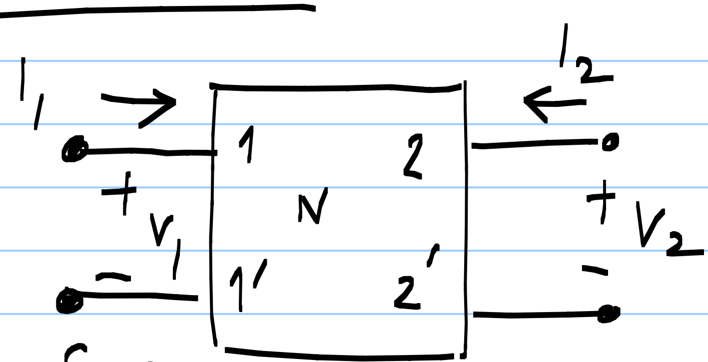
Lecture 15



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Y parameters (conductance)

$(\ )^{-1}$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Z parameters (resistance)

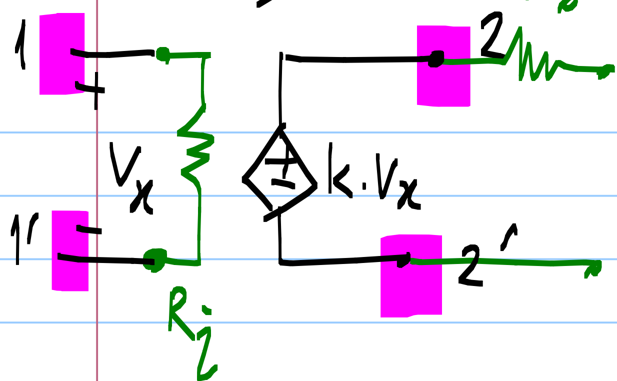
[h] → hybrid parameters

$(\ )^{-1}$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

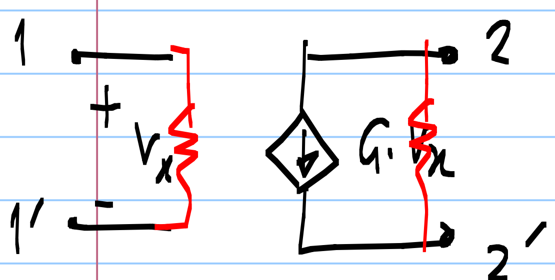
VCVS

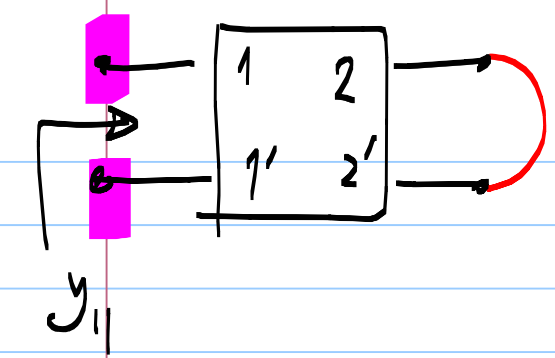


	[Y]	[Z]	[H]	[G]
VCVS	$\begin{bmatrix} 0 & 0 \\ \infty & \infty \end{bmatrix}$ ✗	✗	$\begin{bmatrix} & \\ & \end{bmatrix}$ ✗	$\begin{bmatrix} 0 & 0 \\ k & 0 \end{bmatrix}$ → $\begin{bmatrix} 1/R_i & 0 \\ k & R_o \end{bmatrix}$
VCCS	$\begin{bmatrix} 0 & 0 \\ G & 0 \end{bmatrix}$	✗	✗	✗

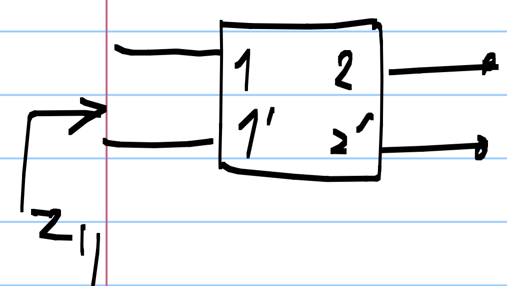
<u>Unilateral:</u> Only one of $\{h_{21}, h_{12}\}$ is non zero

VCCS

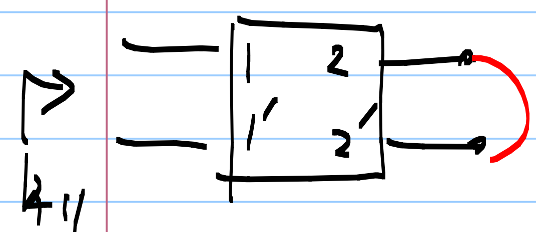




y_{11} : conductance looking into port 1 (1–1') with port 2 shorted



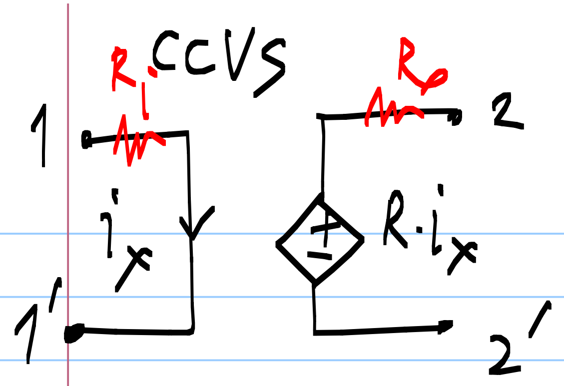
z_{11} : resistance looking into port 1 with port 2 open



h_{11} : resistance looking into port 1 with port 2 shorted

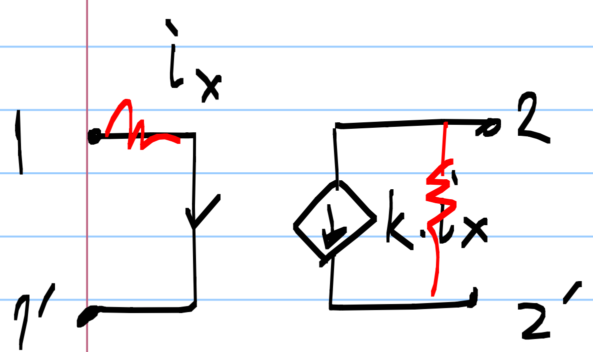
g_{11} : Conductance into port 1 (port 2 open)

CCVS

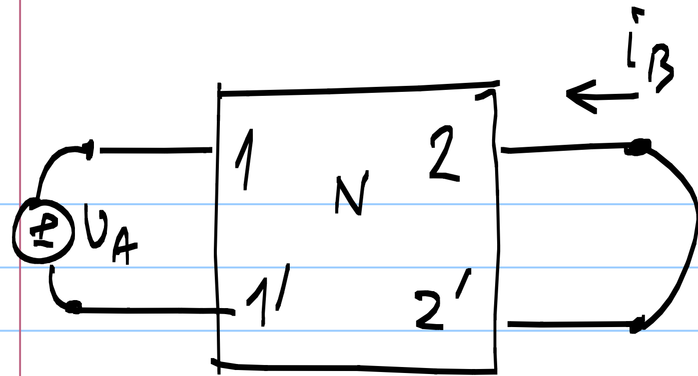


$$[Z] = \begin{bmatrix} 0 & 0 \\ R & 0 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} 1/R_i & 0 \\ \frac{-R}{R_i \cdot R_o} & \frac{1}{R_o} \end{bmatrix}$$

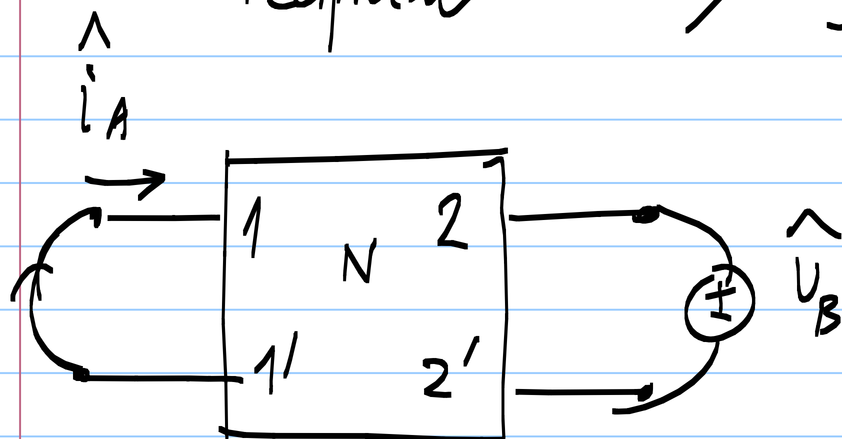


$$[H] = \begin{bmatrix} 0 & 0 \\ k & 0 \end{bmatrix}$$



$[Y] = \begin{bmatrix} & \\ & \end{bmatrix}$

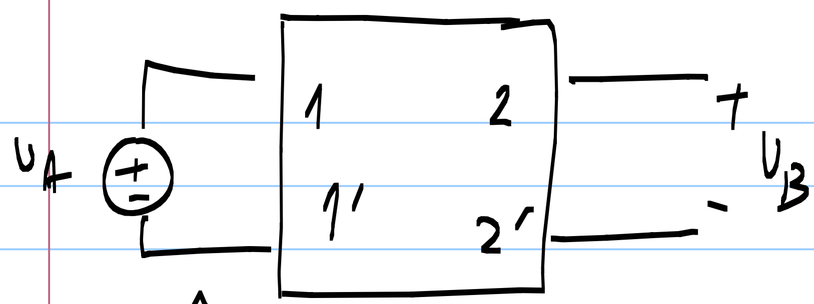
reciprocal $\Longrightarrow$ $y_{12} = y_{21}$; $z_{12} = z_{21}$

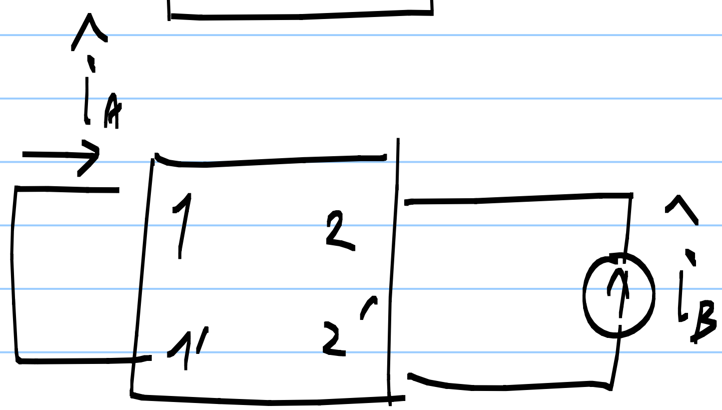


reciprocal

$$\frac{i_B}{v_A} = \frac{\hat{i}_A}{\hat{v}_B}$$

y_{21} $\quad$ y_{12}



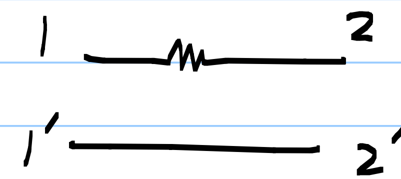


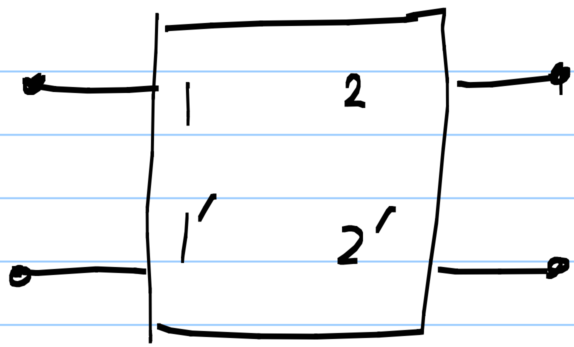
$$\frac{v_B}{v_A} = -\frac{\hat{i}_A}{\hat{i}_B}$$

$$g_{21} = -g_{12}$$

$$h_{12} = -h_{21}$$

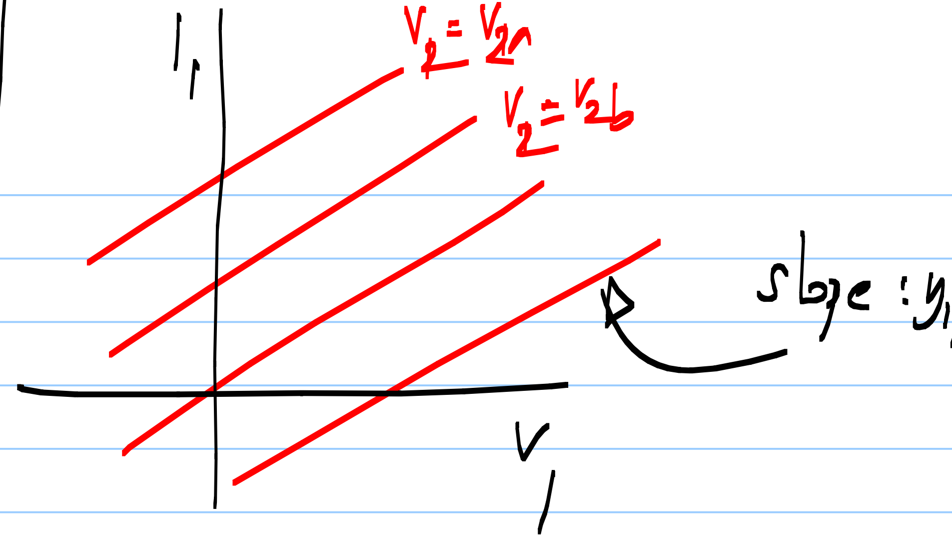
reciprocity



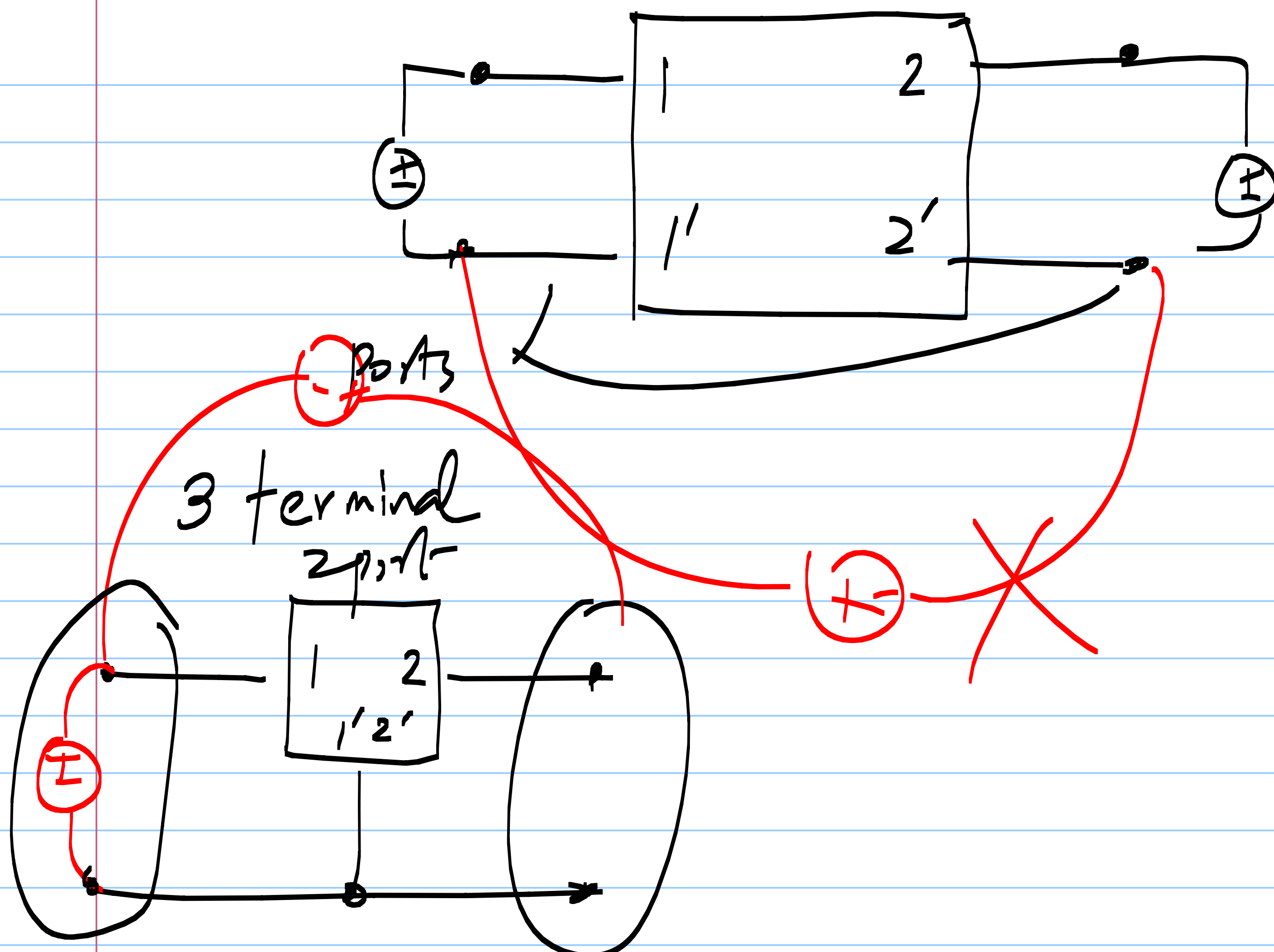


$$I_1 = y_{11} V_1 + y_{12} V_2$$

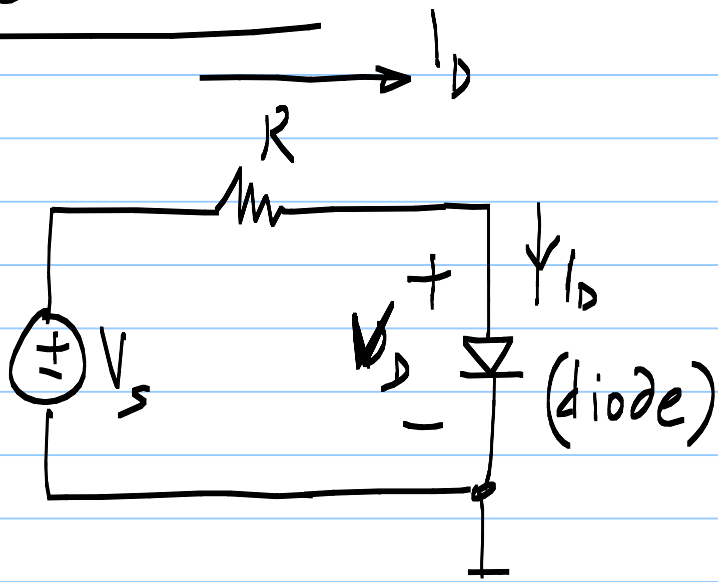
$$I_2 = y_{21} V_1 + y_{22} V_2$$



I_1 vs. V_1 for diff. V_2


1
2
1'
2'
Ports
3 terminal
2 port
1
2
1'2'

Lecture 16 :



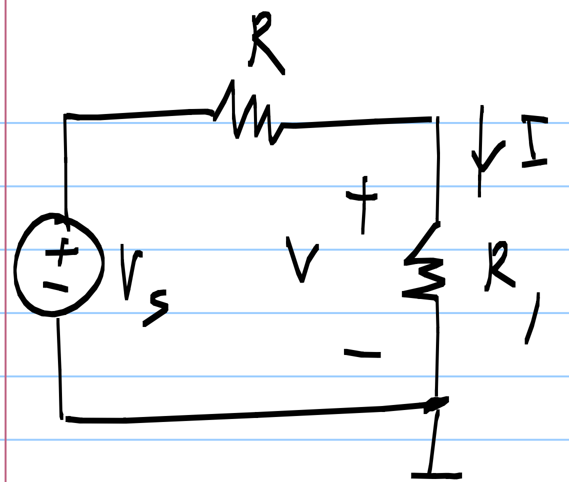
$$I_D = I_s\left(\exp\left(\frac{V_D}{V_t}\right) - 1\right)$$

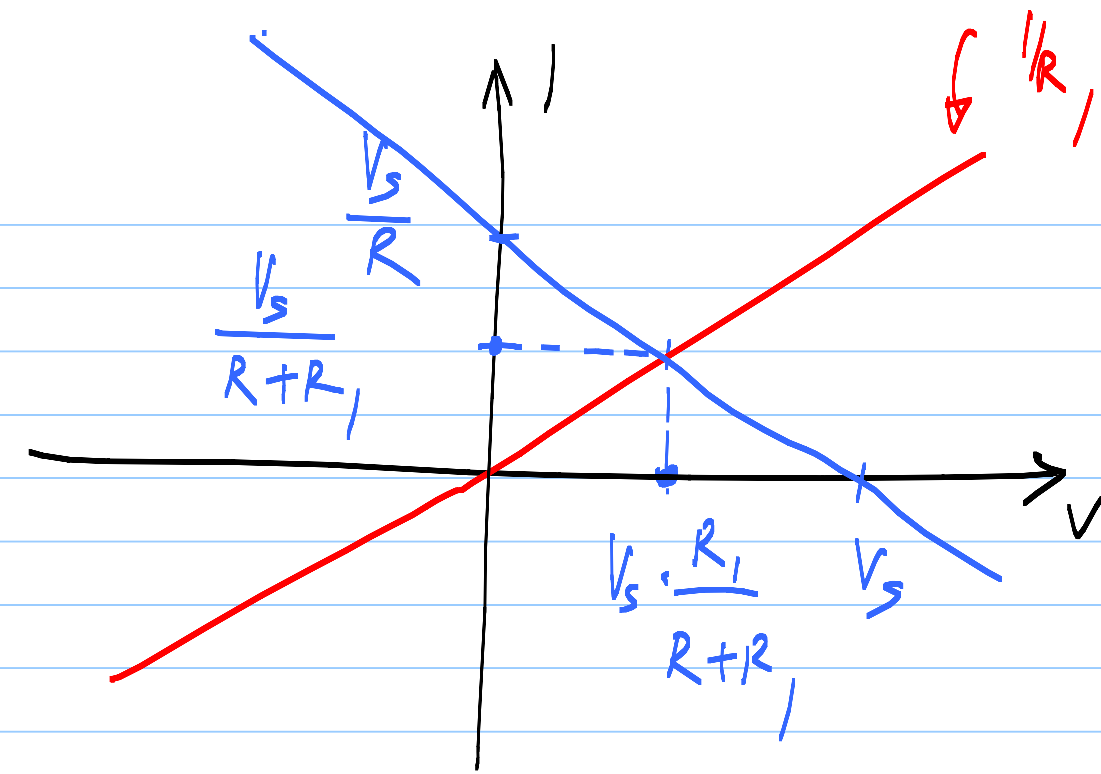
$$I_D = I_s\left[\exp\left(\frac{V_D}{V_t}\right) - 1\right] \quad — ②$$

$$V_s = I_D \cdot R + V_D \quad — ①$$

$$= \left\{ I_s \cdot R\left[\exp\left(\frac{V_D}{V_t}\right) - 1\right] + V_D \right.$$

I_s, V_t : constants

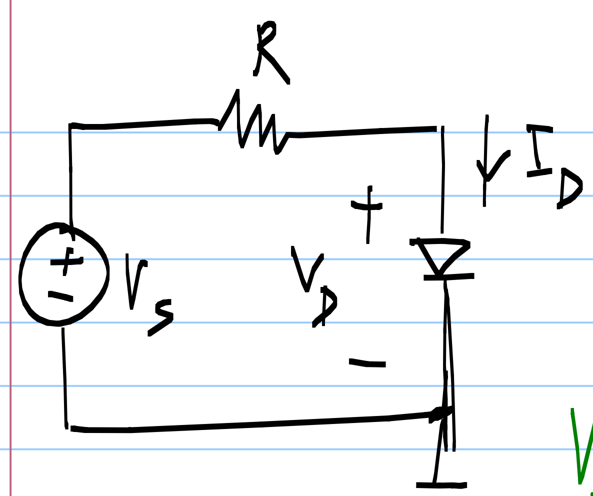


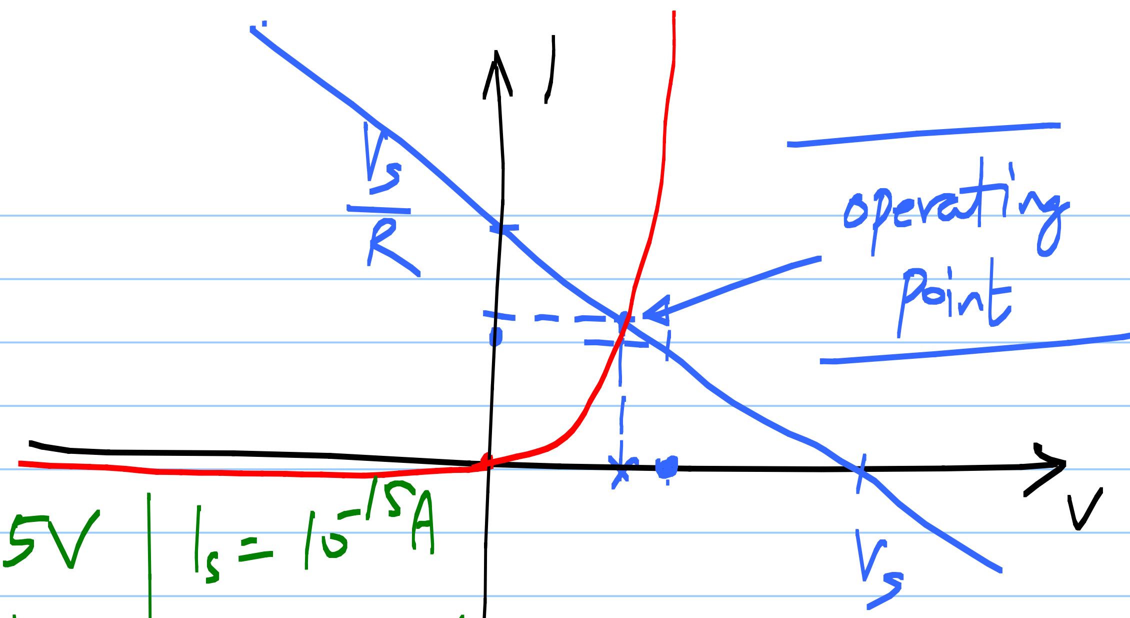


$$I = \frac{V}{R_1}$$

$$V_s = I \cdot R + V \quad - \text{②}$$

$$I = \frac{V}{R_1} \quad - \text{①}$$





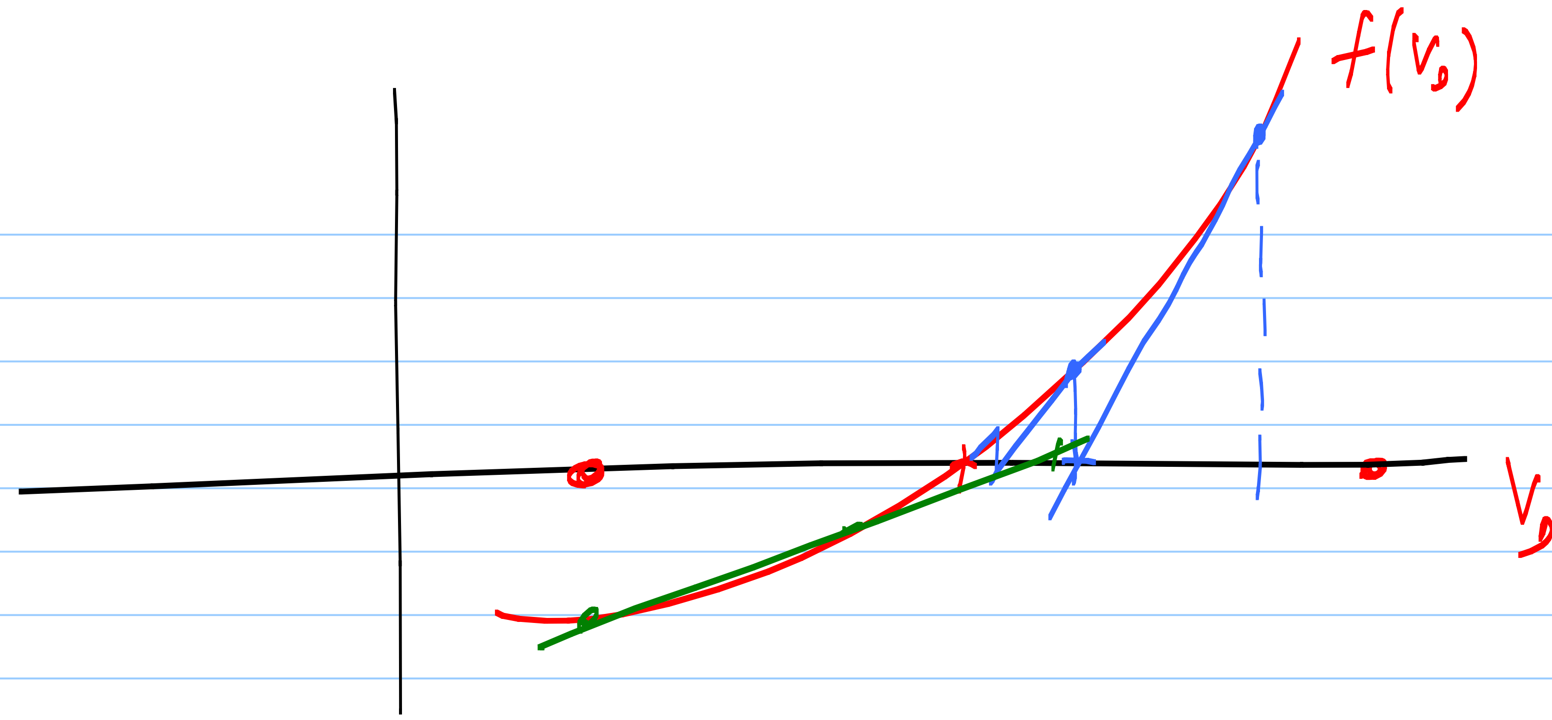
$V_s = 5V$	$I_s = 10^{-15}A$
$R = 1k\Omega$	$V_t = 25mV$

$$I = \frac{V}{R}$$

$$V_s = I \cdot R + V \quad - \text{②}$$

$$I_D = I_s\left(\exp\left(\frac{V_D}{V_t}\right) - 1\right) \quad - \text{①}$$

$$f(V_D) = I_s R\left[\exp\left(\frac{V_D}{V_t}\right) - 1\right] + V_D - V_s = 0$$



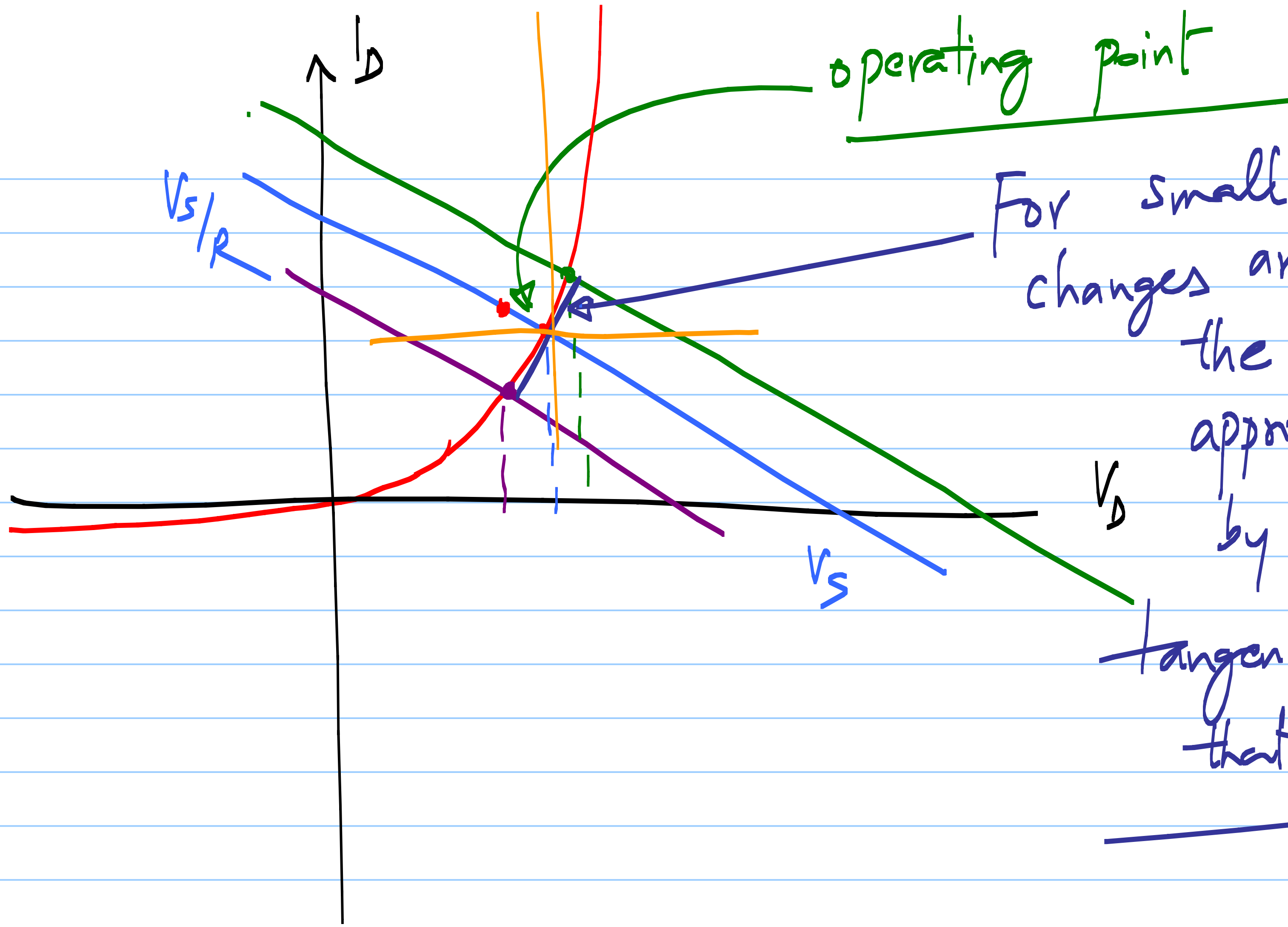
$f(V_D) = 0$

$f(V_D) = 0$ F

$g(V_D) \equiv V_D$ $V_D = 1$ step #1

$g(1) \neq 1$ $V_D = g(1)$

$g(g(1)) \neq g(1)$



For small changes around the op. point, approximated by the tangent at that point

$$f(V_D) = I_s R \left[\exp\left(\frac{V_D}{V_t}\right) - 1 \right] + V_D - V_s = 0 \quad (1)$$

V_s : changes to $V_s + \Delta V_s$ ($\Rightarrow$ V_D changes to $V_D + \Delta V_D$)

$$I_s R \left[\exp\left(\frac{V_D + \Delta V_D}{V_t}\right) - 1 \right] + (V_D + \Delta V_D) - V_s - \Delta V_s = 0$$

$$I_s R \left[\exp\left(\frac{V_D}{V_t}\right) - 1 + \frac{1}{V_t} \cdot \exp\left(\frac{V_D}{V_t}\right) \cdot \Delta V_D + \cdots \right] \quad (2)$$

$$+ (V_D + \Delta V_D) - V_s - \Delta V_s = 0$$

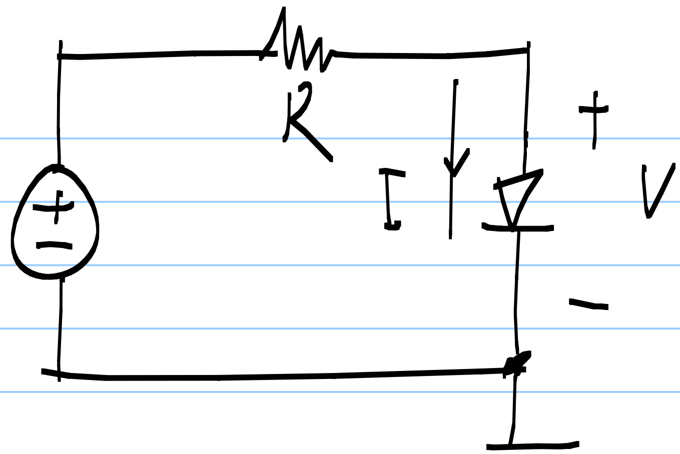
$$g(v)\Big|_{v=V_D} = g(V_D) + g'(v)\Big|_{v=V_D} \cdot (v - V_D) + \frac{g''(v)}{2!}\Big|_{v=V_D} (v-V_D)^2$$

$$+$$

②-①:

$$\frac{I_s R}{V_T} \exp\left(\frac{V_D}{V_T}\right) \cdot \Delta V_D + \Delta V_D - \Delta V_S = 0$$

$$\frac{I_s R}{V_T} \exp\left(\frac{V_D}{V_T}\right) \cdot \Delta V_D + \Delta V_D = \Delta V_S$$



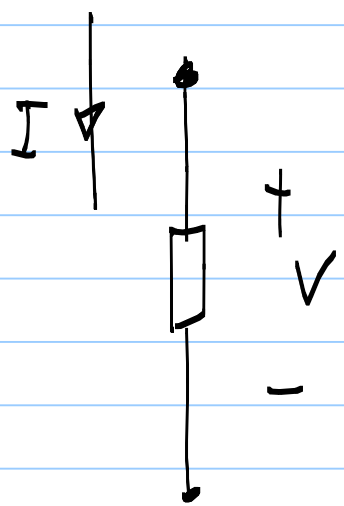
op. point: (I_0, V_0)

$$I_0 = f(V_0)$$

$$I_0 + \Delta I = f(V_0 + \Delta V)$$

$$\simeq f(V_0) + f'\big|_{V_0} \cdot \Delta V$$

(higher order terms neglected)

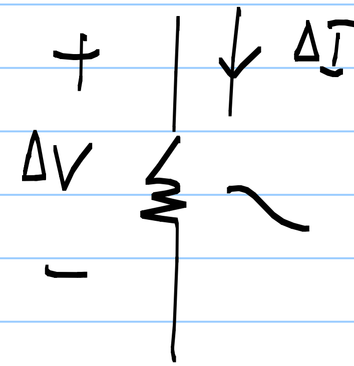


$$I = h(V)$$

$$\Delta I = f'\big|_{V_0} \cdot \Delta V$$

Linear relationship between incremental values

$$\Delta I = f'\Big|_{V_0} \cdot \Delta V \Rightarrow$$

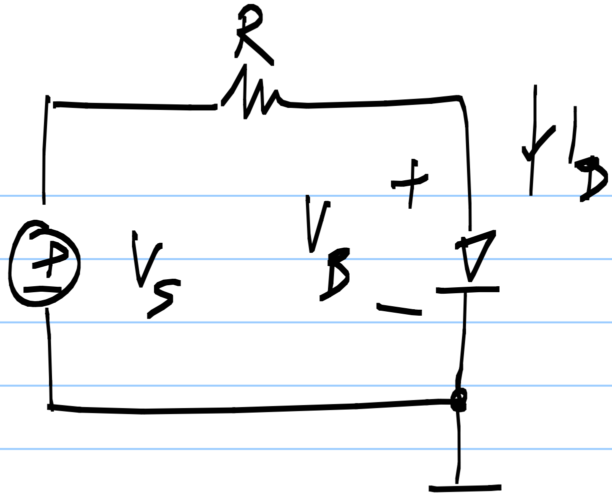


$$\sim r = \frac{1}{f'\Big|_{V_0}}$$

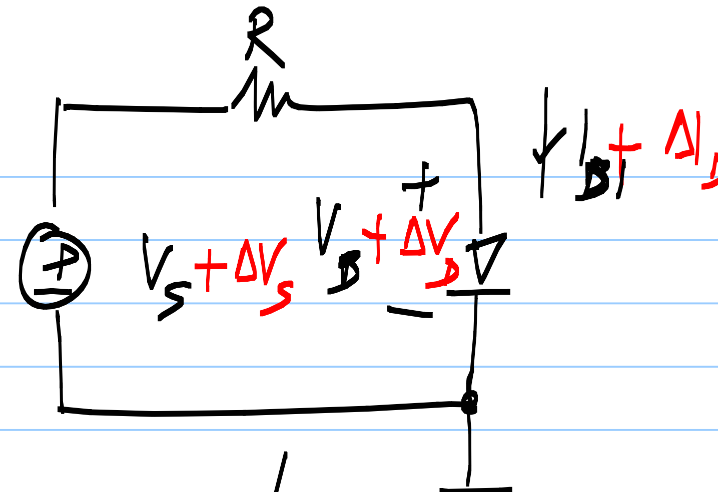
Incremental resistance

$$I_D = I_S\left(\exp\left(\frac{V_D}{V_T}\right) - 1\right) \simeq I_S \exp\left(\frac{V_D}{V_T}\right)$$

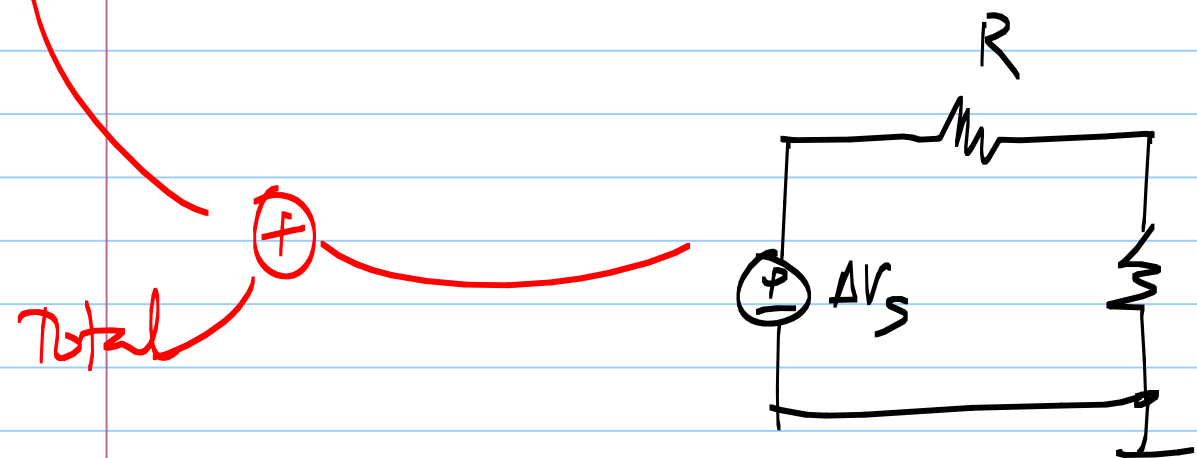
$$r_D = \frac{1}{\frac{I_S}{V_T}\exp\left(\frac{V_D}{V_T}\right)} = \frac{V_T}{I_D}$$



1. Calculate the op. point



form an incremental ckt: @ op. point



$$\frac{1}{f'|_{V_D}} = \frac{V_T}{I_D}$$