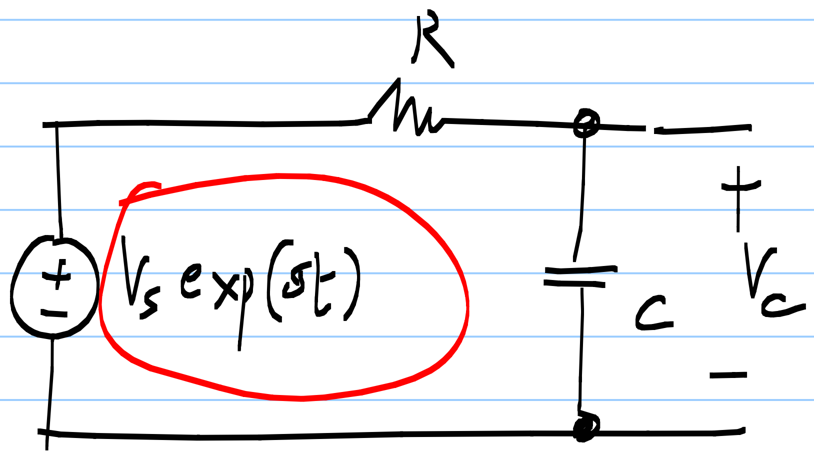
Lecture 23



$$RC\frac{dV_c}{dt} + V_c = V_s \exp(st)$$

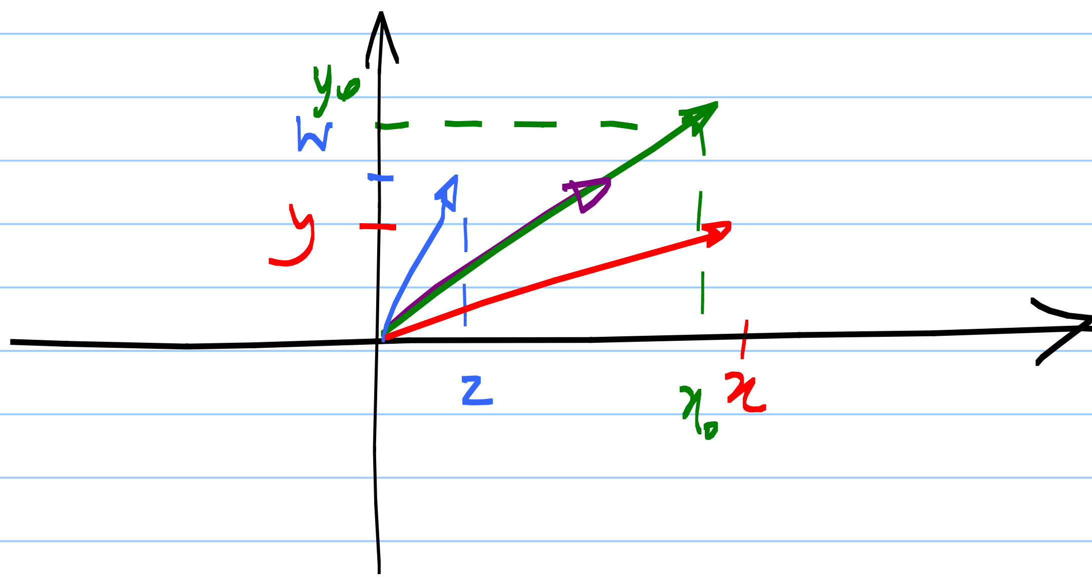
$$V_c(t) = V_0 \cdot \exp\left(-t/_{RC}\right)$$

$$+ \; V_s \exp(st) \cdot \left[\frac{1}{1+sCR}\right]$$

Transient / Natural

Steady state / Forced response

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} z \\ w \end{bmatrix}$$



$$RC \cdot \frac{dV_c}{dt} + V_c = V_s \exp(st)$$

$$V_{c_1} = V_c - V_s \exp(st)$$

$$\frac{dV_{c_1}}{dt} = \frac{dV_c}{dt} - V_s \cdot s \cdot \exp(st)$$

$$RC \cdot \frac{dV_{c_1}}{dt} + sCR \cdot V_s \cdot \exp(st) + V_{c_1} + \cancel{V_s \exp(st)} = \cancel{V_s \exp(st)}$$

$\times sCR$ $\times 1$

+

$$V_c(t) = V_0 \cdot \exp\left(-t/_{RC}\right)$$

$$+ \; V_s \exp(\underline{\underline{s}}t) \cdot \frac{1}{1 + \underline{\underline{s}}CR}$$

$$V_0 : \; V_c(0) - \frac{V_s}{1+sCR}$$

$$s = \frac{\delta - 1}{CR}$$

$$1 + sCR = \delta$$

$$\lim_{\delta \to 0}$$

$$\left(V_c(0) - \frac{V_s}{1+sCR}\right) \exp\left(-t/_{RC}\right) + \frac{V_s \exp(st)}{1+sCR}$$

$$\frac{(1+sCR) V_c(0) \exp\left(-t/_{RC}\right) - V_s \exp\left(-t/_{RC}\right) + V_s \exp(st)}{(1+sCR)}$$

$$\frac{(1+sCR)V_c(0)\exp(-t/RC) - V_s\exp(-t/RC) + V_s\overbrace{\exp(st)}^{\exp\left(\frac{\delta t}{RC}\right)\exp(-t/RC)}}{(1+sCR)}$$

$$\delta = 1 + sCR$$

$$\delta \longrightarrow 0$$

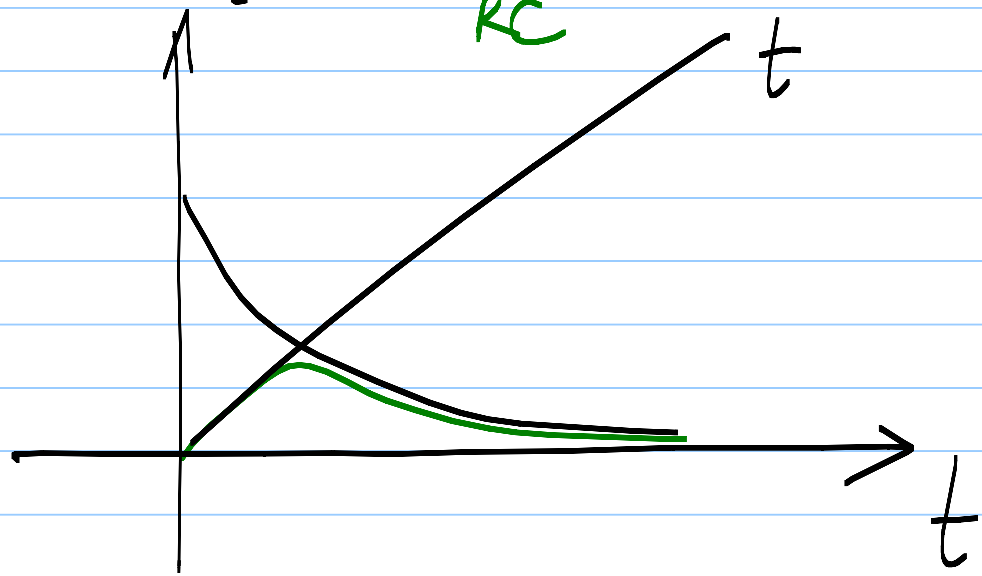
$$s = \frac{\delta - 1}{CR}$$

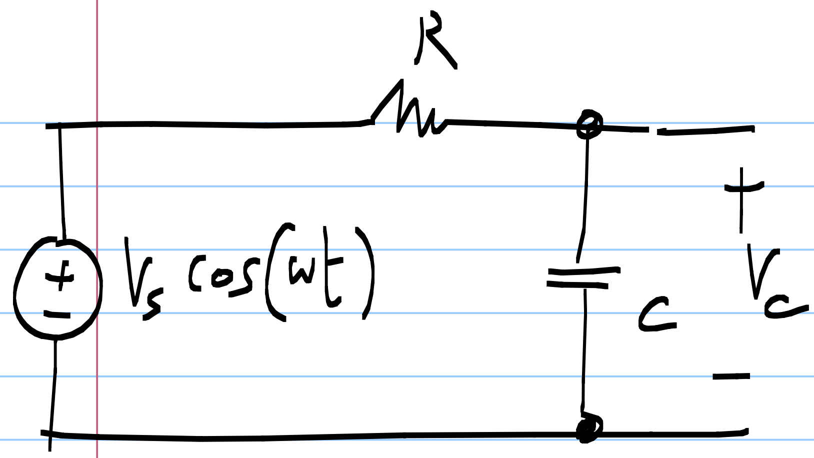
$$V_s\left(\cancel{1} + \frac{\delta t}{RC} + \left(\frac{\delta t}{RC}\right)^2 \frac{1}{2} + \ldots\right)$$

$$\frac{\delta V_c(0) - \cancel{V_s} + V_s\exp(\delta t/RC)}{\delta} \cdot \exp(-t/RC)$$

$$\left(V_c(0) + V_s \cdot \frac{t}{RC}\right)\exp(-t/RC)$$

$$\left(V_c(s) + V_s \cdot \frac{t}{RC} \right) \exp\left(-t/RC\right)$$





$$RC \frac{dV_c}{dt} + V_c = V_s \cdot \cos(wt)$$

$$V_c =$$

$$j = \sqrt{-1}$$

$$\cos(wt) = \frac{\exp(jwt) + \exp(-jwt)}{2}$$

$$\sin(wt) = \frac{\exp(jwt) - \exp(-jwt)}{j2}$$

In $\longrightarrow$ $\boxed{\text{(Linear system)}}$ $\longrightarrow$ Out

real coeff

$Re(\exp(j\omega t))$ $\qquad$ $Re(\text{Resp. to } \exp(j\omega t))$

$\alpha \cos\omega t + \beta \sin \omega t$

$= \gamma \cos(\omega t + \phi)$

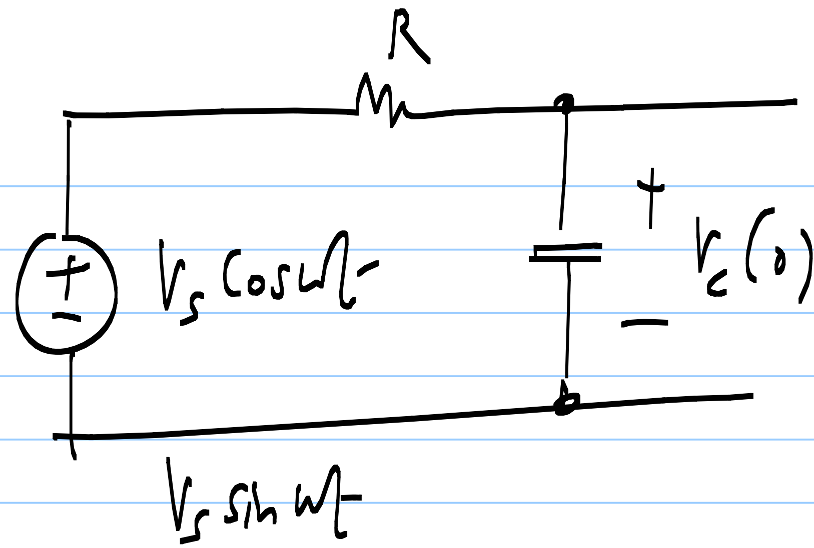
$\exp(j(\omega t + \phi))$

$\cos(\omega t)$ $\qquad$ Response to $\cos \omega t$

$\sin(\omega t)$ $\qquad$ Response to $\sin \omega t$

$\times 1$, $\times j$ $\qquad$ $\times 1$, j

$\exp(j\omega t)$ $\qquad$ Response to $\exp(j\omega t)$



$s = j\omega$

$$\frac{(1+sCR)V_c(0)\exp(-t/RC) - V_s\exp(-t/RC) + V_s\exp(st)}{(1+sCR)}$$

$$V_c(0)\exp(-t/RC) - \left(\frac{V_s}{1+j\omega CR}\right)\exp(-t/RC) + \frac{V_s\exp(j\omega t)}{1+j\omega CR}$$

$$V_c(0)\exp(-t/RC) - \left(\frac{V_s}{1+j\omega CR}\right)\exp(-t/RC) + \frac{V_s\exp(j\omega t)}{\quad}$$

$$V_c(0)\exp(-t/RC) - \frac{V_s}{1+(\omega CR)^2}\exp(-t/RC) \qquad \frac{V_s(1-j\omega CR)}{1+(\omega CR)^2}$$

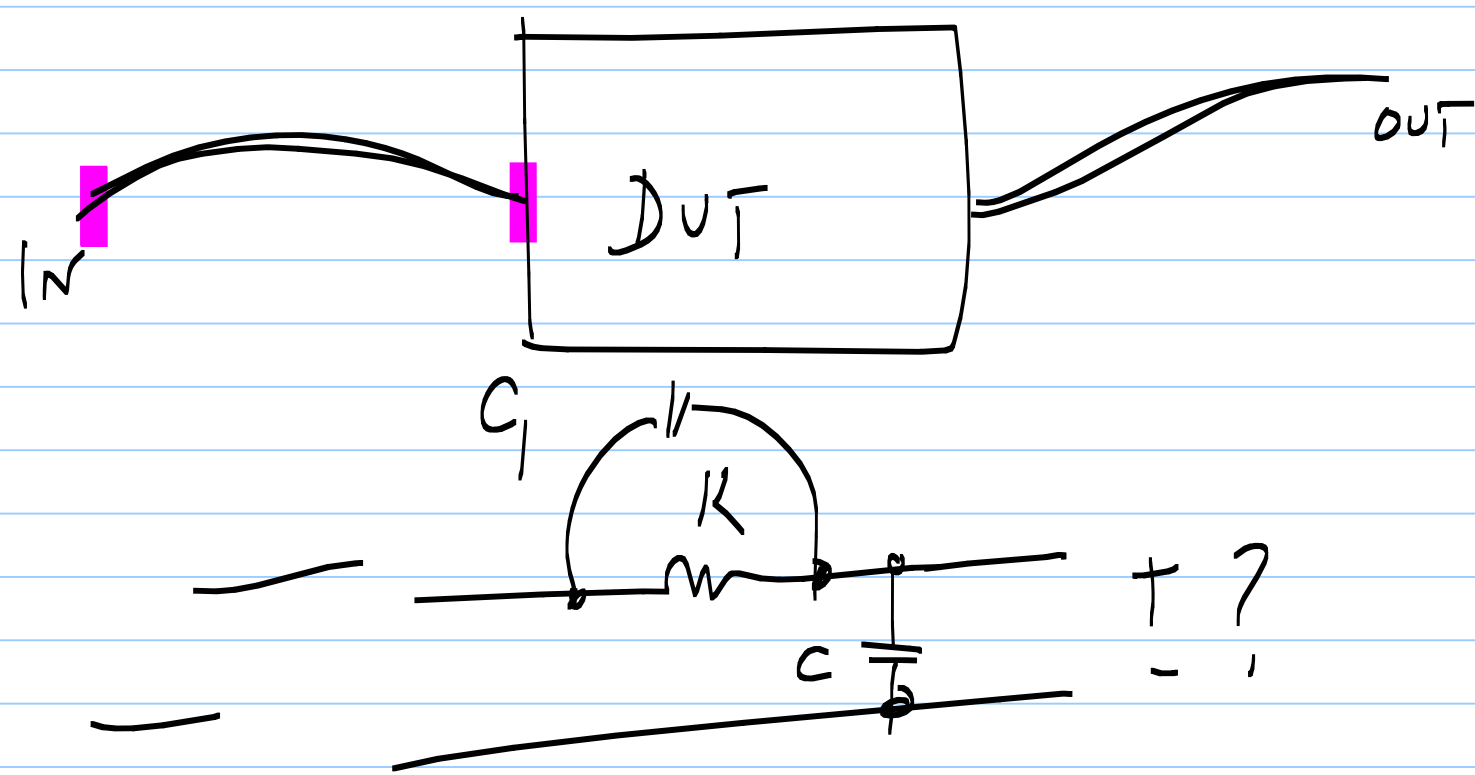
$$+\frac{V_s}{1+(\omega CR)^2}\left[\underbrace{\cos\omega t + (\omega CR)\sin(\omega t)}_{\cos(\omega t+\phi)}\right]$$

$$\frac{1}{1+j\omega CR} = A\exp(j\phi) \qquad \frac{1}{\sqrt{1+(\omega CR)^2}}\exp\left(-j\tan^{-1}(\omega CR)\right) \qquad -\tan^{-1}(\omega CR)$$

$$\frac{V_s \exp(j\omega t)}{1 + j\omega CR} = V_s \cdot \overbrace{\frac{1}{\sqrt{1+(\omega CR)^2}} \cdot \exp\left(-j \tan^{-1}(\omega CR)\right)}^{A \exp(j\phi)}.$$

$$\exp(j\omega t)$$

$$\boxed{\frac{V_s}{\sqrt{1+(\omega CR)^2}} \cdot \cos\left(\omega t - \tan^{-1}(\omega CR)\right)}$$


IN
DUT
OUT
C_1
R
C
+
-
?