An Analysis of Inverter Crystal Oscillators

Improving Performance of a Common Circuit

By Leonard L. Kleinberg
NASA Gordon Space Flight Center

The ordinary linearly biased inverter is commonly employed in crystal oscillator circuits, such as the Pierce oscillator, up to several megahertz. However, by utilizing the inverter in a less conventional configuration, this frequency may be extended by a decade. The crystal may be either a fundamental or overtone cut.

The circuits are easily assembled since they have few components, are low cost, and exhibit better stability than the customary Pierce oscillator. The required supply is approximately 0.5 mA per megahertz.

The most commonly employed method of analyzing oscillator circuits is the feedback method. If an amplifier's gain is $k(\omega)$ and the feedback ratio is $B(\omega)$, the condition of oscillation is:

$$B(\omega)\, k(\omega) = 1 \tag{1}$$

For the circuit in Figure 1, which may be obtained by successive applications of the Thevenin/Norton theorem on a more common circuit,

$$B(\omega) = \frac{Z_2}{Z_1 + Z_2} = \frac{1}{k(\omega)} \tag{2}$$

Another method for analyzing oscillators is to look into the input terminal and equate the driving point admittance (DPA) to zero.

$$Y_{in} = DPA = \frac{1}{Z_2} + \frac{1 - k(\omega)}{Z_1} = 0 \tag{3}$$

$$DPA = \frac{1}{Z_2} + \frac{1}{Z_1} = \frac{k(\omega)}{Z_1} \tag{4}$$

Note that equation 4 is identical to equation 2. This second method of analysis is what will be used for the gate oscillators.

The circuit of Figure 2(a) is the most used gate crystal oscillator, where the crystal is operated in the inductive mode (L_x). Figure 2(b) depicts the approximate equivalent circuit used in determining the driving point admittance at the input terminal. The addition of R_1 is required, since the output resistance of the inverter is usually very small compared to the reactance of C_1. Direct application of equation 4 shows that there is no negative conductance in the DPA, and no oscillations can take place.

The DPA is given by:

$$DPA = \left[1 + \left(\frac{k}{R_1}\right)\left(\frac{1}{J\omega C_1}\right)\right]\left(\frac{1}{J\omega L_x + 1/(J\omega C_1)}\right) + \left(\frac{1+k}{R_B}\right)$$

$$= \frac{1}{J\omega L_x + 1/(J\omega C_1)} + \frac{k}{R_1(1 - \omega^2 L_x C_1)} + \frac{1 + k}{R_B} \tag{5}$$

To complete the DPA, C_2 is added in parallel to term 1 in equation 5:

(Condition 1)

$$J\omega C_2 + \frac{1}{J\omega L_x + 1/(J\omega C_1)} = 0 \tag{6}$$

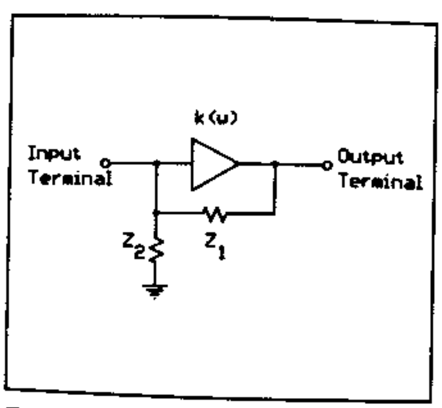

Figure 1. A simplified feedback circuit.

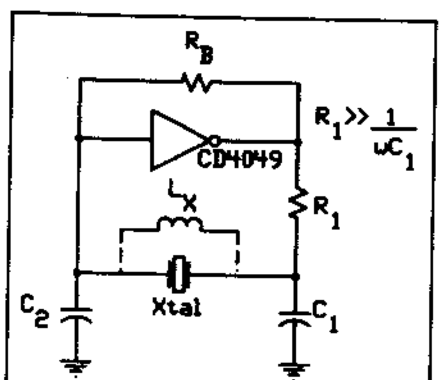

Figure 2(a). A commonly used gate crystal oscillator.

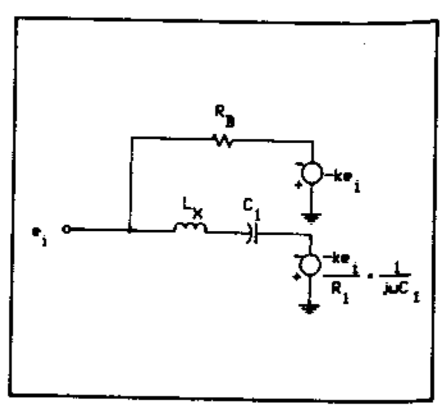

Figure 2(b). Approximate equivalent circuit to determine DPA.

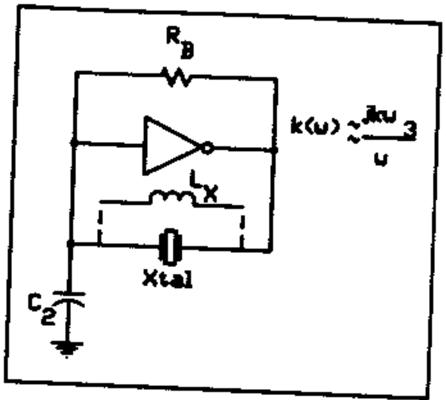

Figure 3(a). Circuit with 90 degrees phase shift.

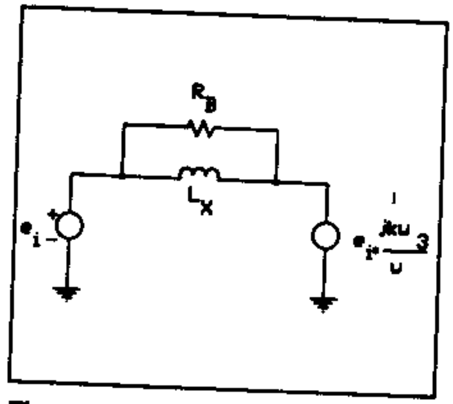

Figure 3(b). Equivalent circuit for Figure 3(a).

This yields the expected expression for ω^2:

$$\omega^2 = \frac{C_1 + C_2}{L_x C_1 C_2} \qquad (7)$$

Substituting equation 7 into term 2 of equation 5 yields a negative conductance (G):

(Condition 2)

$$G = -\frac{kC_2}{R_1C_1} = -\frac{1+k}{R_B} \qquad (8)$$

$$\frac{kC_2}{R_1C_1} + \frac{1+k}{R_B} = 0 \qquad (8a)$$

It is interesting to note that if the circuit of Figure 2(a) is analyzed conventionally, a value of $B(\omega)$ equal to $-C_1/C_2$ is obtained. This implies that as C_2 is increased, the amount of feedback is reduced. However, equation 8 indicates the contrary — as C_2 is increased, the amount of feedback is increased. The latter case is the correct one.

The negative conductance, or positive feedback, was obtained because the voltage across C_1 experienced a 90 degree phase shift after the initial amplifier inversion. At low frequencies the R_1–C_1 elements are essential to oscillation, while at frequencies considerably above the 3 dB cut-off frequency R_1–C_1 may be omitted from the circuit, as the amplifier introduces the required 90 degree phase shift. The circuit is shown in Figure 3(a) and the equivalent circuit in Figure 3(b).

The DPA is given by:

$$DPA = \left(1 - \frac{Jk\omega_3}{\omega}\right)\left(\frac{1}{J\omega L_X} + \frac{1}{R_B}\right) = \frac{1}{J\omega L_x} - \frac{k\omega_3}{\omega^2 L_x} + \frac{1}{R_B} - \frac{Jk\omega_3}{\omega R_B} \qquad (9)$$

For the case where R_B is very large (>1 Megohm), term 4 may be neglected noting that 4 is an inductive term and will be considered later. By adding C_2 at the input terminal, the design of the oscillator is completed.

$$J\omega C_2 + \frac{1}{J\omega L_x} = 0 \;;\; \omega^2 = \frac{1}{L_x C_2} \qquad (10)$$

$$G = -\frac{k\omega_3}{\omega^2 L_x}$$

$$\frac{k\omega_3}{\omega^2 L_x} = \frac{1}{R_B} = k\omega_3 C_2 \qquad (11)$$

The gain vs. frequency response was plotted for the CD4007 and CD4049 (inverters) and for the CD4011 (2 input NAND), as a function of supply voltage. The values are shown in Tables 1, 2 and 3. The basic circuit using the CD4011 is depicted in Figure 4.

If an overtone crystal is used in the basic circuit (Figure 5), it will oscillate at the fundamental frequency. By adding a capacitor in series with the crystal, overtone operation is achieved. With today's newer crystals, in most cases a fundamental cut crystal may be obtained.

Another method of obtaining overtone operation, without C_1, is to parallel C_T with an inductor (L_1) such that:

$X_{L1} > X_{CT}$ at fundamental
$X_{L1} < X_{CT}$ at third overtone

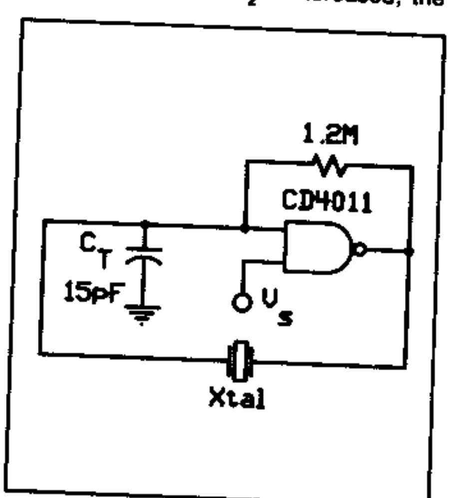

Figure 4. Basic circuit using the 4011.

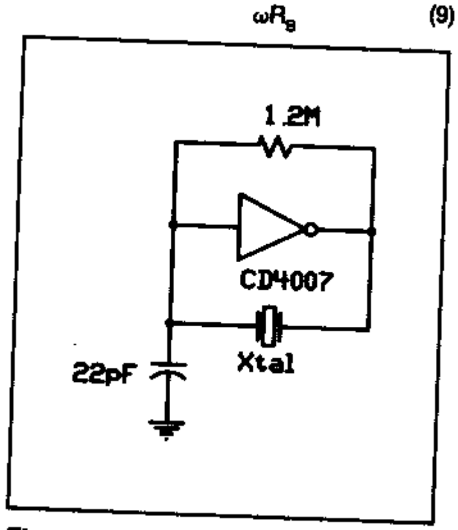

Figure 5. Basic circuit using the 4007.

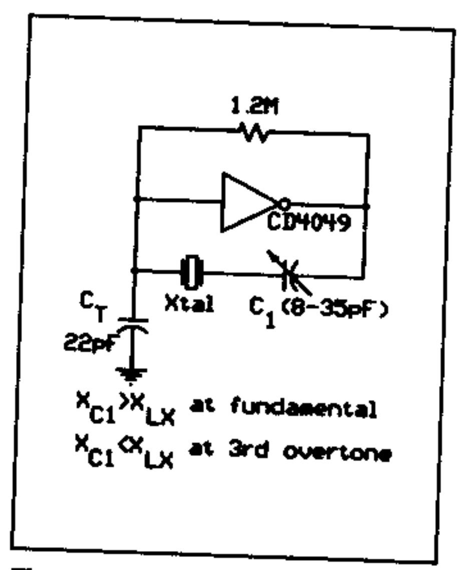

Figure 6. Circuit using the 4049.

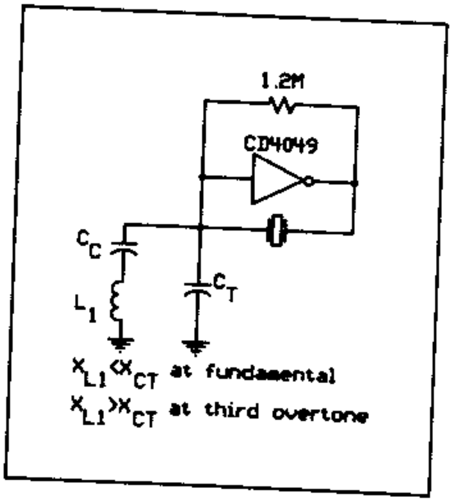

Figure 7. This circuit obtains overtone operation with the use of a parallel inductor.

Using this condition of adding L_1, and referring to the circuit of Figure 3(a), the analysis of the DPA (equation 9) indicates that the fourth term $(-Jk\,\omega_3)/(\omega R_B)$ is an inductive admittance. By selecting R_B properly, it may serve the purpose of L_1 in Figure 7. The conditions are:

$$\frac{\omega_1 R_B}{\omega_3 k} < \frac{1}{\omega_1 C_T} \quad \omega_1 \text{ is the fundamental crystal} \tag{12}$$

$$\frac{3\omega_1 R_B}{\omega_3 k} > \frac{1}{3\omega_1 C_T} \tag{13}$$

The circuit shown in Figure 8 oscillated at the third overtone of a 25 MHz and a 50 MHz crystal. Notice the simplicity of the circuit.

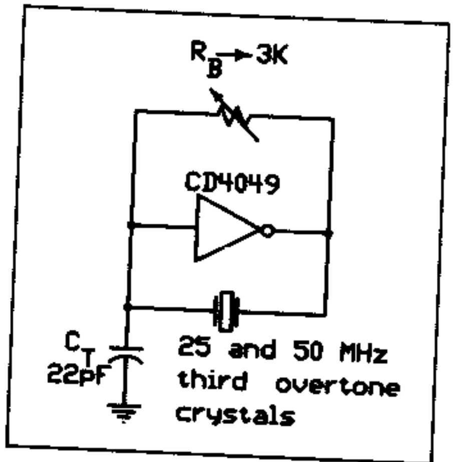

Figure 8. A simple third overtone oscillator.

Summary

By biasing a CMOS inverter in its linear region and by using the device considerably beyond its 3 dB frequency, a simple, inexpensive and stable crystal oscillator — fundamental or overtone — may be quickly designed and assembled. The analysis describes the conditions of oscillation (i.e., the DPA is zero), and considers the gain of the inverter at high frequencies to be a 90 degree phase shifter. The technique is not limited to gate inverters, but includes op amps, transistors and FETs.

For a reasonably good time-base generator, the CD4049 with a 5 V supply and 2 MHz crystal (Figure 5) is approximately one part in 10^8 (1 sec). The 5 V supply should be derived by using a zener diode or a three terminal regulator. This regulation is good practice for this class of oscillators. For higher frequency oscillators the author finds it easier to work with fundamental cut crystal.

V_{Supply}	k	F_3	Xtal Freq
5	50	300 kHz	5 MHz
10	25	2 MHz	10-20 MHz
15	20	2.2 MHz	20-50 MHz

Table 1. Gain vs. frequency response for the CD4049.

V_{Supply}	k	F_3	Xtal Freq
5	42	100 kHz	0.5-1 MHz
10	35	100 kHz	2 MHz
15	24	600 kHz	5 MHz

Table 2. Gain vs. frequency response for the CD4007.

V_{Supply}	k	F_3	Xtal Freq
5	20	800 kHz	5 MHz
10	20	2.0 MHz	10 MHz
15	20	2.2 MHz	15 MHz

Table 3. Gain vs. frequency response for the CD4011.

About the Author

Len Kleinberg is an electronic engineer at the NASA Goddard Space Flight Center, Code 728, Greenbelt, MD 20771. He can be reached by telephone at (301) 286-5683.