An Analysis of Inverter Crystal Oscillators

Improving Performance of a Common Circuit

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The ordinary linearly biased inverter is commonly employed in crystal oscillator circuits, such as the Pierce oscillator, up to several megahertz. However, by utilizing the inverter in a less conventional configuration, this frequency may be extended by a decade. The crystal may be either a fundamental or overtone cut.

The circuits are easily assembled since they have few components, are low cost, and exhibit better stability than the customary Pierce oscillator. The required supply is approximately 0.5 mA per megahertz.

The most commonly employed method of analyzing oscillator circuits is the feedback method. If an amplifier's gain is $k(\omega)$ and the feedback ratio is $B(\omega)$, the condition of oscillation is:

$$B(\omega) k(\omega) = 1 \quad (1)$$

For the circuit in Figure 1, which may be obtained by successive applications of the Thévenin/Norton theorem on a more common circuit,

$$B(\omega) = \frac{Z_2}{Z_1 + Z_2} = \frac{1}{k(\omega)} \quad (2)$$

Another method for analyzing oscillators is to look into the input terminal and equate the driving point admittance (DPA) to zero.

$$Y_{in} = \text{DPA} = \frac{1}{Z_2} + \frac{1 - k(\omega)}{Z_1} = 0 \quad (3)$$

$$\text{DPA} = \frac{1}{Z_2} + \frac{1}{Z_1} = \frac{k(\omega)}{Z_1} \quad (4)$$

Note that equation 4 is identical to equation 2. This second method of analysis is what will be used for the gate oscillators.

The circuit of Figure 2(a) is the most used gate crystal oscillator, where the crystal is operated in the inductive mode ($L_2$). Figure 2(b) depicts the approximate equivalent circuit used in determining the driving point admittance at the input terminal. The addition of $R_s$ is required, since the output resistance of the inverter is usually very small compared to the reactance of $C_1$. Direct application of equation 4 shows that there is no negative conductance in the DPA, and no oscillations can take place.

The DPA is given by:

$$\text{DPA} = \left[1 + \left(\frac{k}{R_1}\right)\left(\frac{1}{\omega C_1}\right)\left(\frac{1}{j\omega L_2 + 1/\omega C_1}\right)\right]^{-1}$$

$$= \frac{1}{\frac{1}{j\omega L_2 + 1/\omega C_1} + \frac{k}{R_s}}$$

$$= \frac{1}{\frac{1}{R_s(1 - \omega_0^2 C_1)} + \frac{1 + k}{R_s}} \quad (5)$$

To complete the DPA, $C_2$ is added in parallel to term 1 in equation 5:

$$(\text{Condition 1})$$

$$\omega C_2 + \frac{1}{j\omega L_2 + 1/\omega C_1} = 0 \quad (6)$$

Figure 1. A simplified feedback circuit.

Figure 2(a). A commonly used gate crystal oscillator.

Figure 2(b). Approximate equivalent circuit to determine DPA.
Figure 3(a). Circuit with 90 degrees phase shift.

This yields the expected expression for \( \omega^2 \):

\[
\omega^2 = \frac{C_1 + C_2}{L_1C_1C_2}
\]

(7)

Substituting equation 7 into term 2 of equation 5 yields a negative conductance (G):

(Condition 2)

\[
G = -\frac{kC_2}{R_1C_1} = -\frac{1 + k}{R_B}
\]

(8)

\[
\frac{kC_2}{R_1C_1} + \frac{1 + k}{R_B} = 0
\]

(8a)

It is interesting to note that if the circuit of Figure 2(a) is analyzed conventionally, a value of \( B(\omega) = -C_1C_2 \) is obtained. This implies that as \( C_2 \) is increased, the amount of feedback is reduced. However, equation 8 indicates the contrary — as \( C_2 \) is increased, the amount of feedback is increased. The latter case is the correct one.

The negative conductance, or positive feedback, is obtained by putting the voltage across \( C_1 \) and \( 90 \) degree phase shift after the initial amplifier inversion. At low frequencies the \( R_1-C_1 \) elements are essential to oscillation, while at frequencies considerably above the 3 dB cut-off frequency \( R_1-C_1 \) may be omitted from the circuit, as the amplifier introduces the required 90 degree phase shift. The circuit is shown in Figure 3(a) and the equivalent circuit in Figure 3(b).

The DPA is given by:

\[
DPA = \left(1 - \frac{jk\omega_2}{\omega}\right) \left(\frac{1}{Jo\omega_2 L_x} + \frac{1}{R_B}\right)
\]

\[
= \frac{1}{\omega L_x} - \frac{jk\omega_2}{\omega L_x} + \frac{1}{R_B} - \frac{jk\omega_2}{\omega R_B}
\]

(9)

For the case where \( R_B \) is very large (>1 Megohm), term 4 may be neglected noting that 4 is an inductive term and will be considered later. By adding \( C_2 \) at the input terminal, the design of the oscillator is completed.

\[
\omega_0 C_2 + \frac{1}{\omega_0 L_x} = 0 ; \quad \omega_0^2 = \frac{1}{L_1C_2}
\]

(10)

\[
G = -\frac{k\omega_2}{\omega L_x}
\]

\[
\frac{k\omega_2}{\omega L_x} = \frac{1}{R_B} = k\omega_2 C_2
\]

(11)

The gain vs. frequency response was plotted for the CD4007 and CD4049 (inverters) and for the CD4011 (2 input NAND), as a function of supply voltage. The values are shown in Tables 1, 2 and 3. The basic circuit using the CD4011 is depicted in Figure 4.

If an overtone crystal is used in the basic circuit (Figure 5), it will oscillate at the fundamental frequency. By adding a capacitor in series with the crystal, overtone operation is achieved. With today's newer crystals, in most cases a fundamental cut crystal may be obtained.

Another method of obtaining overtone operation, without \( C_1 \), is to parallel \( C_T \) with an inductor \( L_x \) such that:

\[ X_{LT}X_{CT} \text{ at fundamental} \]

\[ X_{LT}X_{CT} \text{ at third overtone} \]
Oscillator Design Handbook

Figure 7. This circuit obtains overtone operation with the use of a parallel inductor.

Using this condition of adding \( L_1 \), and referring to the circuit of Figure 3(a), the analysis of the DPA (equation 9) indicates that the fourth term \((-j \omega L_1 \omega R_0)\) is an inductive admittance. By selecting \( R_0 \) properly, it may serve the purpose of \( L_1 \) in Figure 7. The conditions are:

\[
\frac{\omega L_1 R_0}{\omega^2 k} < \frac{1}{\omega^2 L_1 C_T} \quad (12)
\]

\[
\frac{3\omega L_1 R_0}{3\omega^2 k} = \frac{1}{3\omega L_1 C_T} \quad (13)
\]

The circuit shown in Figure 8 oscillated at the third overtone of a 25 MHz and a 50 MHz crystal. Notice the simplicity of the circuit.

Figure 8. A simple third overtone oscillator.

Summary

By biasing a CMOS inverter in its linear region and by using the device considerably beyond its 3 dB frequency, a simple, inexpensive and stable crystal oscillator — fundamental or overtone — may be quickly designed and assembled. The analysis describes the conditions of oscillation (i.e., the DPA is zero), and considers the gain of the inverter at high frequencies to be a 90 degree phase shifter. The technique is not limited to gate inverters, but includes op amps, transistors and FETs.

For a reasonably good time-base generator, the CD4049 with a 5 V supply and 2 MHz crystal (Figure 5) is approximately one part in \( 10^4 \) (1 sec). The 5 V supply should be derived by using a zener diode or a three terminal regulator. This regulation is good practice for this class of oscillators. For higher frequency oscillators the author finds it easier to work with fundamental cut crystal.

Table 1. Gain vs. frequency response for the CD4049.

<table>
<thead>
<tr>
<th>( V_{\text{Supply}} )</th>
<th>( k )</th>
<th>( F_3 )</th>
<th>Xtal Freq</th>
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</thead>
<tbody>
<tr>
<td>5</td>
<td>50</td>
<td>300 kHz</td>
<td>5 MHz</td>
</tr>
<tr>
<td>10</td>
<td>25</td>
<td>2 MHz</td>
<td>10-20 MHz</td>
</tr>
<tr>
<td>15</td>
<td>20</td>
<td>2.2 MHz</td>
<td>20-50 MHz</td>
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Table 2. Gain vs. frequency response for the CD4007.

<table>
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<th>( k )</th>
<th>( F_3 )</th>
<th>Xtal Freq</th>
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<td>0.5-1 MHz</td>
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<tr>
<td>10</td>
<td>35</td>
<td>100 kHz</td>
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<tr>
<td>15</td>
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<td>600 kHz</td>
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Table 3. Gain vs. frequency response for the CD4011.

<table>
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<th>( k )</th>
<th>( F_3 )</th>
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<tr>
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About the Author

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