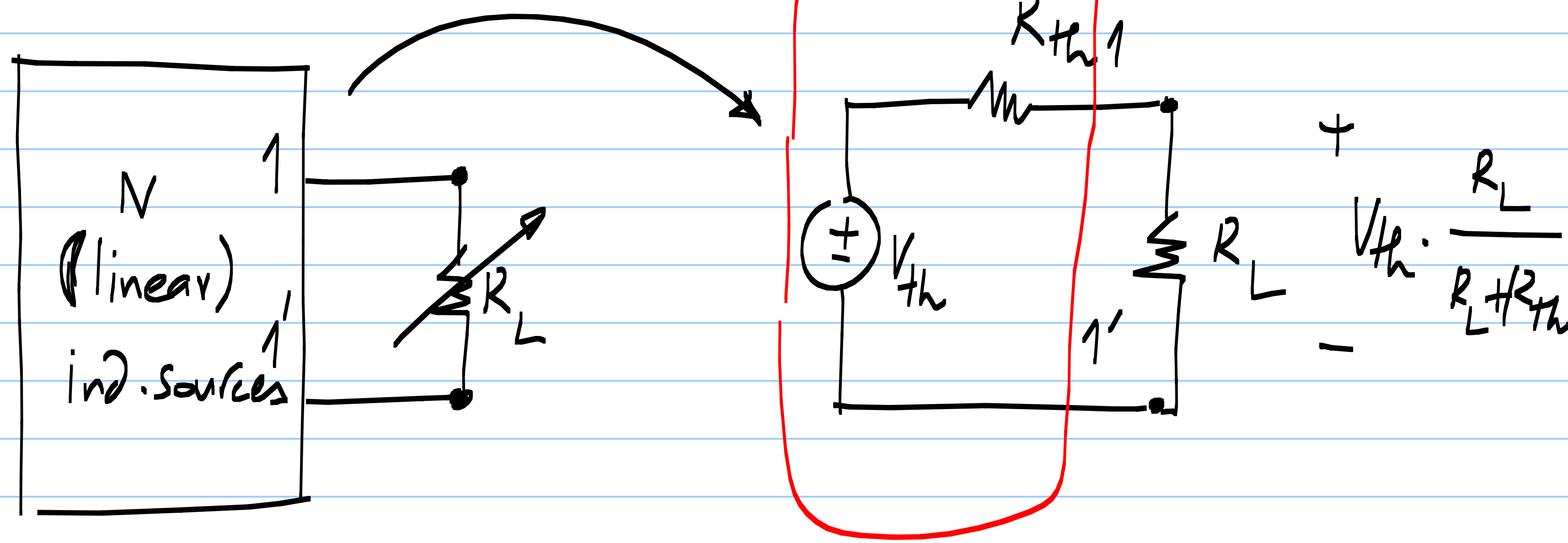
Lecture #11



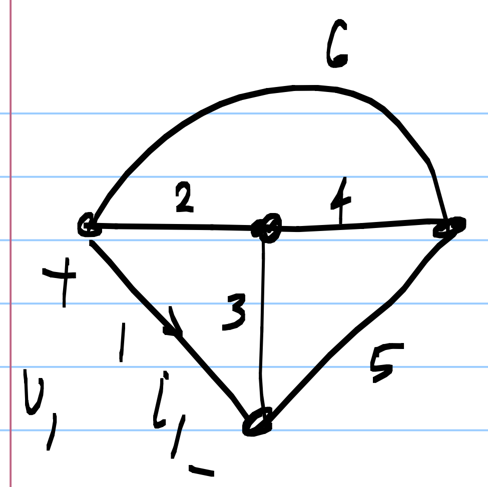
$$P_L = V_{th}^2 \cdot \frac{R_L^2}{(R_L + R_{th})^2} \cdot \frac{1}{R_L}$$

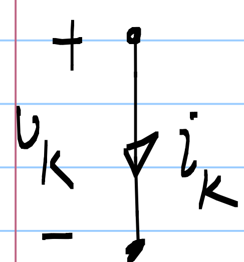
Maximum power transfer theorem

$$P_L = V_{th}^2 \cdot \frac{R_L^2}{(R_L + R_{th})^2} \cdot \frac{1}{R_L} = \left(\frac{V_{th}^2}{R_{th}}\right) \cdot \frac{R_L \cdot R_{th}}{(R_L + R_{th})^2}$$

$$= \frac{V_{th}^2}{R_{th}} \left(\frac{\sqrt{R_L R_{th}}}{R_L + R_{th}}\right)^2 = \left(\frac{V_{th}^2}{R_{th}}\right) \cdot \frac{1}{\left(\sqrt{\frac{R_L}{R_{th}}} + \sqrt{\frac{R_{th}}{R_L}}\right)^2}$$

$$P_{L,max} = \frac{V_{th}^2}{4R_{th}} \quad \text{when } \underline{R_L = R_{th}}$$



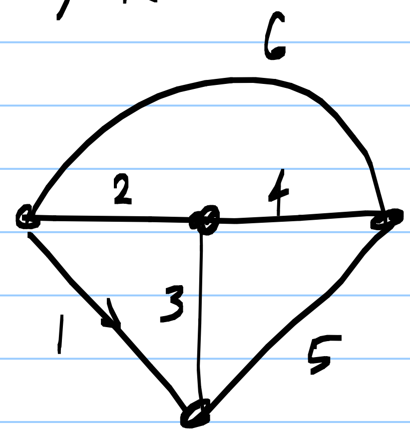


$v_k \cdot i_k$: power dissipated in the k^{th} branch

$$\sum_{\text{all branches}} v_k i_k = 0$$

Network N

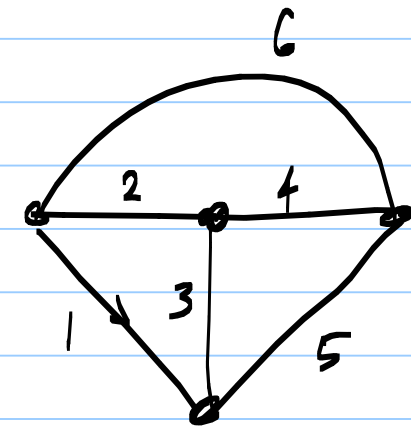
v_k, i_k



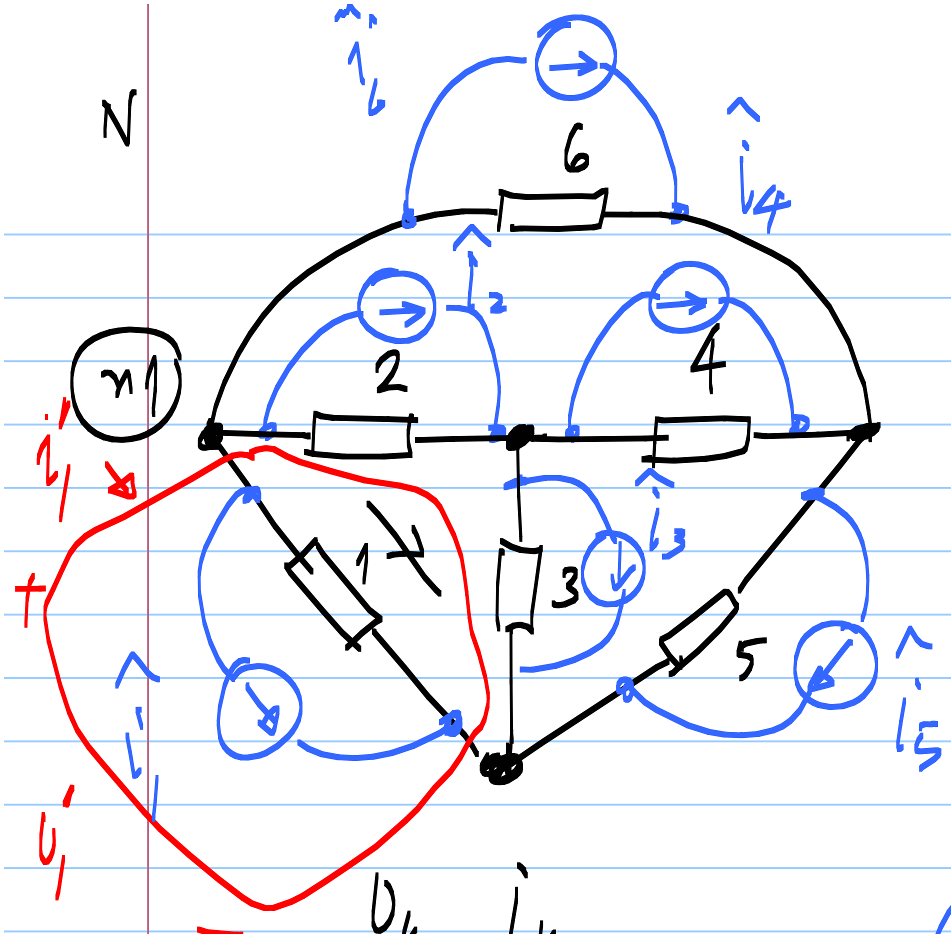
$$\sum_k v_k i_k = 0$$

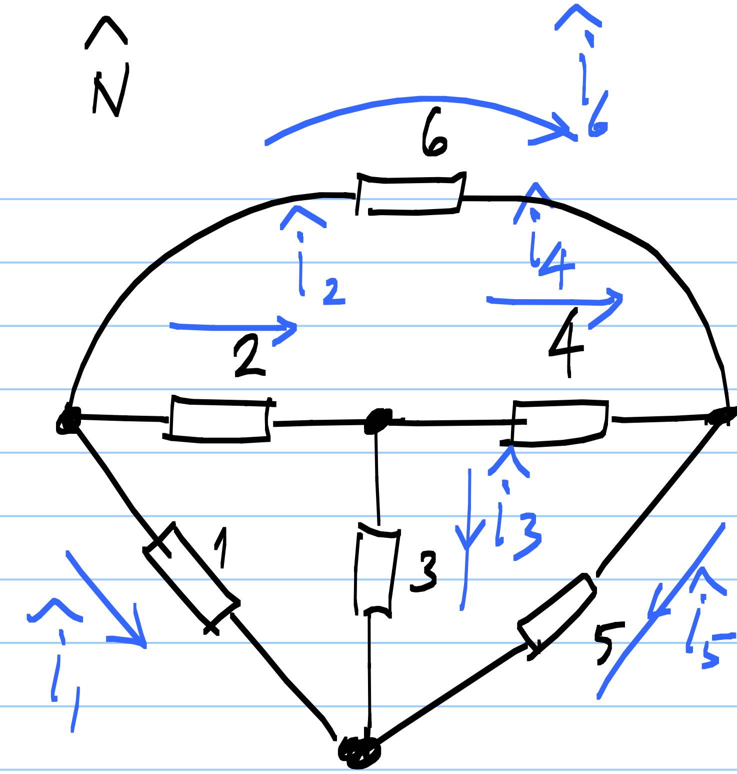
Network $\hat{N}$

$\hat{v}_k, \hat{i}_k$



$$\sum_k \hat{v}_k \cdot \hat{i}_k = 0$$





v_k, i_k'

$= -(\hat{i}_1' + \hat{i}_2' + \hat{i}_6')$

$i_k' + \hat{i}_k'$

n1 $\quad i_1' + i_2' + i_6' = 0$

$$\sum_k v_k' \, i_k' = 0 \qquad \sum \hat{v}_k \cdot i_k' = 0$$

$$\sum_k v_k \cdot (i_k + \hat{i}_k) = 0$$

$$\underbrace{\cancel{\sum_k v_k \cdot i_k}}_{=0} + \sum_k v_k \cdot \hat{i}_k = 0$$

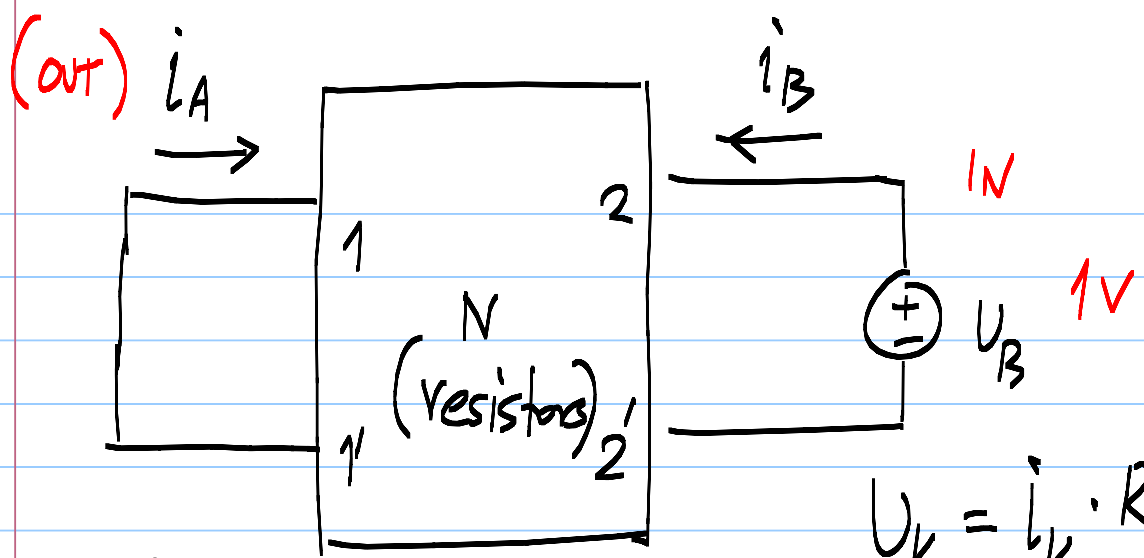
$$\boxed{\sum_k v_k \, \hat{i}_k = 0}$$

$$\sum_k v_k(t_1)\, i_k(t_2) = 0$$

$$\sum_k v_k(t_1) \cdot \hat{i}_k(t_2) = 0$$

Tellegen's theorem

N



$U: N; \quad i: \hat{N}$

$$U_B(-\hat{i}_B) + \sum_k U_k \cdot \hat{i}_k = 0$$

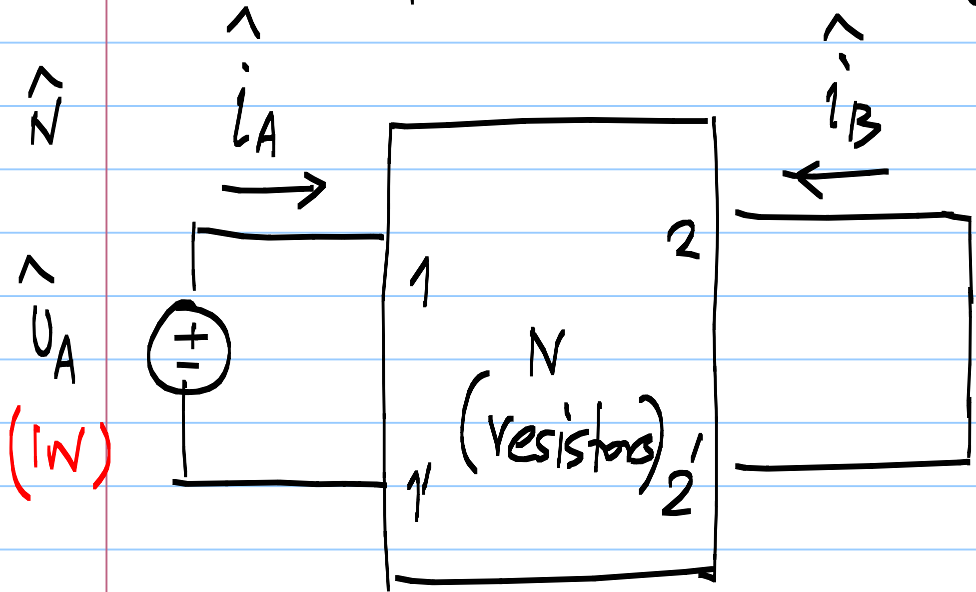
$$U_k = i_k \cdot R_k \qquad U: \hat{N}; \quad i: N$$

$$\hat{U}_k = \hat{i}_k \cdot R_k$$

$$\hat{U}_A \cdot (-i_A) + \sum_k \hat{U}_k \cdot i_k = 0$$

$\hat{N}$

$\hat{U}_A$



$$v_B(-\hat{i}_B) + \sum_k v_k \hat{i}_k = 0$$

[R]

$$v_B(-\hat{i}_B) + \sum_k R_k i_k \hat{i}_k = 0$$

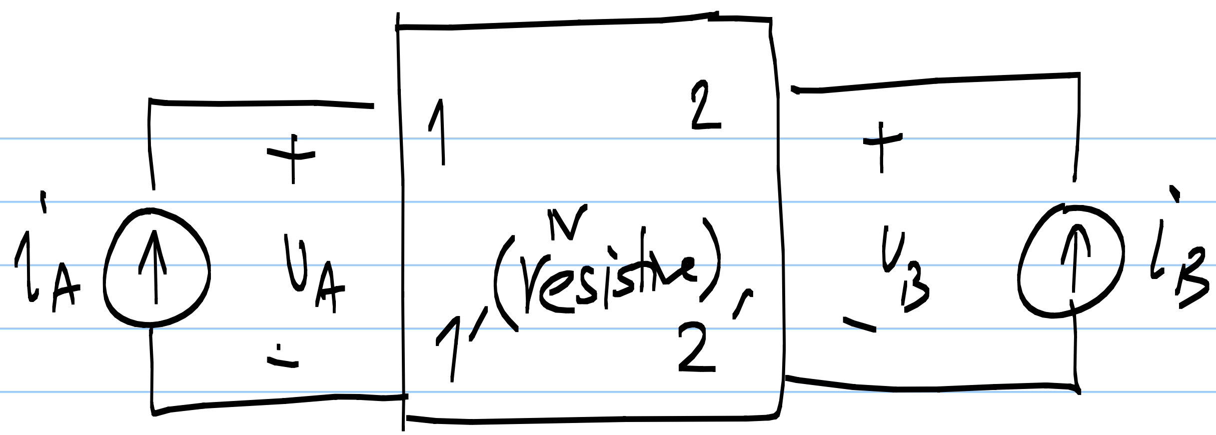
$$\hat{v}_A(-i_A) + \sum_k \hat{v}_k i_k = 0$$

[R]

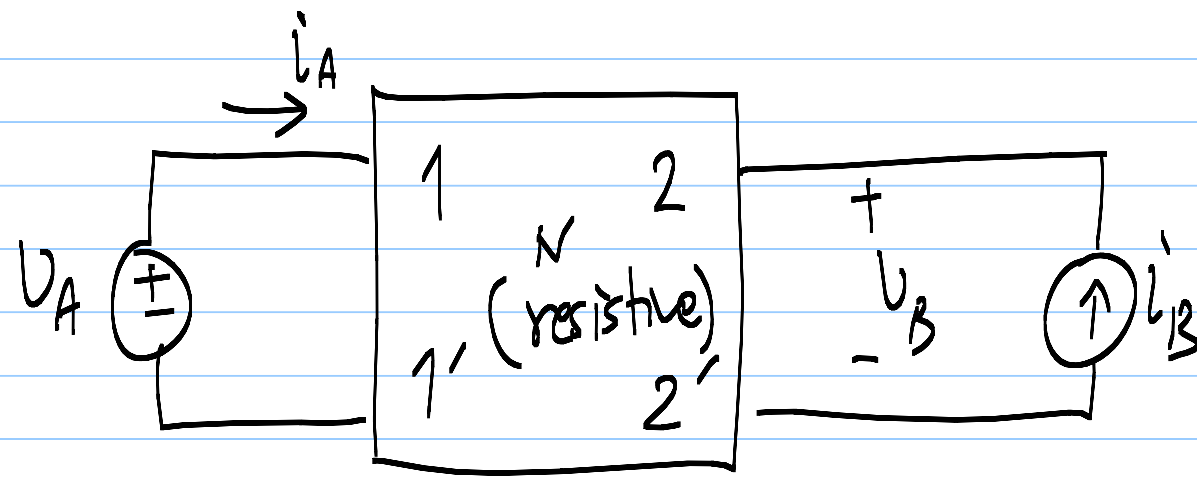
$$\hat{v}_A(-i_A) + \sum_k R_k \hat{i}_k \cdot i_k = 0$$

$$v_B\left(-\hat{i}_B\right) = \hat{v}_A\left(-i_A\right)$$

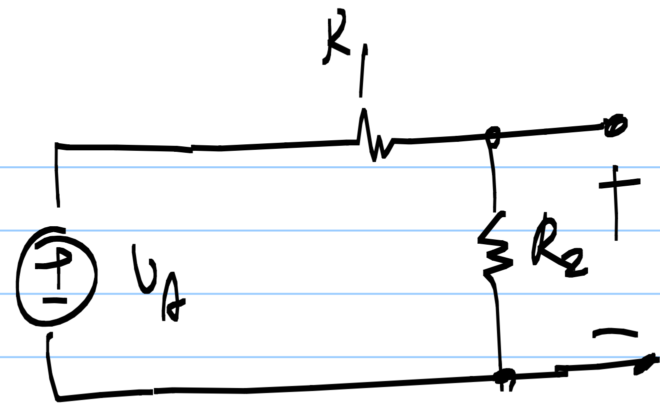
$$\frac{i_A}{v_B} = \frac{\hat{i}_B}{\hat{v}_A}$$



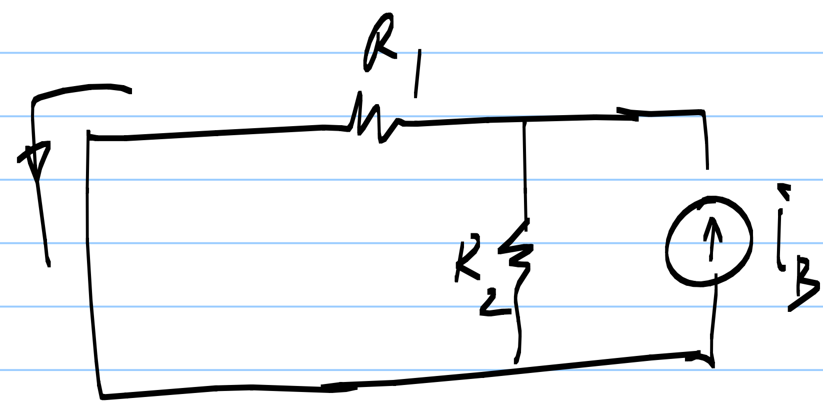
$$\left.\frac{v_A}{i_B}\right|_{i_A=0} \quad \& \quad \left.\frac{v_B}{i_A}\right|_{i_B=0}$$



$$\left.\frac{i_A}{i_B}\right|_{v_A=0} \quad \& \quad \left.\frac{v_B}{v_A}\right|_{i_B=0}$$



$$u_A \frac{R_2}{R_1 + R_2}$$



$$i_B \frac{R_2}{R_1 + R_2}$$