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E6316: Analog systems in VLSI: Midterm solutions

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1. Residue $\left(= V_{LSB1} = \frac{V_{ref}}{2^4}\right)$ should be amplified to the range $(= V_{ref})$ of the second A/D converter.

$$G = 2^4 = 16$$

(a) $\cdot \left|\Delta V_{,amp}\right| \leq V_{LSB}/2 \cdot G = \frac{V_{ref}}{2^5} = 31.25\,mV$

(V_{LSB}: LSB of the overall converter)

(b) $\cdot \left|\Delta V_{,amp}\right| = \left|V_{in,amp,max}\right| \cdot \Delta G_{,max} \leq \frac{V_{LSB}}{2} \cdot G$ [from (a)]

$$= \frac{V_{ref}}{2^4} \cdot \Delta G_{,max} \leq \frac{V_{ref}}{2^9} \cdot G$$

$$\therefore \Delta G_{,max} = \frac{1}{2}$$

Amplifier gain with an opamp whose gain is A_0 is:

$$G' = \frac{G_0}{1 + \frac{G_0+1}{A_0}}$$

$$\Delta G = G' - G_0 = -\frac{G_0(G_0+1)/A}{1 + G_0+1/A_0}$$

$$\therefore A_0 \geq 2G_0(G_0+1) - (G_0+1) = 527 \quad (= 54.4\,dB)$$

(c)

$$\Delta V_{,amp} = V_{os} \cdot (1+G) \leq \frac{V_{LSB}}{2} \cdot G \quad \text{(from (a))}$$

$$V_{os} \leq \frac{V_{LSB}}{2} \cdot \frac{G}{1+G} = \frac{V_{ref}}{2^9} \cdot \frac{G}{1+G} = \underline{1.84\,mV}$$

(d) Transfer function of the amplifier

$$= \frac{G_0}{1 + \frac{s}{w_u}(G_0+1)}$$

$$\text{Ideal } \cancel{\text{input}} \text{ output} = V_{in,amp} \cdot G_0$$

$$\text{Actual} = V_{in,amp}\, G_0 \left(1 - e^{-\frac{w_u \cdot t}{G_0+1}}\right)$$

$$|\text{Error}| = V_{in,amp} \cdot G_0\, e^{-\frac{w_u \cdot t}{G_0+1}} \leq \cancel{V_{LSB}}\ \frac{V_{ref}}{2^5} \quad \text{(from (a))}$$

$$V_{in,amp,max} = V_{LSB_1} = \frac{V_{ref}}{2^4}$$

$$\therefore\ e^{-w_u t/G_0+1} \leq \frac{1}{2^5}, \quad t = \frac{T_s}{2} = 2.5\,ns$$

$$\therefore\ \omega_u \geq \frac{G_0+1}{(T_s/2)} \cdot 5\ln(2) = 23.6\ \text{Grad/s}$$

Unity gain frequency of the opamp = 3.75 GHz!

2. Accuracy of the D/A used in the 2 step converter is $\underline{\underline{8 \text{bits}}}$. i.e error in the o/p voltage can be at most $\frac{V_{ref}}{2^9} \cdot \left(\frac{V_{LSB}}{2}\right)$

Ideal output: $V_{o,ideal} = -V_{ref}\left[\frac{b_3}{2R} + \frac{b_2}{4R} + \frac{b_1}{8R} + \frac{b_0}{16R}\right] \cdot R$

Actual output:

$$V_{o,actual} = -V_{ref}\left[\frac{b_3}{2R(1+\alpha_3)} + \frac{b_2}{4R(1+\alpha_2)} + \frac{b_1}{8R(1+\alpha_1)} + \frac{b_0}{16R(1+\alpha_0)}\right] R$$

$$\left|Error\right| \simeq V_{ref}\left[\frac{b_3}{2} \cdot \alpha_3 + \frac{b_2}{4} \cdot \alpha_2 + \frac{b_1}{8} \alpha_1 + \frac{b_0}{16} \cdot \alpha_0\right]$$

Note that, in a D/A converter used in a 2 step converter, the $\underline{\text{absolute error}}$ matters. i.e $V_o\Big|_{D_{in} = 2^M - 1}$ is of importance. In a stand alone D/A converter, this merely influences the gain

Assume $\alpha_3 = \alpha_2 = \alpha_1 = \alpha_0$.

Worst case error $(b_3 b_2 b_1 b_0 = 1111)$ $= V_{ref} \cdot \frac{15}{16} \cdot \alpha \leq \frac{V_{LSB}}{2} = \frac{V_{ref}}{2^9}$

$$\boxed{\therefore \quad \alpha \leq \frac{1}{15 \cdot 2^5} = 0.21\,\%}$$

Worst case error due to offset at $b_3 b_2 b_1 b_0 = 1111$

$$\Delta V_{out} = V_{os} \cdot \left(1 + \frac{15}{16}\right) \leq \frac{V_{ref}}{2^9}$$

$$\therefore \quad V_{os} \leq \frac{V_{ref}}{2^5 \cdot 31} = 1\,mV$$

③ (a) $$V_{LSB} = \frac{V_{ref}}{2^6} = 15.6\,mV$$

Minimum input to preamplifiers $= \frac{V_{LSB}}{2}$, which must be amplified to 50mV

$$\therefore \quad \underline{Gain = 6.4}$$

(b) For an input @ $f_s/2$, error due to jitter $\Delta V = 2\pi \cdot V_{in,peak} \cdot \frac{f_s}{2} \cdot \Delta t_{jitter} \leq \frac{V_{LSB}}{2}$

$$V_{in,peak,max} = \frac{V_{ref}}{2}$$

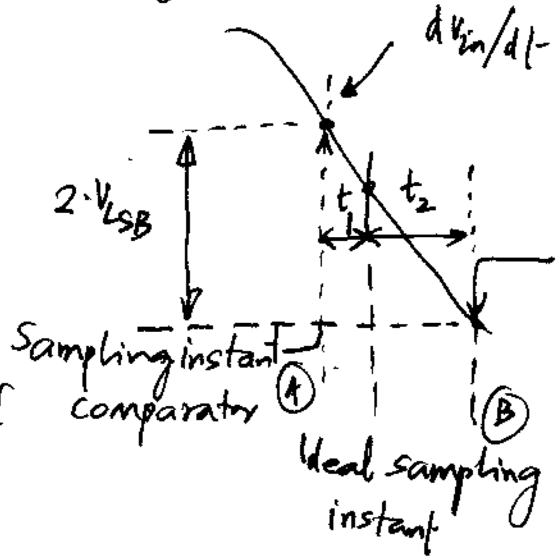
$$\Delta t_{,jitter} = \frac{V_{ref}/2^7}{2\pi \cdot \frac{V_{ref}}{2} \cdot \frac{f_s}{2}} = \frac{1}{f_s} \cdot \frac{1}{\pi \cdot 2^6}$$

$$= \underline{2.5\,ps}$$

(c) The logic looks for a '011' pattern in the output of the comparators to eliminate single bubbles in the ~~say~~ thermometer code.

	Ideal differential Input to preamplifiers		Ideal Comparator o/p	Non ideal Comparator o/p
$m+3$	$-5 \cdot V_{LSB}/2$		0	0
$m+2$	$-3 \cdot V_{LSB}/2$		0	0
$m+1$	$-V_{LSB}/2$	(A)	0	1
mV_{LSB}	$+V_{LSB}/2$		1	1
$m-1$	$+3V_{LSB}/2$	(B)	1	0
$m-2$	$+5V_{LSB}/2$		1	1
$m-3$	$+7V_{LSB}/2$		1	1

If the skews are such that comparator (A) sees an input $V_{LSB}/2$ more than what it should be and (B) sees an input $\frac{3V_{LSB}}{2}$ less than what it should be, a ... 011 011 ... pattern is seen at the o/p of the comparators, i.e an uncorrectable bubble.



t_1 : skew A

t_2 : skew B

A net skew $t_1 + t_2 = \dfrac{2V_{LSB}}{dV_{in}/dt}$ results in an uncorrectable bubble.

$$\left.\frac{dV_{in}}{dt}\right|_{max} = 2\pi \cdot \frac{f_s}{2} \cdot \frac{V_{ref}}{2}$$

$\therefore$ A skew of $\dfrac{1}{f_s} \cdot \dfrac{1}{\pi \cdot 2^4} = 10ps$ can result in a bubble.

[Maximum slope above holds if the skew is in the comparators near the middle of the ladder have a skew].

(d) Input to the 3 comparators in the 2 circuits in Fig. 3(b)

No Interpolation

top: $(V_{in} - (m+1) V_{LSB})G$

mid: $(V_{in} - m \cdot V_{LSB})G$

bot: $(V_{in} - (m-1) V_{LSB})G$

With 2x Interpolation

top: $(V_{in} - (m+1) V_{LSB})G$

mid: $\{(V_{in} - (m+1) \cdot V_{LSB})G + (V_{in} - (m-1) V_{LSB}) \cdot G\} \frac{1}{2}$

$= \underline{(V_{in} - m \cdot V_{LSB})G}$

bot: $(V_{in} - (m-1) V_{LSB})G$

The required gains are identical. $G = 6.4$