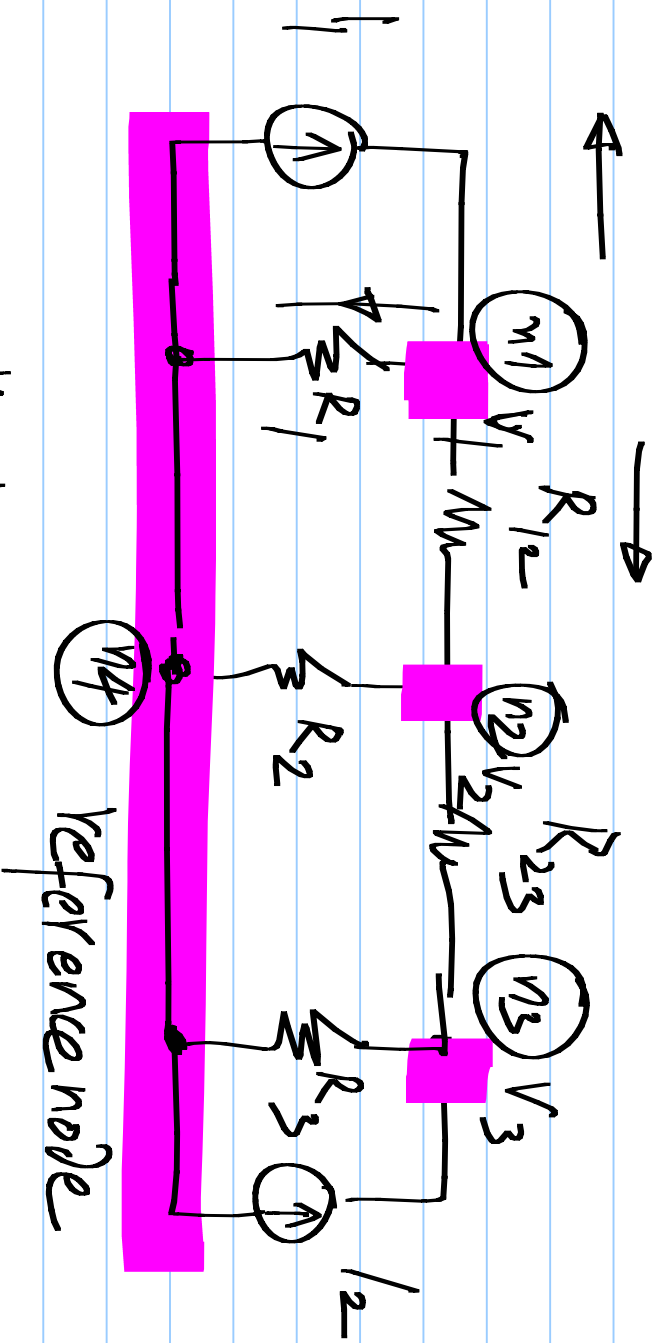


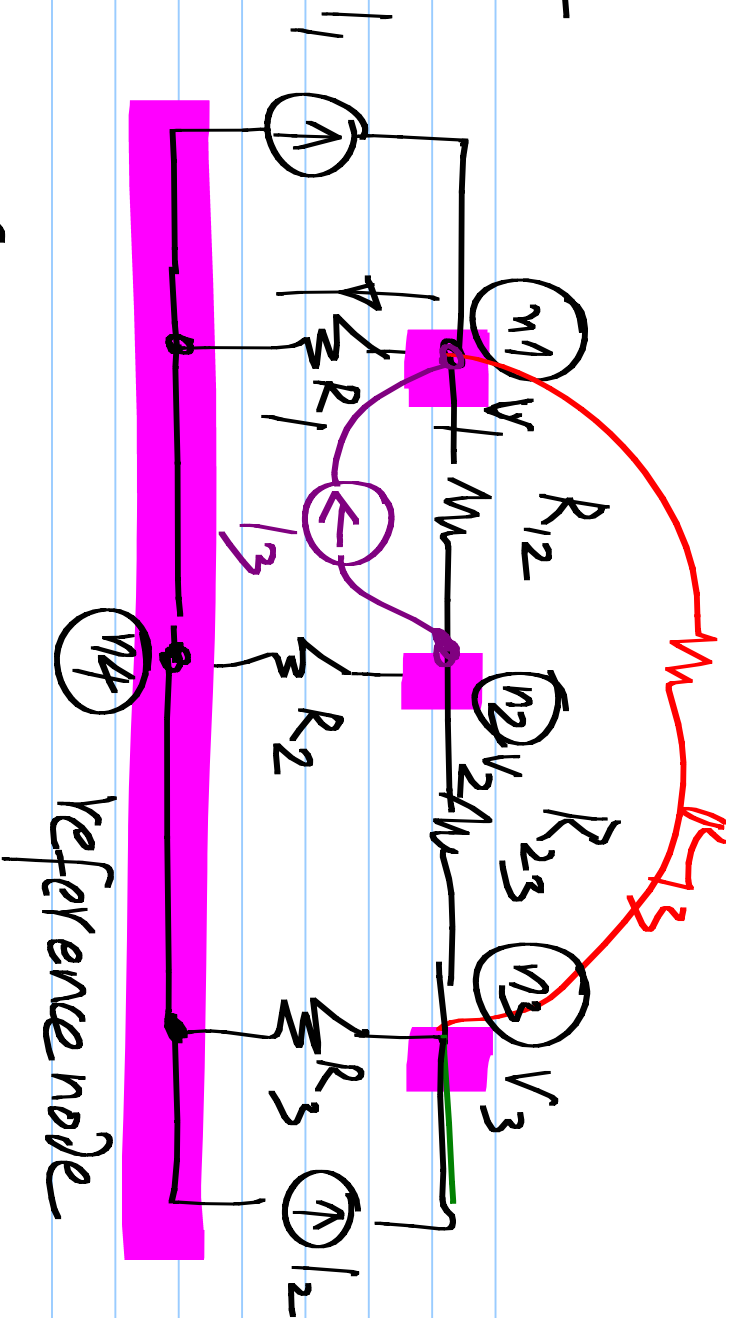
Nodal analysis :



$$V_1/R_1 + \frac{V_1 - V_2}{R_{12}} = I_1$$

$$V_2/R_2 + \frac{V_2 - V_3}{R_{23}} + \frac{V_2 - V_1}{R_{12}} = 0$$

Lecture 6



$$n1: V_1 \cdot G_1 + (V_1 - V_2) G_{12} = I_1$$

$$n2: V_2 \cdot G_2 + (V_2 - V_1) G_{12} + (V_2 - V_3) G_{23} = 0$$

$$n3: V_3 \cdot G_3 + (V_3 - V_2) G_{23} = I_2$$

$$n_1: (g_1 + g_{12})V_1 - g_{12} \cdot V_2 = 1_1$$

$$n_2: -g_{12}V_2 + (g_2 + g_{12} + g_{23})V_2 - g_{23}V_3 = 0$$

$$n_3: -g_{23} \cdot V_2 + (g_{23} + g_3)V_3 = 1_2$$

$$\begin{bmatrix} g_1 + g_{12} + g_{13} & -g_{12} & -g_{13} \\ -g_{12} & g_2 + g_{12} + g_{23} & -g_{23} \\ -g_{13} & -g_{23} & g_3 + g_{23} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1_1 \\ 1_3 \\ 1_2 \end{bmatrix}$$

Current sources & resistances:

$$\underline{V} = [G]^{-1} \underline{I}$$

Nodal equations (N-1) nodes

$$\left. \begin{array}{l} \text{Conductance} \\ \text{matrix} \end{array} \right\} [G] \cdot \left. \begin{array}{l} \text{variable vector} \\ \text{node voltages} \end{array} \right\} \underline{V} = \left. \begin{array}{l} \text{Source vector} \\ \text{wrt. reference node} \end{array} \right\} \underline{I}$$

* Symmetric

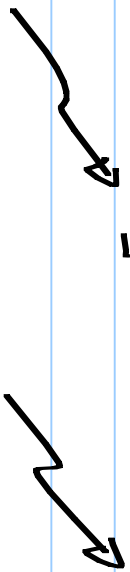
* Diagonal elements: total node conductance

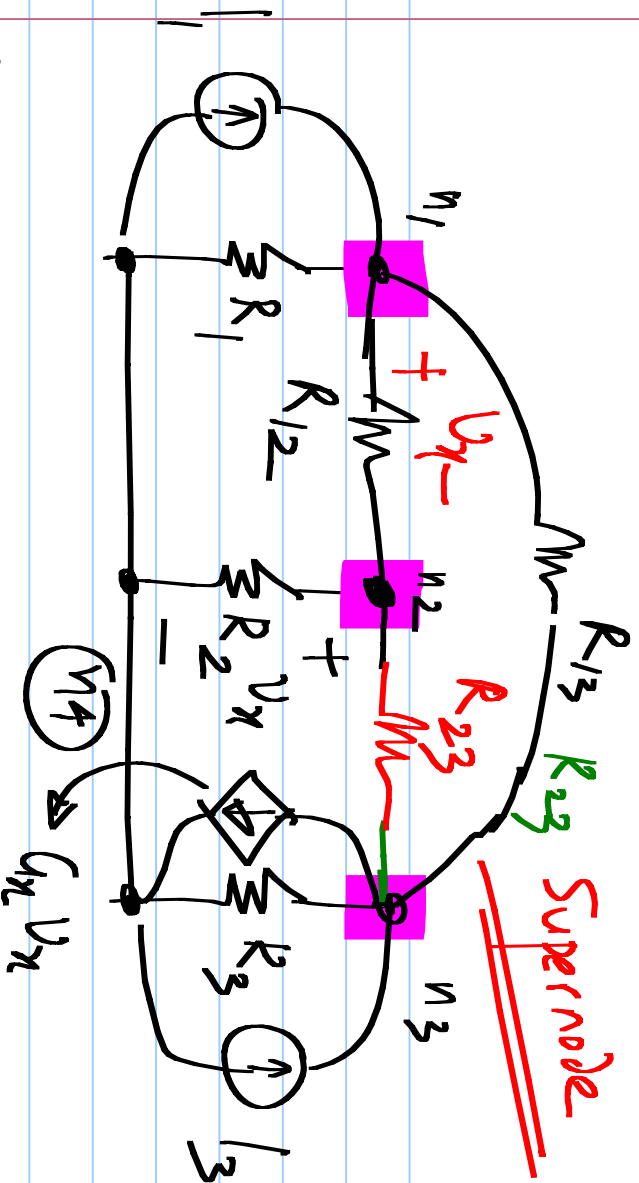
* off diagonal: — (conductance between nodes)

$$V_1 = \frac{|G_{11}|}{|G|}$$

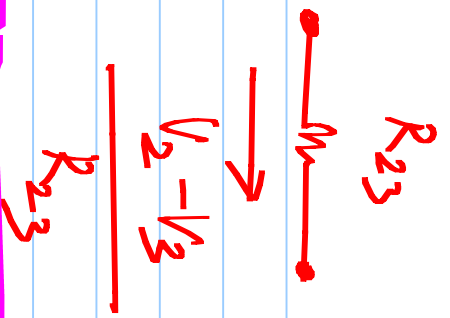
$$= \begin{vmatrix} 1 & -g_{12} & 0 \\ 0 & g_{22} + g_{12} + g_{23} & -g_{23} \\ 1/2 & -g_{23} & g_{33} + g_{23} \end{vmatrix}$$

$$\underline{V} = G^{-1} \begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}$$

$$= G^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + G^{-1} \begin{bmatrix} 0 \\ 0 \\ 1/2 \end{bmatrix}$$




Supernode



✓ n1:

$$n2: -G_{12}V_1 + (G_2 + G_{12})V_2 + I_x = 0$$

$$n3: -G_{13}V_1 - I_x + (G_3 + G_{13})V_3 = I_3$$

✓ Voltage source: $V_2 - V_3 = V_{23}$

$$\begin{aligned}
 & -(G_{12} + G_{13})V_1 \\
 & + (G_2 + G_{12}) \cdot V_2 \\
 & + (G_3 + G_{13})V_3 \\
 & = I_3
 \end{aligned}$$

$$\begin{array}{l}
 v_1: \\
 v_2: \\
 v_3:
 \end{array}
 \begin{bmatrix}
 g_1 + g_{12} + g_{13} & -g_{12} & -g_{13} \\
 -g_{12} - g_{13} & g_2 + g_{12} & g_3 + g_{13} \\
 0 & 1 & -1
 \end{bmatrix}
 \begin{bmatrix}
 v_1 \\
 v_2 \\
 v_3
 \end{bmatrix}
 =
 \begin{bmatrix}
 1 \\
 1/3 \\
 v_{23}
 \end{bmatrix}$$

$$\begin{bmatrix} G_1 + G_{12} + G_{13} & -G_{12} & -G_{13} \\ -G_{12} & G_2 + G_{12} + G_{23} & -G_{23} \\ -G_{12} & G_3 + G_{23} + G_{13} & G_3 + G_{23} + G_{13} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$-G_{13} \cdot V_1 + (G_2 - G_{23}) \cdot V_2 + (G_3 + G_{23} + G_{13}) V_3 = 1/3$$