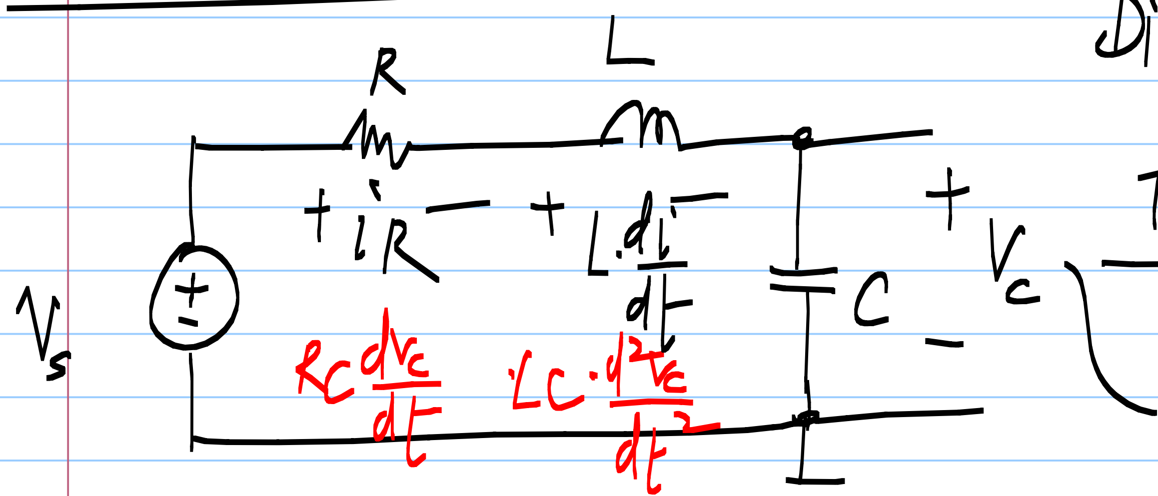
Lecture 26



Differential equation for this circuit (V_c)

$$LC\frac{d^2V_c}{dt^2} + RC\frac{dV_c}{dt} + V_c = V_s(t)$$

Homogeneous equation:

$$LC\frac{d^2v_c}{dt^2} + RC\frac{dv_c}{dt} + v_c = 0$$

$$LC\cdot p^2 + RC\cdot p + 1 = 0$$

Two values p_1, p_2

$$\downarrow$$

$$k_1\cdot \exp(p_1 t) + k_2\cdot \exp(p_2 t)$$

$$\exp(p\cdot t)$$

$$RC\frac{dv_c}{dt} + v_c = 0$$

$$RC\cdot p\cdot \exp(pt) + \exp(pt) = 0$$

$$RC\cdot p + 1 = 0$$

$$p = -\frac{1}{RC}$$

Characteristic equation

$$LC \cdot p^2 + RC \cdot p + 1 = 0$$

→ Two values p_1, p_2

$$p_{1,2} = \frac{-RC \pm \sqrt{R^2C^2 - 4LC}}{2LC}$$

$$= -\frac{R}{2L} \pm \sqrt{\underbrace{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}_{>0}}$$

Real, distinct roots:

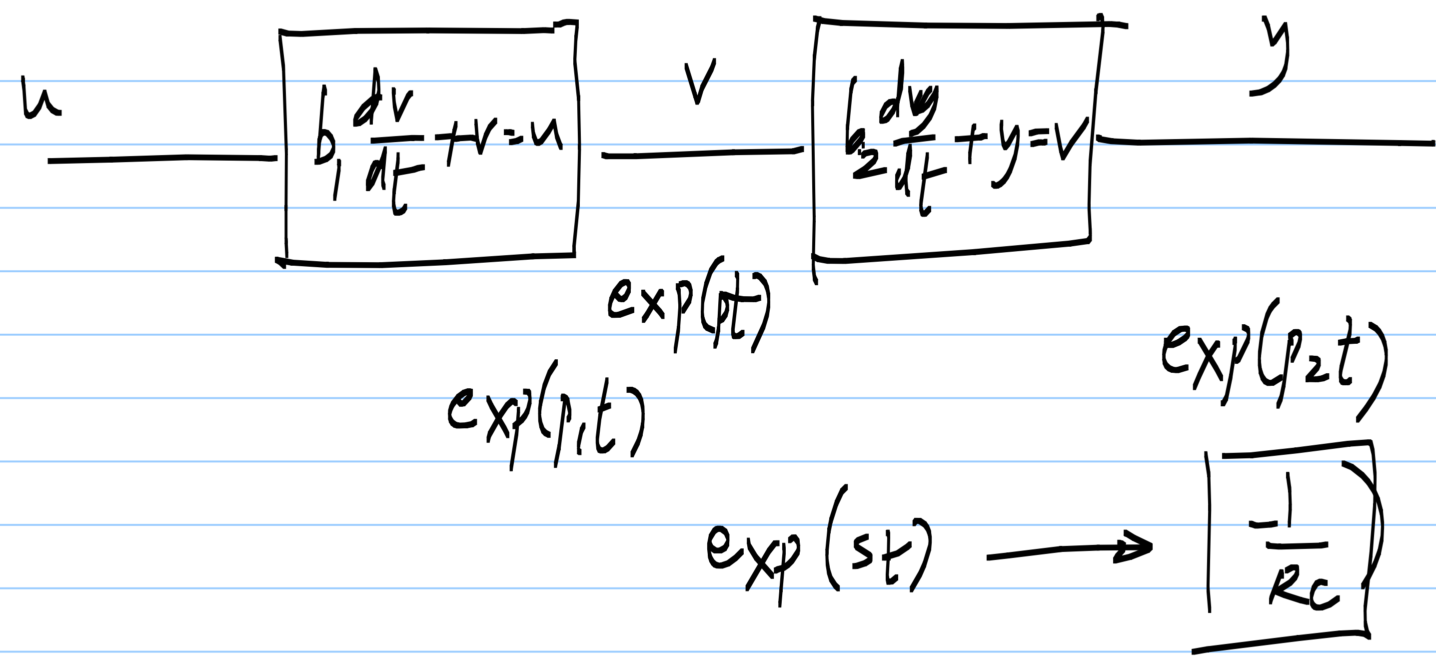
Solution. $k_1 \exp(p_1 t) + k_2 \exp(p_2 t)$

$$\left(\frac{R}{2L}\right)^2 - \frac{1}{LC} > 0$$

Identical roots: $(k_1 + k_2 t)\exp(p_1 t)$

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

$$\frac{R}{2}\sqrt{\frac{C}{L}} > 1 \quad ; \quad \frac{1}{R}\sqrt{\frac{L}{C}} < \frac{1}{2}$$



u
$b_1 \frac{dv}{dt} + v = u$
v
$b_2 \frac{dy}{dt} + y = v$
y
exp(pt)
exp($p_1 t$)
exp($p_2 t$)
exp(st)
$-\frac{1}{RC}$

Complex conjugate roots:

$$p_{1,2} = -\frac{R}{2L} \pm j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = p_r \pm j\, p_i$$

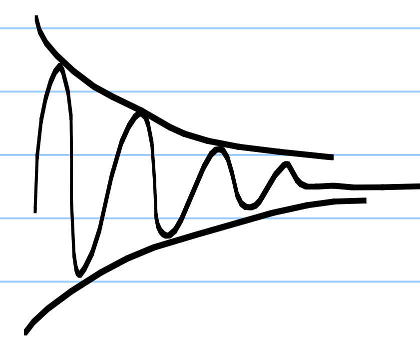
$$v_c(t) = k_1 \exp(p_1 t) + k_2 \exp(p_2 t)$$

$$= k_0 \exp(p_r t) \left[\exp(j\phi_k) \cdot \exp(j p_i t) + \exp(-j\phi_k) \exp(-j p_i t) \right]$$

$$k_1 = k_0 \cdot \exp(j\phi_k)$$

$$k_2 = k_1^* = k_r \exp(-j\phi_k)$$

$$= 2 \cdot k_0 \cdot \exp(p_r \cdot t) \cdot \cos(p_i \cdot t + \phi_k)$$



Natural response:

ξ : Damping factor

$k_1 \exp(p_1 t) + k_2 \exp(p_2 t)$ — overdamped • real, distinct, $\xi > 1$

$(k_1 + k_2 t) \exp(p_1 t)$ — critically damped. • identical $\xi = 1$

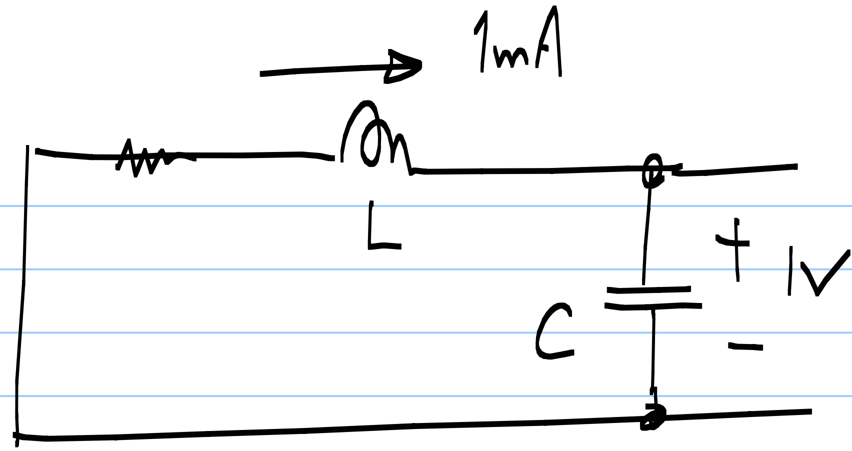
$2k_0 \exp(p_r t) \cos(p_i t + \phi_k)$ — underdamped. • complex conj. $\xi < 1$

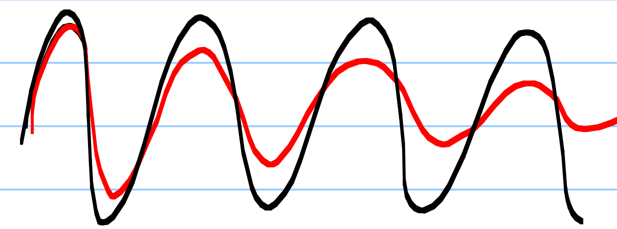
Q : Quality factor

$LC \cdot p^2 + RC \cdot p + 1 = 0$ real, distinct $Q < 1/2$

$\omega_n^2 \cdot p^2 + 2\xi \omega_n \cdot p + 1 = 0$ identical $Q = 1/2$

$\omega_n^2 \cdot p^2 + \frac{\omega_n \cdot p}{Q} + 1 = 0$ complex conj. $Q > 1/2$


1mA
L
C
+
1V
-



(Natural) (Forced)

Total solution = $\underbrace{\text{Transient response}}$ + Steady state response

$k_1 \exp(p_1 t) + k_2 \exp(p_2 t)$

OR

$(k_1 + k_2 t)\exp(p_1 t)$

OR

$2 \cdot k_0 \exp(p_r t) \cos(p_i t + \phi_k)$

<u>Input :</u>

Constant — Constant

$\exp(st)$ — $(\quad)\exp(st)$

$\exp(j\omega t)$