

E6316: Analog Systems in VLSI

Distortion Analysis of Sampled Signals

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Discrete Fourier Transform (DFT) is used for analyzing the frequency components of a sampled signal. The output of a track-and-hold (T/H) or a sample-and-hold (S/H) circuit is an example of a sampled signal. The continuous time fourier transform of the output of a sample-and-hold signal contains components related to the transients of the signal before the output settles to its “held” value. If the circuit following the S/H is sensitive only to the “held” value, what is relevant is the DFT of the sequence of sampled signals and not the continuous-time fourier transform of the signal.

Cadence’s waveform calculator provides the discrete fourier transform function. The output $X(k/N f_s)$ (where $0 \leq k \leq N - 1$ and f_s is the sampling frequency) is related to the input $x(nT_s + T_o)$ (where $0 \leq n \leq N - 1$, $T_s = 1/f_s$ is the sampling period and T_o is the time of the first sample in the sequence) by the following equation

$$X\left(\frac{k}{N}f_s\right) = \sum_{n=0}^{N-1} x(nT_s) \exp\left(j2\pi \frac{k}{N}nT_s\right)$$

The input frequency must be a carefully chosen fraction of the sampling frequency because

- The output of a DFT is a sequence of signal strengths at a discrete set of frequencies $k/N f_s$, $0 \leq k \leq N - 1$. The frequency f_{in} of the sinusoid being analyzed must belong to this discrete set, i.e. be an integer multiple of f_s/N .
- Sampling causes aliasing. To avoid overlap of aliased frequencies (e.g. an aliased harmonic frequency being the same as the fundamental. Third harmonic of $f_{in} = f_s/2$ aliases to $f_s/2$, making it impossible to distinguish between the two).

Therefore, the input frequency must be $f_{in} = K/N f_s$ where K and N are mutually prime.

The way the DFT is implemented in Cadence, N must be an integer power of 2. So to satisfy all conditions, it is best to choose $N = 2^{n1}$, $K = 2^{n2} \pm 1$ where $n1 < n2$. e.g. $f_s = 1$ GHz. $f_{in}/f_s = 31/64$ for a 64 point FFT. In this case, the DFT will contain frequencies $\{0, 1/64 \text{ GHz}, \dots, 32/64 \text{ GHz}\}$ (i.e. from 0 to half the sampling frequency)

	before aliasing	after aliasing
Fundamental	31/64 GHz	31/64 GHz
2nd Harmonic	62/64 GHz	2/64 GHz
3rd Harmonic	93/64 GHz	29/64 GHz

and so on ...

Refer to Fig. 1 for a graphical depiction.

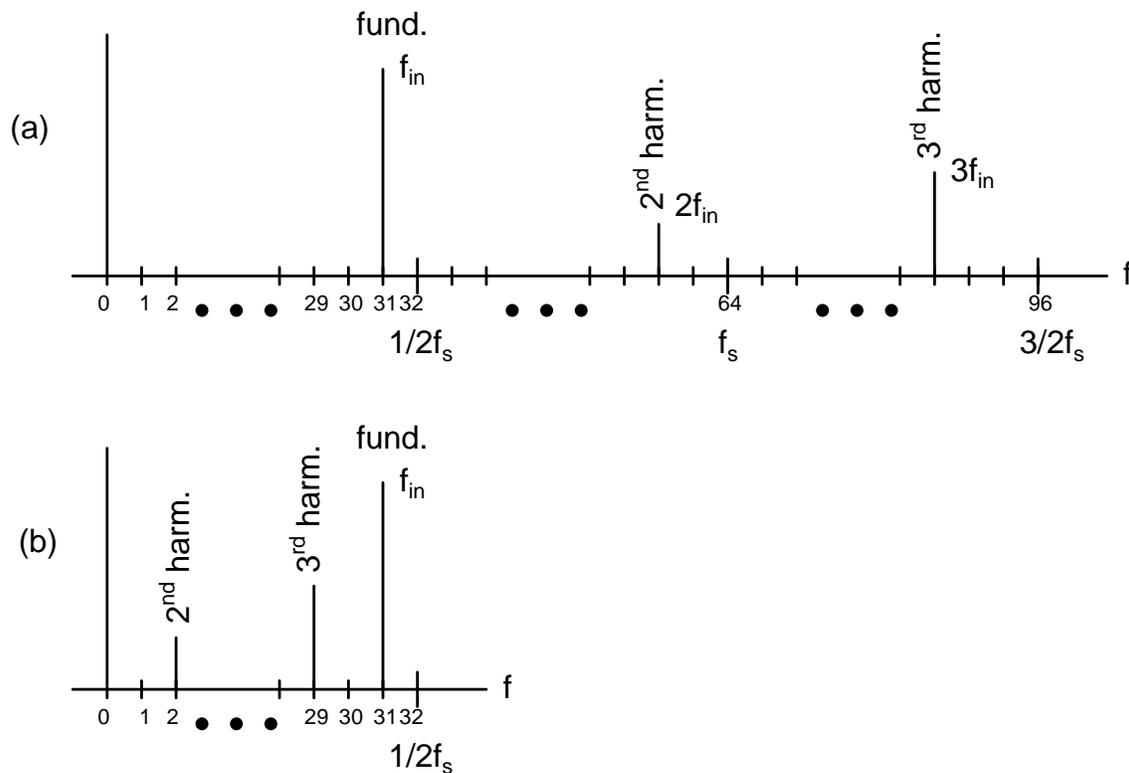


Figure 1: Distortion components before and after aliasing

In summary, to analyze the frequency content of a sampled signal in Cadence,

- Choose N . A larger N results in a finer resolution of frequencies. Say $N = 64$. Choose K to be mutually prime to N .
- Simulate the circuit for $T_o + NT_s$. T_o should be sufficient for the circuit to settle and the initial transients of the circuits to settle down and should be such that the signal is sampled near the end of the sampling phase.
- Calculate the DFT using the function `dft`
 Syntax: `dft(vtest, tstart, tstop, npts, 'Rectangular', 1, 1)`. This gives you the dft of `vtest` with the first sample at `tstart` ($= T_o$) and `npts` ($= N$) samples spaced $(tstop - tstart) / npts$ apart. Don't change the last 3 arguments of the function. For more extensive documentation, consult `cadence help` or type `ocnHelp('dft')` in Cadence main window.