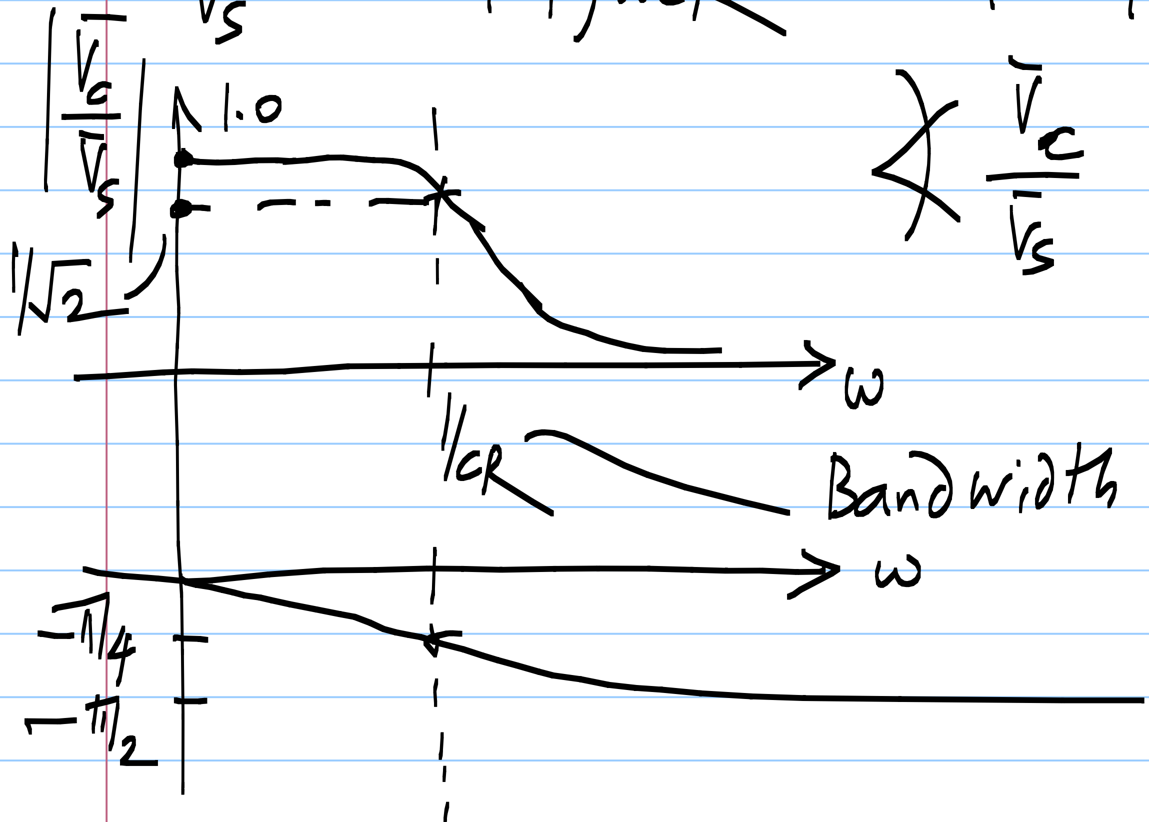
$$\bar{V}_c = \frac{Z_2}{Z_1 + Z_2} \cdot \bar{V}_s$$

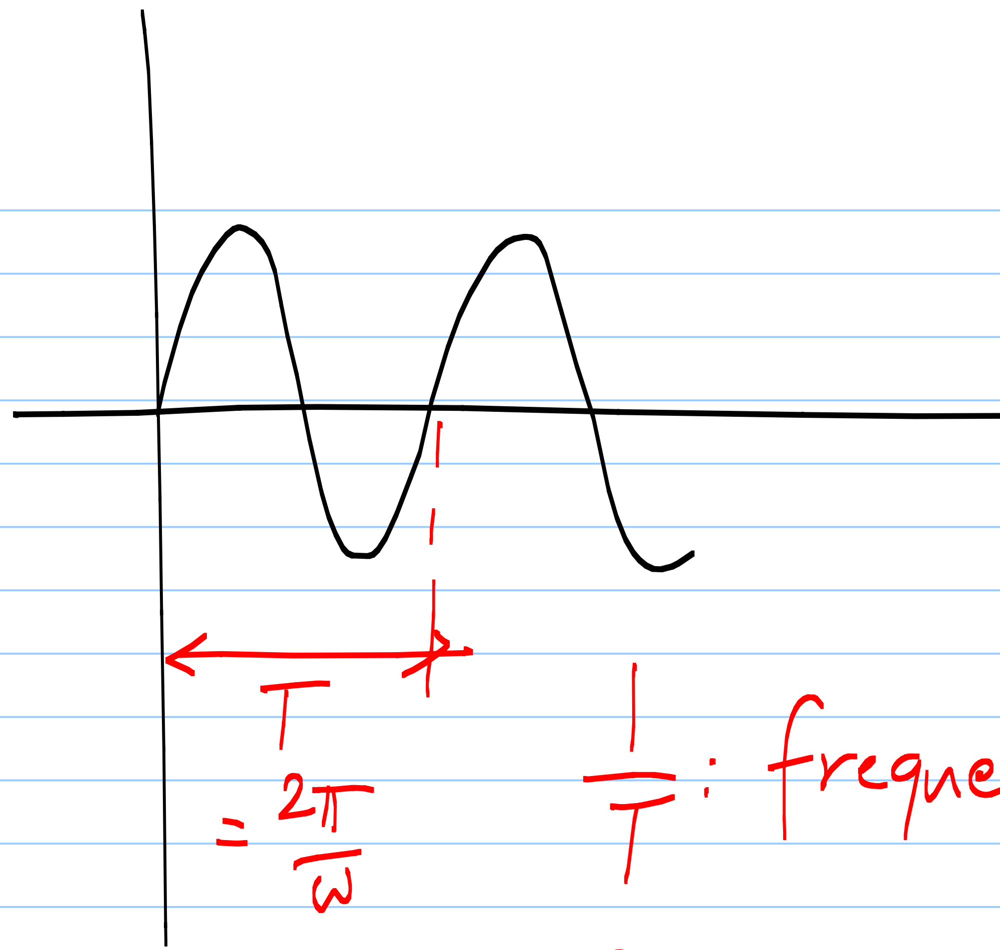
$V_s = V_p \cos\omega t$

$$= \frac{-j/\omega C}{R - j/\omega C} = \frac{1}{1 + j\omega CR}$$

$$\frac{\bar{V}_c}{\bar{V}_s} = \frac{1}{1+j\omega CR}, \quad \left|\frac{\bar{V}_c}{\bar{V}_s}\right| = \frac{1}{\sqrt{1+(\omega CR)^2}}$$

$$\angle \frac{\bar{V}_c}{\bar{V}_s} = -\tan^{-1}(\omega CR)$$





$T = \frac{2\pi}{\omega}$

$\frac{1}{T}$: frequency (Hz) $= f$

$\frac{2\pi}{T}$: freq. (rad/s) ω