Downlink Resource Allocation under Time-varying Interference: Fairness and Throughput Optimality

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Abstract

We address the problem of downlink resource allocation in the presence of time-varying interference. We consider a scenario where users served by a base station face interference from a neighbouring base station that uses the same channels. We model the interference from the neighbouring base station as an ON/OFF renewal process, that arises due to its idle and busy cycles. The users feedback their downlink SINR values to their base station, but these values are outdated. In this setting, we characterize how Layer 2 can optimally exploit the reported SINR values, which could be unreliable due to time-varying interference. In particular, we propose resource allocation policies in two well-known paradigms. First, we address the problem of $\alpha$—fair scheduling, and propose a policy that ensures asymptotic convergence to the optimal $\alpha$—fair throughput. Second, we propose a throughput optimal resource allocation policy, i.e., a policy that can stably support the largest possible set of traffic rates under the interference scenario considered. Estimating the outage probability from the outdated SINR values plays an important role in both scheduling paradigms, and we accomplish this using tools from renewal theory.

I. INTRODUCTION

In this paper, we tackle the problem of downlink resource allocation, in a scenario where the users suffer time-varying interference from neighbouring base stations.

It is well known that a user equipment (UE) served by a base station does not suffer significant interference from transmissions to other UEs in the same cell. This is because UEs in the same cell are typically served on orthogonal resources. For example, in a Time Division Multiple

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Access (TDMA) system, different UEs are served during different time-slots, whereas in a multi-carrier system such as Orthogonal Frequency Division Multiple Access (OFDMA), different UEs in a cell are served on different frequency sub-bands. On the other hand, a given UE does suffer interference from neighbouring base stations that use the same resources to transmit to UEs in the neighbouring cells. In addition, this interference is usually time-varying, due to the underlying idle and busy cycles of the interfering base stations on that particular channel. Since this time-varying interference from neighbouring base stations can have a detrimental effect on the throughput obtained by the UEs – particularly the cell-edge UEs – it is important to understand its implications on resource allocation.

The main idea behind opportunistic resource allocation [2] is to preferentially transmit on channels with favourable conditions. In order to enable opportunistic resource allocation, the UEs have to feedback their instantaneous channel quality information (CQI) to the base station from time to time. However, the CQI available at the base station is usually outdated by a few time-slots due to propagation and processing delays, or due to other practical reasons. In a scenario with time-varying interference, delayed CQI implies that it is possible for the channels to have changed drastically between the time that the CQI is measured and resource allocation is performed. For example, consider a UE that reports a relatively strong channel to the base station when an interferer is idle. When the user is scheduled however, the interferer could turn active on that channel, thereby causing a transmission outage. This clearly indicates that it is not desirable to directly use the outdated CQI to perform resource allocation.

In this paper, we specify how Layer 2 should interpret the delayed CQI in the presence of time-varying interference, in order to perform resource allocation efficiently. Specifically, we exploit the statistics of the time-varying interference to design an optimal resource allocation policy for this scenario. We consider a base station serving several UEs. Each UE suffers time-varying interference from a neighboring base station. We model the interference from the neighboring base station using an ON/OFF renewal process [3, Section 3.4.1]. We view the ON/OFF interference from the neighboring base station as arising due to the underlying busy and idle cycles of transmissions to a neighboring cell user.

In this setting, we use tools from renewal theory to compute the conditional distribution of the current channel quality, given the delayed CQI available at the base station. This conditional distribution allows us to compute the conditional outage probability of each user, for any given rate of transmission. We then use this conditional outage probability to devise a fair scheduling
algorithm and invoke the theory of Stochastic Approximation [4] to prove its optimality under the $\alpha$-fair scheduling regime. Next, to derive a throughput optimal resource allocation policy for the interference scenario considered, we combine our outage probability expression with a Lyapunov stability framework.

A. Related work:

The problem of resource allocation in a wireless network has been widely studied and throughput optimal and $\alpha$-fair resource allocation policies have been established in various scenarios.

The most commonly studied algorithm in the $\alpha$-fair regime is the Gradient algorithm which was first proven to be optimal in [5] using the theory of Stochastic Approximation. The algorithm has subsequently been studied in an OFDM framework for downlink scheduling and resource allocation in [6] and other similar works. The $\alpha$-fair scheduling paradigm has been studied from a cross layer perspective in [7] for a network with uncertain channel state information.

The Lyapunov stability framework for designing throughput optimal scheduling policies was introduced in [8]. Subsequently, [9] addressed the issue of throughput optimal scheduling with delayed network state information and time-varying channels. Similarly, [10], [11] and [12] address scenarios with partial, infrequent and sparse channel information, respectively. Although the above papers do consider the effect of CQI uncertainties, the channel is often considered to be independent and identically distributed (i.i.d) across slots. A few recent papers [13], [14], [15] have also considered exploiting temporal correlation, using Markov models for the channel state.

The problem of throughput-optimal resource allocation with delayed network state information has been studied in [16] for channel capacities that vary according to a finite-state Markov chain and a collision model based interference. Opportunistic multiuser downlink scheduling under a two-state Markov chain channel model for throughput maximization is done using a partially observable Markov decision process (POMDP framework) in [17]. Similarly, scheduling under partial channel state information has also been studied in [18].

As we observe in this paper, the value of the conditional outage probability plays a critical role in the resource allocation policy design. Computation of outage (coverage) probability under time-varying interference has been studied in [19]. However, in our scenario, the computation is to be performed using delayed information and to this end, we make use of an algorithm that is similar in structure to the Prediction by Partial Match (PPM) [20] algorithm.
A distinguishing feature of our paper is that we capture the temporal correlations in the channel by explicitly modeling the interference using ON/OFF renewal processes. We believe that a renewal process represents a more natural/realistic framework to model the idle/busy cycles of an interfering basestation, compared to simpler iid models used elsewhere in the literature [16], [20]. We then show how the statistical properties of the interference can be exploited to construct $\alpha$-fair and throughput optimal resource allocation policies.

II. SYSTEM MODEL

A. Interference

Consider a time-slotted downlink system comprising of a base station $B_0$ serving $K$ UEs. We assume that the UEs experience interference from a single neighbouring base station $B_1$ (see Fig. 1). We also assume that the base stations have synchronized slot boundaries. Let the transmit power of the interfering base station at time $t$, be given by $P_1(t)$ and that of base station $B_0$ be $P_0(t)$. For simplicity, suppose that the base stations transmit with unit power whenever they transmit.

Fig. 1: Base Station Layout: Mobiles experience interference from transmissions of the neighboring base station

The idle and busy cycles of transmission of the interfering base station are assumed to constitute an arithmetic ON/OFF renewal process [3, Section 3.4.1] of span 1. Our renewal
model is motivated by the fact that the idle and busy cycles of fairly general buffering systems (such as a G/G/1 queue, for example) constitute renewal processes. The OFF and ON periods of the interferer are assumed to be distributed according to the random variables $Z$ and $Y$ respectively. We assume that their joint probability mass function (pmf), given by $\mathbb{P}_{Z,Y}(\cdot)$, is known to $B_0$.

**B. Fading**

We consider a frequency-flat block fading channel that offers a constant fading gain to the UEs over a block of size $M$ slots. For ease of computation, we assume that all the UEs experience a Rayleigh fading channel from $B_0$ and $B_1$. The average fading power gain to user $i$ from $B_0$ and $B_1$ are assumed to be known to $B_0$ and are denoted by $\Gamma_{0,i} = \mathbb{E}[g_{0,i}(t)]$ and $\Gamma_{1,i} = \mathbb{E}[g_{1,i}(t)]$ respectively. Here, $g_{0,i}(t)$ and $g_{1,i}(t)$ are the time varying fading power gains to user $i$ from $B_0$ and $B_1$ respectively.

![Interferer Power Cycle](image1)

![Capacity variation as a function of time](image2)

*Fig. 2: Time-varying Interference and Capacity: Fluctuations of capacity as a result of block fading and ON/OFF Interference*
The ON/OFF interference and the ensuing fluctuations in Capacity of the channel owing to interference and block fading effects are depicted in Figure 2. The dotted lines represent the fade boundaries. The Channel capacity is computed based on the expression given in (2). The figure clearly depicts the extent of variation in the Channel capacity over time thereby highlighting the effect the delay can potentially have on resource allocation.

C. Channel Quality Information

The signal to interference plus noise ratio (SINR) for UE $i$ at time $t$ is given by

$$\gamma_i(t) = \frac{g_{0,i}(t)P_0(t)}{N_0 + g_{1,i}(t)P_1(t)},$$

where $N_0$ is the average additive white Gaussian noise (AWGN) power for the channel. We assume that the capacity of the channel to user $i$ at time $t$ is given by

$$C_i(t) = \log_2(1 + \gamma_i(t)).$$

We assume that the UEs follow an aperiodic CQI feedback scheme, wherein the channel CQI is reported to $B_0$ whenever a change in the CQI value is observed in a slot. Owing to delays due to propagation and processing, the information sent by user $i$ is received at the base station with a constant, known delay of $\delta_i$ slots.

We assume in this paper that the CQI reported by user $i$ is the SINR, $\gamma_i(t)$, observed by the user over slot $t$. The user observes the pilot symbols, computes the channel gains from both $B_0$ and $B_1$ and reports the SINR to the base station $B_0$. Thus, $B_0$ observes $\gamma_i(t - \delta_i)$ at slot $t$. Since the CQI at every change point is reported, the Base Station $B_0$ is aware of the SINR values for all slots, albeit with a delay.

The renewal cycles represent the busy and idle cycles of the interfering base station. They are expected to be fairly large in comparison to the CQI feedback delay experienced by the users. Thus, we make the following assumption on the distribution of the renewal period :

$$\mathbb{P}(X \leq \delta_{\text{max}}) \leq \chi << 1,$$

where $\delta_{\text{max}}$ is an upper bound to the feedback delay experienced by the users. We also assume that the delays apply to all information, including ACKs which are fed back by the UEs. Thus, the success or failure of a transmission attempted for UE $i$ at time $t$ is learnt only at $t + \delta_i$. 
III. Preliminaries

In this paper, we address two well-known resource allocation paradigms: Throughput Optimal and $\alpha$–fair Resource Allocation. In this section, we will introduce some notations and preliminaries with regard to each of the resource allocation policies considered.

As was described in Section II, the idle and busy cycles of the interfering base station are assumed to be a span 1 ON/OFF renewal process. That is, the transmission cycle of the interferer renews at every transition from ON to OFF. Thus, the interferer power, $P_1(t)$ is distributed independently across these renewal periods. Let the $n^{th}$ renewal epoch of the ON/OFF renewal process be denoted as $S_n$. Let $N(t)$ denote the number of renewals observed until time $t$.

A. $\alpha$–fair Resource Allocation

The $\alpha$– fair resource allocation policy aims to ensure fair usage among all users, by maximizing a concave utility function of long term throughputs. In this setting, we assume that the system is “saturated”, i.e, every user has data to be served at all times.

Let $\mu_i(t)$ be the number of packets of user $i$ successfully transmitted by $B_0$ in slot $t$, which is given by:

\[ \mu_i(t) = a_i r_i(t) I_i(t). \] (4)

In (4), $r_i(t)$ is the transmit rate assigned for user $i$, $a_i$ denotes the fraction of slot $t$ that is allocated to the user $i$ and $I_i(t) = 1$, if $C_i(t) \geq r_i(t)$ and is 0 otherwise. The transmission at a rate of $r_i$ is said to be in outage if the attempted rate of transmission exceeds the capacity of the channel in that slot.

The average downlink throughput obtained by user $i$ till slot $t$ is

\[ \theta_i(t) = \frac{1}{t} \sum_{\tau=1}^{t} \mu_i(\tau). \] (5)

The objective of the $\alpha$–fair paradigm is to maximize a concave utility function,

\[ U(\tilde{\theta}) = \sum_{i=1}^{K} U_i(\theta_i), \] (6)

where, the individual user utilities are given by

\[ U_i(\theta_i) = \begin{cases} \frac{1}{\alpha} (\theta_i)^\alpha, & \alpha \leq 1, \alpha \neq 0, \\ \log(\theta_i), & \alpha = 0. \end{cases} \] (7)
The case of $\alpha = 0$ is the criterion that defines the Proportional Fair Scheduler [5]. In Section IV, we describe an algorithm that ensures that the steady-state throughputs asymptotically converge to a solution that maximizes the desired utility function.

B. Throughput Optimal Resource Allocation

The throughput optimal class of resource allocation policies make decisions such that the queues in the network are stabilized for the largest supportable set of arrival rates.

We assume that the base station maintains separate downlink buffers for each UE. Exogenous arrivals for UE $i$ are characterized by the arrival process $A_i(t)$. We assume that the arrivals are independent and identically distributed (i.i.d) from slot to slot and are independent of the channel realizations. Let $\mathbb{E}[A_i(t)] = \lambda_i$ and $\mathbb{E}[A_i^2(t)] = \tilde{A}_i < \infty$. Let $Q_i(t)$ be the length of the downlink buffer corresponding to user $i$. We can characterize the dynamics of the queue corresponding to user $i$ as:

$$Q_i(t+1) = (Q_i(t) - \mu_i(t))^+ + A_i(t), \quad (8)$$

where, $(x)^+ = \max\{x, 0\}$.

The queuing system comprising of $K$ queues is said to be strongly stable, if $\forall 1 \leq i \leq K$,

$$\limsup_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T} \mathbb{E}[Q_i(t)] < \infty.$$

Let $\Pi$ be the class of all possible resource allocation algorithms operating under the fading and information delay constraints mentioned above. The stability region, $\Lambda$, of the system is defined as the set of all arrival rate vectors, $\bar{\Lambda}$, such that there exists a policy $\pi \in \Pi$ that stabilizes the queuing system. A policy $\pi^* \in \Pi$ is said to be throughput optimal if it maintains strong stability in the queuing system for all arrival rates in the interior of the stability region $\Lambda$.

Thus, during each slot, the base station has to determine the users to transmit to, the fraction of the slot to be allocated to each user and the optimal rate of transmission for each user using the available delayed CQI and the statistics of the time-varying interference. In Section V, we first develop a provably throughput optimal policy for single-carrier systems and then extend it to multi-carrier systems.

IV. $\alpha$-FAIR RESOURCE ALLOCATION

In this section, we consider the regime of $\alpha$-fair resource allocation and propose the optimal policy under the system model being studied. For this section, we assume that the transmit rate for any user in a slot has an upper limit of $R_{max}$. 
Algorithm 1 Optimal $\alpha$–fair Policy, $\pi^*_\alpha$

\begin{algorithm}
\For{$i = 1$ to $K$} {
\State Compute $p_i(r)$ (Section VI)
\State $r^*_i(t) \leftarrow \arg \max_{0 \leq r \leq R_{\text{max}}} \{G_i(r)\}$
\EndFor
\State $k(t) \leftarrow \arg \max_{1 \leq i \leq K} \{\nabla U_i(\theta_i(t - \delta_i))G_i(r^*_i(t))\}$
\State Transmit to UE $k(t)$ at rate $r^*_k(t)$
\end{algorithm}

Let $p_i(r)$ be the conditional outage probability for a transmission of rate $r$ to UE $i$, given the delayed SINR history. Let $\bar{\gamma}_i(t) = \{\gamma_i(\tau) : \tau \leq t - \delta_i\}$ and let $\bar{\gamma}(t) = \cup_i \bar{\gamma}_i(t)$. Then,

$$p_i(r) = P(C_i(t) \leq r | \bar{\gamma}(t)).$$

In Section VI, we describe how this outage probability can be computed by the base station $B_0$. Let us define the goodput for UE $i$ as $G_i(r_i) = r_i(1 - p_i(r_i))$, which is the average successful rate of transmission.

For each slot $t$, the base station performs Algorithm 1. At each time slot, the BS determines the optimal rate of transmission for each user as the rate that maximizes the goodput and transmits at the corresponding rate to that user who maximizes the goodput-gradient utility product. The method to determine the outage probability function, given the delayed SINR information is elaborated in Section VI.

We now prove the optimality of the proposed policy. First, we invoke the theory of Stochastic Approximation to study the convergence of the time average throughputs. We then prove that our policy ensures convergence of the throughput vector to the maximizer of (6). For simplicity, we start with a static network that has constant channel gains, and then generalize to the block fading scenario.

A. Proportional Fairness under Static Channel Gains

Consider the problem of $\alpha$-fair scheduling with time varying interference in a network with constant channel gains. That is, $\forall t$, $g_0(t) = g_0, g_1(t) = g_1$. Let $\theta_i(t)$ be the average downlink throughput of user $i$ at time $t$. 
Lemma 1: Under $\pi^*_\alpha$, there exists a Lipschitz continuous function $h(\bar{\theta})$ such that the time average throughputs, $\bar{\theta}(t)$, converge to an equilibrium point of the ODE

$$\dot{\Sigma}(\tau) = h(\Theta(\tau)),$$

almost surely. \hspace{1cm} (9)

Proof: Let us define a reward process $\phi_i(t)$ for user $i$ on the ON/OFF renewal process as

$$\phi_i(t) = \begin{cases} 0, & \text{if } S_n \leq t < S_n + \delta_{\text{min}} \\ \mu_i(t), & \text{o/w} \end{cases} \hspace{1cm} (10)$$

Let $\mathbb{E}[\Phi_i]$ be the expected accumulated reward over a renewal period.

Define $\xi_i(t) = \mu_i(t) - \phi_i(t)$. Thus, $\xi_i(t)$ is the effective rate of transmission to UE $i$ during the first $\delta_{\text{min}}$ slots of each renewal period. These time slots constitute the period during which the rate of transmission is decided using prior renewal periods while the channel conditions are determined by the current renewal interval.

$\xi_i(t)$ is dependent only on the interferer states in the interval $[S_N(t)-\delta_{\text{max}}, S_N(t)+1]$. Let

$$V^r_i(t) = \sum_{\tau=S_N(t)-k}^{S_N(t)-k+1-1} \xi_i(t).$$

Next, (5) can be written as:

$$\theta_i(t+1) = \theta_i(t) + a(t)(\mu_i(t+1) - \theta_i(t)), \hspace{1cm} (11)$$

where $a(t) = \frac{1}{t+1}$. We will rewrite this expression as

$$\theta_i(t+1) = \theta_i(t) + a(t)(h_i(\theta_i(t)) + \epsilon_i(t) + M_i(t)),$$

where

$$h_i(\theta_i(t)) = \frac{1}{X} \left\{ \mathbb{E}[\Phi_i] + \sum_{k=1}^{\delta_{\text{max}}+1} \mathbb{E}[V^r_i(t)] - \theta_i(t) \right\},$$

$$\epsilon_i(t) = \mathbb{E}[\mu_i(t+1)|\mathcal{F}_t] - \frac{1}{X} \left\{ \mathbb{E}[\Phi_i] + \sum_{k=1}^{\delta_{\text{max}}+1} \mathbb{E}[V^r_i(t)] \right\},$$

where $\mathcal{F}_t = \sigma(\theta(\tau), M(\tau), \tau \leq t)$ is the filtration corresponding to the history of attempted transmissions. Finally,

$$M_i(t) = \mu_i(t+1) - \mathbb{E}[\mu_i(t+1)|\mathcal{F}_t].$$

$M_i(t)$ forms a Martingale Difference Sequence (MDS) and is square-integrable. Thus, from [4, Theorem 1.9, Section 2.2], we observe that the time average throughputs converge to the equilibrium points of (9).
**Theorem 1:** The equilibrium point $\theta^*$ of the ODE (9) is stable and unique. Hence, under $\pi_\alpha^*$, the throughput vector converges to the optimal $\alpha-$fair solution.

**Proof:** Let us first define the set of all possible average throughput vectors that can be asymptotically achieved. This set is governed by the set of all possible resource allocation schemes, $\Pi$, under the given constraints. Then,

$$\bar{R} = \{ \bar{r} : \exists \pi \in \Pi \text{ such that } \bar{r} = \bar{R}^\pi \},$$

where $\bar{R}^\pi$ is the value that the time average throughput converges to, upon application of a resource allocation policy $\pi$.

**Lemma 2:** Under $\pi_\alpha^*$, in a static network, for all throughput vectors, $\bar{\theta}$,

$$\bar{R}_\alpha^* \in \left\{ \bar{r} : \bar{r} = \arg \max_{\bar{r} \in \bar{R}} \left\{ \sum_{i=1}^{K} \frac{\partial U(\theta_i)}{\partial \theta_i} r_i \right\} \right\}.$$ 

The proof of the lemma has been relegated to Appendix A. The above lemma shows that the point of asymptotic convergence of throughputs upon application of $\pi_\alpha^*$ maximizes the gradient-rate product. An equilibrium point, $\Theta$, of (9) is governed by $\dot{\Theta} = 0$, thus implying that $\Theta = \bar{R}^\pi$. As a result, we have the following lemma.

**Lemma 3:** The equilibrium point, $\theta^*$, of the ODE governed by the application of policy $\pi_\alpha^*$, maximizes (6) over the set of all possible throughputs, $\bar{R}$.

Again, we shall reserve the proof of the Lemma to Appendix A. We now make use of the lemmas to prove stability and uniqueness of the equilibrium point. Consider the utility function as a Lyapunov function. We then have

$$\frac{dU(\Theta(\tau))}{dt} = \sum_{i=1}^{K} \frac{\partial U(\Theta_i(\tau))}{\partial \Theta_i(\tau)} \frac{d\Theta_i(\tau)}{d\tau}$$

$$= \sum_{i=1}^{K} \frac{\partial U(\Theta_i(\tau))}{\partial \Theta_i(\tau)} (R_i^* - \Theta_i(\tau))$$

$$\geq \max_{\bar{r} \in \bar{R}} \left( \sum_{i=1}^{K} \frac{\partial U(\Theta_i(\tau))}{\partial \Theta_i(\tau)} r_i \right) - \sum_{i=1}^{K} \frac{\partial U(\Theta_i(\tau))}{\partial \Theta_i(\tau)} \Theta_i(\tau) \geq 0.$$}

Here, (15) follows from (9) and (16) follows from Lemma 2. (16) holds with equality only for $\Theta(\tau) = \theta^*$ thereby proving that the solution is unique and stable in the Lyapunov sense [21, Section 5.1].

We have thus shown that the equilibrium point is stable and from Lemma 3 we know that it maximizes the utility. Further, using the result from Lemma 1, we can see that under $\pi_\alpha^*$, the throughputs converge to the optimal $\alpha-$fair solution. 

\[\square\]
From the above theorem, we note that Alg. 1 is $\alpha$-fair under time varying interference for channels with static gains. We now extend the result to the model with block fading.

B. Proportional Fairness under Block Fading

Let us now consider the block fading channel model defined in Section II. The convergence and optimality of the time-average downlink throughputs can be proved as was done for the static case, with a few modifications as elicited below.

Define a derived arithmetic renewal process of span $M$ defined by the renewals of the ON/OFF process that occur at fade boundaries. That is, $\tilde{S}_n = t$ is a renewal epoch of the derived renewal process, if $\exists m$, such that $S_m = t$ and $\exists k$, such that $t = kM$. The number of renewals of this derived process, $\tilde{N}(t)$ represents the number of renewals of the interference process up to time $t$ that occur at fade boundaries. Let $\tilde{X}_1, \tilde{X}_2, \ldots$ be the renewal periods of the derived renewal process. This derived renewal process is depicted in Fig. 3.

Fig. 3: Interference and derived renewal process: The figure shows the transmit power of the interferer over time in a block fading environment and depicts the derived and interferer renewal processes.
Lemma 4: Under $\pi_\alpha^*$, there exists a Lipschitz continuous function $\tilde{h}(\theta)$ such that the time average throughputs converge to an equilibrium point of the ODE
\[ \dot{\Theta}(\tau) = \tilde{h}(\Theta(\tau)), \] almost surely. \hfill (17)

Proof: Let $\tau = \tilde{S}_{\tilde{N}(t)}$. Define
\[ \tilde{\xi}_i(t) = \begin{cases} \mu_i(t), & \forall \tau \leq t < S_{\tilde{N}(t)+1} + \delta_{\text{min}} \\ 0, & \text{o/w} \end{cases} \]
and $\tilde{\phi}_i(t) = \mu_i(t) - \tilde{\xi}_i(t)$. Thus, $\tilde{\phi}_i$ forms a reward process on the derived renewal process.

Having obtained the reward process on the derived renewal process, the proof of convergence is similar to that of Lemma 1. Specifically, let $\tilde{\Phi}_i$ be the random variable corresponding to the reward for user $i$ and
\[ \tilde{V}_i^{(k)}(t) = \sum_{\tau = \tilde{S}_{\tilde{N}(t)-k}}^{\tilde{S}_{\tilde{N}(t)-k+1}-1} \tilde{\xi}_i(\tau). \]

Then, for
\[ \tilde{h}_i(\theta_i(t)) = \frac{1}{\mathbb{E}[X]} \left\{ \mathbb{E}[\tilde{\Phi}_i] + \mathbb{E} \left[ X + \delta_{\text{max}} + 1 \sum_{k=1}^{X+\delta_{\text{max}}+1} \tilde{V}_i^{(k)}(t) \right] \right\} - \theta_i(t), \]
$h(\bar{\theta}) = (h_1(\theta_1), \ldots, h_K(\theta_K))$ is Lipschitz continuous and thus, following similar arguments from Stochastic Approximation, the convergence result follows.

Theorem 2: The equilibrium point $\theta^*$ of the ODE (17) is stable and unique. Hence, under $\pi_\alpha^*$, the throughput vector converges to the optimal $\alpha-$fair solution.

Here we provide a sketch of the proof of the above optimality result as it follows directly from the proof of Theorem 1. Note that Lemma 4 asserts the convergence of trajectories of the average throughput vector to the ODE (17). Next, we can define the set of all possible achievable throughput vectors similar to (13). For an appropriately defined achievable rate region, the claims of Lemma 2 and 3 can be extended using the same arguments and the finite expectation of the renewal period. Thus, the optimality result for the block fading scenario follows.

Thus, we have shown that the policy proposed above is optimal under the $\alpha-$fair regime. We will now look at the second class of resource allocation schemes, namely the throughput optimal resource allocation paradigm.

V. THROUGHPUT OPTIMAL RESOURCE ALLOCATION

In this section, we propose a scheduling policy that is provably throughput optimal for the time-varying interference model considered in Section II.
A. Throughput Optimal Policy - Single Carrier System

During each slot $t$, $B_0$ observes $\bar{\gamma}_i(t)$ and $Q_i(t-\delta_i)$ of each UE $i$ and executes Algorithm 2.

Algorithm 2 Throughput Optimal Policy, $\pi^*_MW$

\begin{algorithm}
\begin{algorithmic}
\FOR{$i = 1$ \TO $K$}
\STATE Compute $p_i(r)$ (Section VI)
\STATE $r^*_i(t) \leftarrow \arg\max_{r \geq 0} \{G_i(r)\}$
\ENDFOR
\STATE $k(t) \leftarrow \arg\max_{1 \leq i \leq K} \{Q_i(t-\delta_i)G_i(r^*_i(t))\}$
\STATE Transmit to UE $k(t)$ at rate $r^*_{k(t)}(t)$
\end{algorithmic}
\end{algorithm}

In each slot, $\pi^*_MW$ transmits to the UE that maximizes the goodput-backlog product with ties broken at random.

B. Lyapunov Drift Analysis

Let us define the quadratic Lyapunov function as:

$$L(\bar{Q}(t)) = \sum_{i=1}^{K} (Q_i(t))^2.$$  

The Lyapunov drift is defined as:

$$\Delta(t) = \mathbb{E}[L(\bar{Q}(t+1)) - L(\bar{Q}(t))|\bar{y}(t-\bar{\delta})].$$  

(18)

where $\bar{y}(t-\bar{\delta}) = (Q_1(t-\delta_1), Q_2(t-\delta_2), ..., Q_K(t-\delta_K))$. We know the expression for the queue dynamics from (8). Also, by our assumption in Section II, the arrival processes for each UE have bounded second moments. Further, we know that the transmit rate for the base station is upper bounded by the capacity offered, i.e, $\mu_i(t) \leq C_i(t)$. We also know that

$$\mathbb{E}[\mu_i(t)|\bar{y}(t-\bar{\delta})] = a_iG_i(r_i(t)).$$

Using these and the fact that arrivals are independent of the queue lengths, we bound (18) as

$$\Delta(t) \leq \sum_{i=1}^{K} \{C_i(t) + A_i\} + 2Q_i(t-\delta_i)(\lambda_i - a_iG_i(r_i))\}$$  

(19)

where $\bar{C}_i = \mathbb{E}[C_i^2(t)|\bar{y}(t-\bar{\delta})] < \infty$ and $\bar{A}_i = \mathbb{E}[A_i^2(t)].$
Our policy minimizes the Lyapunov drift. To see this, consider:

\[
\max_{\{a_i\},\{r_i\}} \sum_{i=1}^{K} Q_i(t - \delta_i)G_i(r_i)a_i,
\]  

subject to

\[
\begin{align*}
\sum_{i=1}^{K} a_i & \leq 1, \\
a_i & \geq 0, \ 1 \leq i \leq K, \\
r_i & \geq 0, \ 1 \leq i \leq K.
\end{align*}
\]

We have assumed here that modulation and coding schemes for any desired rate \(r_i\) are available.

**C. Minimizing Lyapunov Drift**

We will now solve (20). Introduce non-negative Lagrange multipliers \(\eta, \{\alpha_i\}, \{\beta_i\}\) for constraints \((C_1) - (C_3)\) respectively. Thus applying the KKT conditions, we get the following set of conditions that are to be satisfied by the optimal solution \((.,.)^*\) represents optimal solutions):

\[
Q_i(t - \delta_i)G_i(r_i^*) + \alpha_i^* - \eta^* = 0, \ \forall i
\]

\[
Q_i(t - \delta_i)a_i^* \frac{\partial G_i(r_i^*)}{\partial r_i^*} + \beta_i^* = 0, \ \forall i
\]

\[
\eta^*(\sum_{i=1}^{K} a_i^* - 1) = 0
\]

\[
\alpha_i^* a_i^* = 0, \ \forall i
\]

\[
\beta_i^* r_i^* = 0, \ \forall i
\]

**Proposition 1:** The optimal resource allocation scheme for the problem in (20) is that which assigns the channel exclusively to the UE with the largest queue-length goodput product and transmits at the rate that maximizes its goodput.

**Proof:** From (25) and (22), we can see that if \(r_i^* > 0\), then \(\beta_i^* = 0\) and thus \(\frac{\partial G_i(r_i^*)}{\partial r_i^*} = 0\). Thus the optimal rate allotted to UE \(i\) is that which maximizes his goodput.

Similarly, from (24) and (21), if \(a_i^* > 0\), then \(\alpha_i^* = 0\) and thus \(\eta^* = Q_i(t - \delta_i)G_i(r_i^*)\). Also, if \(a_i^* = 0\), then \(\alpha_i^* \geq 0\). Thus, \(\eta^* \geq Q_i(t - \delta_i)G_i(r_i^*)\). This clearly shows that \(\eta^* = \max_{1 \leq i \leq K} Q_i(t - \delta_i)G_i(r_i^*)\). We can thus see that the optimal allocation is to transmit to the UE with the maximum queue length goodput product.

We can thus see that \(\pi_{MW}^*\) minimizes the Lyapunov drift.
D. Throughput Optimality

In this section, we will prove the throughput optimality of the proposed scheme.

Theorem 3: The scheduling policy, $\pi_{MW}^*$, is throughput optimal over the class of all policies constrained by time-varying interference and CQI mechanism described in Section II.

Proof: Consider an arrival rate vector $\bar{\lambda} \in \Lambda$. Then there exists a policy $\pi \in \Pi$ that stabilizes the system for this arrival rate. Consequently, $\exists \bar{\epsilon} = (\epsilon_1, \epsilon_2, \ldots, \epsilon_K)$ with $\epsilon_i > 0, \forall i \in \{1, 2, \ldots, K\}$ such that

$$\lambda_i \leq b_i G_i(\hat{r}_i) - \epsilon_i, \forall i \in \{1, 2, \ldots, K\},$$

(26)

for some $\bar{b} = (b_1, b_2, \ldots, b_K)$ with $b_i \geq 0, \forall i \in \{1, 2, \ldots, K\}$ and $\sum_{i=1}^{K} b_i = 1$. Here, $\hat{r}_i$ is the rate assigned by $\pi$ to UE $i$.

Now consider the expression:

$$\sum_{i=1}^{K} Q_i(t - \delta_i) b_i G_i(r_i)$$

(27)

We already know from Proposition 1 that the optimal assignment that maximizes (27) for the above mentioned set of constraints is achieved by $\pi_{MW}^*$. Hence, we can conclude that

$$\sum_{i=1}^{K} Q_i(t - \delta_i) b_i G_i(r_i) \leq \sum_{i=1}^{K} Q_i(t - \delta_i) a_i G_i(r_i^*), \quad \forall \bar{b}$$

(28)

where $r_i^*$ is the rate assignment chosen for UE $i$ under policy $\pi_{MW}^*$.

From (26) and (28), we can see that

$$\sum_{i=1}^{K} Q_i(t - \delta_i) \lambda_i \leq \sum_{i=1}^{K} Q_i(t - \delta_i) (a_i G_i(r_i^*) - \epsilon_i).$$

(29)

Substituting (29) in (19), we get:

$$\Delta(t) \leq \sum_{i=1}^{K} (\delta_i + 1) (\tilde{A}_i + \tilde{C}_i) - 2 \sum_{i=1}^{K} Q_i(t - \delta_i) \epsilon_i.$$ 

Since $\epsilon_i > 0$, the Lyapunov drift becomes negative when the queues are large. This in turn establishes that $\pi_{MW}^*$ stabilizes the system for any arrival rate of $\bar{\lambda} \in \Lambda$.

We have thus established that $\pi_{MW}^*$ is throughput optimal under the given set of constraints on time-varying interference, fading and CQI delay.
Next, we generalize the policy outlined for the single carrier system to work in a multi-carrier setting. Assume that the base station has \( M \) sub-carriers to serve the users. The throughput optimal policy performs the Algorithm 3 at every slot \( t \).

<table>
<thead>
<tr>
<th>Algorithm 3 Multi-carrier Throughput Optimal Policy, ( \pi^*_{\text{multi}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>for ( j = 1 ) to ( M ) do</td>
</tr>
<tr>
<td>for ( i = 1 ) to ( K ) do</td>
</tr>
<tr>
<td>Compute ( p_{i,j}(r) ) (Section VI)</td>
</tr>
<tr>
<td>( r^*<em>{i,j} \leftarrow \text{arg max}</em>{r \geq 0} { G_{i,j}(r) } )</td>
</tr>
<tr>
<td>end for</td>
</tr>
<tr>
<td>( k^<em><em>j(t) \leftarrow \text{arg max}</em>{1 \leq i \leq K} { Q_i(t - \delta) G_{i,j}(r^</em>_{i,j}(t)) } )</td>
</tr>
<tr>
<td>Transmit to user ( k^<em>_j(t) ) at ( r^</em>_{k^*_j(t),j}(t) ) on sub-carrier ( j ).</td>
</tr>
<tr>
<td>end for</td>
</tr>
</tbody>
</table>

In the above algorithm, \( p_{i,j}(r) \) is the outage probability for a transmission of rate \( r \) to user \( i \) on sub-carrier \( j \). We can prove using Lyapunov drift techniques that the problem of scheduling on a multi-carrier system essentially simplifies to solving \( M \) scheduling problems on single carriers. Hence, the algorithm \( \pi^*_{\text{multi}} \) iterates over the sub-carriers and assigns each sub-carrier to the user with maximal goodput-backlog product. This algorithm can be proved to be throughput optimal using similar techniques to Theorem 3.

VI. ESTIMATION OF OUTAGE PROBABILITY

The outage probability estimate plays a crucial role in both resource allocation policies. Here we describe how the base station computes this conditional outage probability. Since, the temporal correlations in the general block fading case are fairly difficult to analyze, we first consider the case of a static network with constant channel gains. We then analyze the scenario of a fast fading channel where \( B_0 \) only stores the most recent SINR value. Finally we consider the more general scenario described in Section II.

A. Static Network

In a scenario without fading, the interferer state can be inferred from the SINR. We temporarily suppress the user index to simplify notation. Let \( \tilde{P}_1(t) = (P_1(t - \delta), \ldots, P_1(S_{N(t - \delta)})) \), represent
the state of the interferer from the last observed renewal period, to the most recent, observed
time slot. We observe here that $P_1(t)$ is independent of $P_1(\tau), \forall \tau < S_{N(t-\delta)}$, given $\bar{P}_1(t)$. This
follows from the renewal nature of the interference process.

In the absence of fading, only 2 values of SINR are possible (interferer being ON/OFF). Let
the two possible SINR values be $\gamma_0 = \frac{g_0}{N_0}$, $\gamma_1 = \frac{g_0}{N_0+g_1}$ and the corresponding capacities be
$C_0, C_1$ respectively ($C_0 > C_1$). Further, the outage probability satisfies

$$P_{out}(r) = \begin{cases} 0, & \text{if } r \leq C_1 \\ \mathbb{P}(C(t) = C_1 | \bar{\gamma}(t)), & \text{if } C_1 < r \leq C_0 \\ 1, & \text{o/w.} \end{cases}$$

(30)

Since $P_{out}(r)$ is independent of $r$ within each of the three regions defined above, the optimal
transmission scheme only transmits at rates of either $C_0$ or $C_1$. Further, the capacity, $C(t) = C_1$
if and only if $P_1(t) = 1$. Further, given $\bar{\gamma}(t)$, $\bar{P}_1(t)$ can be inferred owing to the lack of fading.
Hence, it is of interest to compute the term $\mathbb{P}(C(t) = C_1 | \bar{\gamma}(t)) = \mathbb{P}(P_1(t) = 1 | \bar{P}_1(t))$.

Let the lengths of the “OFF” and “ON” periods of the most recent transmit cycle be $z$ and
$y$ respectively. Further, let $T = t - S_{N(t-\delta)}$ where $N(t)$ is the number of renewals up to time $t$
and $S_n$ is the $n^{th}$ renewal epoch. Fig. 4 depicts the interferer transmit cycle in two scenarios:
the first with $P_1(t-\delta) = 0$ and the second with $P_1(t-\delta) = 1$. 
If $P_1(t - \delta) = 0$ we have $z = t - \delta - S_{N(t-\delta)} = T - \delta$, $y = 0$. Let $N_\delta$ be the number of renewals in $(t - \delta, t)$. Using the Law of Total Probability, we shall define the probability of the interferer being OFF at $t$ and $\bar{P}_1(t) = \bar{0}$, $p_0$ as follows

$$p_0 = \mathbb{P}(P_1(t) = 0, \bar{P}_1(t) = \bar{0}) = \sum_{n=0}^{\infty} \mathbb{P}(P_1(t) = 0, N_\delta = n, \bar{P}_1(t) = \bar{0})$$

$$= \mathbb{P}(Z > T) + \sum_{\tau = z}^{T} \mathbb{P}_X(\tau) \mathbb{P}(Z > T - \tau) + o(\chi)$$

(31)

where (31) follows from (3).

For the case where $P_1(t - \delta) = 1$, let $z = t_1 - S_{N(t-\delta)}$, $y = t - \delta - t_1$ and $\bar{P}_1(t) = \bar{p}$, as is depicted in Fig. 4. Then, we can define the probability of the interferer being ON at $t$ and $\bar{P}_1(t) = \bar{p}$ as $p_1$. This is derived as follows:

$$p_1 = \mathbb{P}(P_1(t) = 1, \bar{P}_1(t) = \bar{p}) = \sum_{n=0}^{\infty} \mathbb{P}(P_1(t) = 1, N_\delta = n, \bar{P}_1(t) = \bar{p})$$

$$= \mathbb{P}(Z = z, Y > t - z) + \sum_{\tau = y+z}^{T} \mathbb{P}_X(\tau) F_Z(T - \tau) + o(\chi)$$

(32)

where (32) follows from the same arguments made in (31).

Using these probabilities, the outage probability can now be computed as

$$\mathbb{P}(P_1(t) = 1|\bar{P}_1(t) = \bar{p}) = \begin{cases} 1 - \frac{p_0}{\mathbb{P}(P_1(t) = \bar{p})} & \text{if } P_1(t - \delta) = 0, \\
 p_1 \frac{1}{\mathbb{P}(P_1(t) = \bar{p})} & \text{o/w.} \end{cases}$$

(33)

where $\mathbb{P}(\bar{P}_1(t) = \bar{p})$ is obtained using the distribution of the ON/OFF renewal process.

**B. Using most recent CQI in Fast Fading**

In this section, we consider a channel wherein the fading gains are i.i.d across slots. Again, we suppress the user index. Further, we also assume that $B_0$ is constrained to use only the most recent SINR report, $\gamma(t - \delta)$, for resource allocation. For this special case, we will show that the outage probability is given by,

$$P_{out}(r) = F_\gamma(2^r - 1|\gamma(t - \delta)) = \sum_{P_1} \sum_{P_0} F_P(\gamma|P_1) \mathbb{P}(P_1|P_0) \mathbb{P}(P_0|\gamma_0).$$

(34)

**Lemma 5:** If the channel gains vary i.i.d across slots, the base station has access to only the most recent SINR information, and transmitted SINR information has a delay $\delta$, then

1) $\gamma(t)$ is conditionally independent of $\gamma(t - \delta)$ given $P_1(t)$.
2) \( F_{\gamma(t)|P_1(t)}(\gamma|P) = 1 - \left( \frac{\alpha_0}{\gamma \alpha_1 + \alpha_0} \right) \exp \left( \frac{-\gamma}{\alpha_0} \right) \) where \( \alpha_0 = \frac{\Gamma_0 P_0(t)}{N_0} \) and \( \alpha_1 = \frac{\Gamma_1 P_1(t)}{N_0}, \forall P \in \{0,1\}. \)

3) \( \mathbb{P}(P_1(t) = 1) = \rho = \frac{E[Y]}{E[X]} \)

4) \( P_1(t) \) is conditionally independent of \( \gamma(t-\delta) \) given \( P_1(t-\delta) \).

Proof:

1) \( \gamma(t) \) is a function of \( g_0(t), g_1(t) \) and \( P_1(t) \). By the assumptions on fading, \( g_0(t), g_1(t) \) are independent of \( g_0(t-\delta), g_1(t-\delta) \). Thus, given \( P_1(t), \gamma(t) \) is independent of \( \gamma(t-\delta) \).

2) The conditional CDF is obtained as follows:

\[
F_{\gamma(t)|P_1(t)}(\gamma|P) = \mathbb{P}(\gamma(t) \leq \gamma|P_1(t) = 1) = \mathbb{P}(g_0(t) \leq \gamma(N_0 + g_1(t)P)) = 1 - \left( \frac{\alpha_0}{\alpha_0 + \gamma \alpha_1} \right) \exp \left( \frac{-\gamma}{\alpha_0} \right),
\]

where (35), follows from the Rayleigh fading statistics.

3) Since the interferer state follows an ON/OFF renewal process, we invoke the Key Renewal Theorem [3, Theorem 3.4.4] to get \( \mathbb{P}(P_1(t) = 1) = \rho = \frac{E[Y]}{E[X]} \).

4) Since \( g_0(t), g_1(t) \) are independent of \( g_0(t-\delta), g_1(t-\delta) \), \( P_1(t) \) is independent of \( \gamma(t-\delta) \) given \( P_1(t-\delta) \).

As a consequence of claim 2 in the above theorem, we have

\[
f(\gamma|P) = \left( \frac{\alpha_0}{\gamma \alpha_1 + \alpha_0} \right) \exp \left( \frac{-\gamma}{\alpha_0} \right) \left( \frac{\alpha_1}{\gamma \alpha_1 + \alpha_0} + \frac{1}{\alpha_0} \right).
\]

Let \( P_{i,j} = \mathbb{P}(P_1(t) = i|P_1(t-\delta) = j), \forall i,j \in \{0,1\}. \)

Lemma 6: The conditional probability of \( P_1(t) = 1 \) given that \( P_1(t-\delta) = 1 \) is

\[
P_{1,1} = 1 - F_Y^{res}(\delta) + \mathbb{P}(E_1) + o(\chi^2),
\]

where \( \chi \) is from (3) and \( F_Y^{res}(\cdot) \) is the cdf of the residual period of the ON process at \( t-\delta \),

given \( P(t-\delta) = 1 \) with pmf

\[
p_Y^{res}(i) = \frac{1 - F_Y(i)}{E[Y]}.
\]

Here, \( \mathbb{P}(E_1) = \sum_{y=0}^{\delta} \sum_{z=0}^{\delta-y} (1 - F_Y(\delta - z - y)) \mathbb{P}(z)p_Y^{res}(y). \)

Proof: Consider the event \( E \) of having \( P(t) = 1 \), given \( P(t-\delta) = 1 \). Let us assume that \( E_0, E_1, E_2 \) are events such that \( P(t) = 1 \), given \( P(t-\delta) = 1 \) with no renewals, one renewal, and at least two renewals in the interval \( (t-\delta, t) \), respectively. Clearly, \( E = E_0 \cup E_1 \cup E_2 \).

Fig 5 shows sample ON-OFF renewal processes with \( \delta = 5 \). Figures (5 a) and (5 b) show the cases under consideration with one renewal and two renewals respectively in \( (t-\delta, t) \).
From Figure (5 b), we observe that we necessarily need \( Y_{res} < \delta \) and \( X_1 = Y_1 + Z_1 < \delta \) when there are at least two renewals. Thus we have

\[
P(E_2) \leq P(Y_{res} < \delta, X_1 < \delta | P(t - \delta) = 1)
= P(Y_{res} < \delta) P(X_1 < \delta | P(t - \delta) = 1)
\leq \left( \frac{\delta}{E[Y]} \right) \left( \frac{\chi}{\rho} \right) = o(\chi^2),
\]

where, (38) follows from the fact \( X_1 \) and \( Y_{res} \) are independent, owing to the renewal nature of the process. Using (3) and (37), we get \( P(Y_{res} < \delta) \leq \frac{\delta}{E[Y]} = \frac{\delta}{\rho E[X]} \). Neglecting the trivial case of \( \rho = 0 \) and using the fact that propagation delays are much smaller than the duration of the average transmit cycle, \( \frac{\delta}{\rho E[X]} = o(\chi) \). This implies that the event \( E_2 \) is of very low probability.

Let us now consider the case where there is exactly one renewal in \((t - \delta, t)\). Then, we have:

\[
P(E_1) = \sum_{y=0}^{\delta - y} \sum_{z=0}^{\delta - y} F_Y^c(\delta - z - y) P_Z(z) p_{Y_{res}}^{res}(y),
\]

where, \( F_Y^c(.) \) is the complementary cdf of \( Y \) and \( p_{Y_{res}}^{res} \) is given in (37).

For the case of no renewals, we can see that \( P(E_0) = P(Y_{res} > \delta) = 1 - F_Y^res(\delta) \).

The events, \( E_0 \), \( E_1 \) and \( E_2 \) are not just disjoint, but also encompass all possible renewal scenarios in the delay interval. Thus by the law of total probability, we have \( P_{1,1} = P(E_0 \cup E_1 \cup E_2) = 1 - F_Y^res(\delta) + P(E_1) + o(\chi^2) \) where \( P(E_1) \) is given by (40).

From \( P_{1,1} \), we obtain \( P_{0,1} = 1 - P_{1,1} \). Further, \( P_{1,0} = (\frac{\rho}{1-\rho}) P_{0,1} \) and \( P_{0,0} = 1 - P_{1,0} \). Thus, we can use these results to obtain the conditional outage probability according to (34).

C. M-Block Fading

In this section, we focus on obtaining the conditional outage probability for the generalized system model assumed in Section II. In order to compute the outage probability, it is necessary...
to keep track of the renewal epochs in the interference process. We thus devise an algorithm to compute the outage using a set of probable states, each of which corresponds to a possible set of renewals that could result in the observations.

We will say that a time slot $t$ is a ‘change point’, if $\gamma(t) \neq \gamma(t-1)$ or $\exists k \in \mathbb{Z}^+$, such that, $t = kM$ ($t$ corresponds to a fade block boundary). Let $\tau(t) = \max\{\tau \leq t : \tau$ is a change point $\}$, the most recent change point.

Let $\delta_{min} = \min_k \delta_k$. Let us define $S(t) = (\omega(t), z(t), y(t), p(t), \psi(t))$ as a probable state at time $t$ (a set of renewal epochs that could result in the observed set of CQI values). Here,

- $\omega(t)$ is the interferer state at time $t - \delta_{min}$ corresponding,
- $z(t)$ is the observed OFF time of most recent renewal cycle (up to $t - \delta_{min}$),
- $y(t)$ is the observed ON time of most recent renewal cycle (up to $t - \delta_{min}$),
- $p(t) \triangleq \mathbb{P}(S(t) | \gamma(t))$ is the belief of state $S(t)$ at time $t$ and
- $\psi(t) \triangleq p(\tau(t - \delta_{min}))$ is the belief corresponding to the most recent, observed change point.

Let $S(t)$ be the set of all possible states at time $t$ and let $L_S(t)$ be its cardinality.

At any time $t$, suppose we receive the set of SINRs $\{\gamma_1(t_1), \gamma_2(t_2), ..., \gamma_K(t_K)\}$, $t_i = t - \delta_i$. Without loss of generality, we assume that $\delta_i \leq \delta_j$, $\forall i \leq j$. We classify the users into “Delay classes” $\{D_1, \ldots, D_L\}$ such that users with the same delay fall in the same delay class. Let $\bar{\gamma}_D(t) = \{\bar{\gamma}_i(t) : i \in D\}$ and $\gamma_D(t) = \{\gamma_i(t) : i \in D\}$. Let $t_D = t - \delta_k$ for some UE $k \in D$.

Given $S(t)$, the outage probability can be computed as

$$\mathbb{P}_{out}(r) = \sum_{i=1}^{L_S(t)} p_i(t) \mathbb{P}(C(t) < r | S_i(t), \gamma(t)). \quad (41)$$

Hence, we first describe the algorithm to keep track of the set of possible states. We then compute $\mathbb{P}(C(t) < r | S_i(t), \gamma(t))$ using the fading statistics.

We first start with a single user (of delay $\delta_{min}$) and describe the algorithm to update the set of states. Then we will describe the method to incorporate multiple users.

There exist four possible state transitions upon reception of $\gamma_1(t_1 + 1) = \gamma$ at time $t + 1$. These transitions are depicted in Fig 6 and work as follows:

1) Within a fade block, an observation of an increase in SINR points to a renewal. Thus, as shown in Fig 6a, we reduce the set of states to this one deterministic state.

2) Within a fade block, if the SINR value does not change, then we know that the interferer state is maintained as shown in Fig 6b.
3) In a fade block, a decrease in SINR indicates an OFF→ON transition. Observation of such a transition indicates that we can discard all states \( S(t) \) that assumed that the interferer state was ON at time \( t - \delta_{\text{min}} \), i.e., \( \omega(t) = 1 \). Thus the states merge as shown in Fig. 6c.

4) At a fade boundary, it is possible for the interferer to either maintain its state or flip. We account for this by doubling the number of states. We shall assume that the odd indices shall correspond to the OFF state and the even indices for the ON state at time \( t + 1 \).

Beliefs of the states are updated as in Alg. 4 and the expressions are derived in Appendix B.

The belief update expressions are derived in Appendix B. Having obtained the algorithm to update the states and associated beliefs, we now compute \( \mathbb{P}(C(t) < r|S_i(t), \bar{\gamma}(t)) \) using the fading and interference statistics.

Let us assume that we are computing the outage probability for the \( k^{th} \) fading block. If \( B_0 \) observes two distinct values of SINR for any particular user within the block, then each term of the summation in (41) reduces to the computation highlighted for the static network. If \( B_0 \) has no SINR information regarding the current fade block, then the summation terms can be computed as in the fast fading channel.
Algorithm 4: Algorithm to update belief at time $t + 1$

\[
\text{if } \text{mod}(t_1 + 1, M) \neq 0 \text{ then} \\
\quad \text{if } \gamma_1(t_1 + 1) = \gamma_1(t_1) \text{ then} \\
\quad \quad \quad p_i(t + 1) \leftarrow \frac{\psi_i(t) \prod_{l \in \mathcal{L}_S(t_1)} \{P(\omega_l(j + 1) | z_l, y_l(j))\}}{\sum_{k = 1}^{L_S(t_1)} \psi_k(t) \prod_{l \in \mathcal{L}_S(t_1)} \{P(\omega_l(j + 1) | z_l, y_l(j))\}} \\
\quad \quad \text{else if } \gamma_1(t_1 + 1) > \gamma_1(t_1) \text{ then} \\
\quad \quad \quad p_1(t + 1) \leftarrow 1 \\
\quad \quad \text{else} \\
\quad \quad \quad p_i(t + 1) \leftarrow p_{2i-1}(t) + p_{2i}(t) \\
\quad \text{end if} \\
\text{else} \\
\quad p_{2i}(t + 1) \leftarrow \frac{f(\gamma_D_1(t_1)|1)P(S_i(t), 1|\gamma_D_1(t))}{\sum_{P'} f(\gamma_D_1(t_1)|P') \sum_{k = 1}^{L_S(t_1)} P(S_i(t), P'|\gamma_D_1(t))} \\
\quad p_{2i-1}(t + 1) \leftarrow p_i(t) - p_{2i}(t + 1) \\
\text{end if} \\
\text{for } j = 2 \text{ to } L \text{ do} \\
\quad \text{if } t_{D_j} = \tau(t_1) \text{ and } \text{mod}(t_{D_j}, M) = 0 \text{ then} \\
\quad \quad f_j \leftarrow \prod_{k \in D_j} f(\gamma_k(t_{D_j} + 1) | P_1(t_{D_j} + 1) = j), j \in \{0, 1\} \\
\quad \quad \eta \leftarrow \frac{1}{\sum_{i \leq L_{S}(t)(t+1)/2} \sum_{i \leq L_{S}(t)(t+1)/2} p_{2i-1}(t + 1)} \\
\quad \quad p_i(t) \leftarrow \begin{cases} 
\frac{(1+\eta)f_0p_i}{f_0+\eta f_1} & i \text{ is odd} \\
\frac{(1+\eta)f_1p_i}{f_0+\eta f_1} & i \text{ is even} 
\end{cases} \\
\quad \text{and } \psi_i(t) \leftarrow \begin{cases} 
\frac{f_0\psi_i}{\sum_{k \in \text{odd}} f_0\psi_k + \sum_{k \in \text{even}} f_1\psi_k} & i \text{ is odd} \\
\frac{f_1\psi_i}{\sum_{k \in \text{odd}} f_0\psi_k + \sum_{k \in \text{even}} f_1\psi_k} & i \text{ is even} 
\end{cases} \\
\text{end if} \\
\text{end for}
\]

Lemma 7: If $(k - 1)M < t - \delta_1 < t \leq kM$ and $B_0$ has exactly one SINR, $\gamma$, corresponding to this block, then

\[
\mathbb{P}(C(t) < r | P_1(t) = P, S_i(t), k(t)) = \begin{cases} 
1 - e^{-\frac{r}{\alpha_1}} (\frac{g(r)}{g(r)})^{-1} & \text{, if } \omega_i(t) = 1, P = 0 \\
0 & \text{, if } \omega_i(t) = 0, P = 1 \\
e^{-\frac{r}{\alpha_1}} (\frac{g(r)}{g(r)})^{-1} & \text{, otherwise,} 
\end{cases}
\]  \hspace{1cm} (42)

where, $g(r) = 2^r - 1$.

Proof: If $\omega_i(t) = 0$, then $g_0 = \frac{\gamma N_0}{P}$. Thus, if $P_1(t) = 0$, $\gamma(t) = \gamma$, w.p.1. Hence there is no
outage if \( P_1(t) = 0 \) and \( \omega_i(t) = 0 \). If \( P_1(t) = 1 \), we have
\[
\mathbb{P}(C(t) < r | P(t) = 1, S_i(t), \bar{\gamma}(t)) = \mathbb{P}\left(g_1 > \left(\frac{\gamma}{g(r)} - 1\right) N_0\right) = e^{-\frac{1}{\alpha_1}\left(\frac{\gamma}{\bar{\gamma}}-1\right)}
\]

Now, if \( \omega_i(t) = 1 \), then \( g_0 = \gamma(N_0 + g_1) \). Thus, if \( P_1(t) = 1 \), \( \gamma(t) = \gamma \), w.p.1. Hence the conditional outage probability in this case is 0. On the other hand, if \( P_1(t) = 0 \), we have
\[
\mathbb{P}(C(t) < r | Pr_{it}(t) = 0, \bar{\gamma}(t)) = \mathbb{P}\left(\gamma\left(1 + \frac{g_1}{N_0}\right) < g(r)\right) = \begin{cases} 1 - e^{-\frac{1}{\alpha_1}\left(\frac{\gamma}{\bar{\gamma}}-1\right)} & \text{if } g(r) > \gamma \\ 0 & \text{otherwise} \end{cases}
\]

This gives us the conditional outage probabilities. ■

We finally use (41) to compute the outage probability.

**D. Memory Requirement**

In the above mentioned algorithm, the number of possible states in the system doubles at every fade boundary, suggesting a possible exponential growth in storage requirement. We will show that this requirement is in fact relaxed in the case of a slow fading environment. To this end, we will assume that the block fading environment satisfies:
\[
\mathbb{P}_X(kM) \leq \alpha^{k-1}\mathbb{P}_X(M), \quad \forall k \geq 1,
\]
for some \( \alpha < 1 - \mathbb{P}_X(M) \). This assumption on \( \alpha \) is motivated by the assumptions that the renewal process is of span 1 and the renewal period, \( X \), is light tailed. We will now show that, when the fading process is slow enough for (43) to hold, the memory requirement for the outage computation can be relaxed.

**Theorem 4:** When the block fading is slow and satisfies (43) as assumed above and interferer load is as assumed in section II, the probability that renewals of the ON/OFF process occur only at the fade boundaries for at least \( K \) blocks decays exponentially.

**Proof:** Let \( p = \mathbb{P}_X(M) \). Since we are interested in the probability that renewals, if any occur only at fade boundaries for the next \( K \) blocks (referred to as \( P(k) \) here), we have,
\[
P(K) \leq pP(K-1) + \alpha pP(K-2) + \cdots + \alpha^{K-2}P(1) + \beta \theta^K \\
\leq a_1 \phi^K + a_2 \theta^K
\]
where $\theta = e^{-sM} < 1$, $\phi = \alpha + p$, $a_1 = \frac{p\beta\theta}{(\phi-\theta)\phi}$ and $a_2 = \beta\left(1 - \frac{p}{(\phi-\theta)}\right)$. From our system model assumptions on fading, we have $\phi < 1$. Hence the result follows.

The number of states decreases only upon observation of an interferer state transition. Having more than $2^K$ states implies renewals occur only at fade boundaries for at least $K$ blocks. This justifies that an $O(K)$ bit memory overflows with exponentially decreasing probability.

We can see that finite buffer space and computational overheads prevent the accurate computation of the conditional outage probability. However, since the overflow probability decays exponentially, we can get arbitrarily close to the optimal solution by using sufficient memory.

Using the above described algorithm in coherence with the proposed policies we perform optimal resource allocation.

VII. CONCLUSION

We considered the problem of downlink resource allocation in a scenario where the UEs face time-varying interference from a neighboring cell. We exploited the statistics of the interference to compute the conditional outage probability, given the unreliable CQI available at the base station. We then proposed a resource allocation scheme and proved its optimality in the regime of $\alpha$-fair resource allocation using the theory of Stochastic Approximation. We then combined the outage probability expression with a Lyapunov stability framework to obtain a throughput optimal resource allocation policy for the scenario considered.

In essence, this paper provides an analytically tractable model for a practically relevant phenomenon and shows how the stochastic gradient algorithm and the max-weight policies can be adapted to mitigate the effect of time-varying interference.

APPENDIX A

In this appendix we will provide the proofs for the Lemmas that support the optimality proof of the $\alpha$-fair scheduling policy.

We will first provide the proof for Lemma (2). But before that, we will elucidate as to why the time average throughputs converge to $\bar{R}$. From our definitions made earlier, we know that we can rewrite (5) as

$$\theta_i(t) = \frac{1}{t} \sum_{t_1=1}^{t} \phi_i(t_1) + \frac{1}{t} \sum_{t_1=1}^{t} \xi_i(t_1).$$

(44)

Since $\phi_i(t)$ forms a reward process, using time average reward theorem [3], we know that the time average converges to $\frac{E[\phi_i]}{X}$ almost surely. Further, we can see that $\xi_i(t)$ for some $t$ ∈
\([S_n, S_n + \delta_i]\) depends on at most \(\delta_{\text{max}} + 1\) renewal periods. Using this property, we can show that the time average throughputs for scheduling policy \(\pi\) in fact converge to \(\bar{R}^\pi\).

**Proof of Lemma 2:** We can first see that
\[
\mathbb{E}[\bar{\Phi}^\pi] + \sum_{k=1}^{\delta_{\text{max}}+1} \mathbb{E}[\bar{V}^{k,\pi}(t)] = \mathbb{E}\left[ \sum_{\tau=S_N(t)}^{S_N(t)+1-1} \bar{\mu}(\tau) \right]
\]
for some \(n\) from the definition of the functions and the expectations. Further, we can show that for any throughput vector, \(\bar{\theta}\),
\[
\sum_{i=1}^{K} \nabla U_i(\theta_i) \mathbb{E}[\mu_i^{\pi^\ast}(t)] \geq \sum_{i=1}^{K} \nabla U_i(\theta_i) \mathbb{E}[\mu_i^\pi(t)]
\]
for all \(t\) as the decisions made in our policy maximize the gradient-goodput product and consequently, the gradient-expectation product too.

Exploiting this property and the law of iterated expectations, we can show that
\[
\bar{R}^{\pi^\ast} \in \mathcal{M}(\theta)
\]

**Proof of Lemma 3:** From Lemma 2, we know that \(\bar{R}^{\pi^\ast} \in \mathcal{M}(\theta)\), for all \(\theta\). Thus, we know that the limit point, \(\theta^\ast \in \mathcal{M}(\theta^\ast)\).

Hence, we know that
\[
\sum_{i=1}^{K} \frac{\partial U_i(\theta_i^\ast)}{\partial \theta_i^\ast} \{\theta_i - \theta_i^\ast\} \leq 0, \forall \theta \in \bar{\mathcal{R}}
\]

Owing to the concavity of \(U(.)\), from [22, Proposition 2.1.2], we can see that \(\theta^\ast\) in fact maximizes the utility over the given constraint set.

**APPENDIX B**

Here we derive the belief update expressions for the states. We first derive the expressions for a single UE of delay \(\delta_{\text{min}}\) and subsequently incorporate the SINR corresponding to other UEs.

**Lemma 8:** If UE 1 with CQI delay \(\delta_1 = \delta_{\text{min}}\) reports SINR \(\gamma\) at time \(t + 1\), then

1) if \(t_1 + 1 \neq kM, \forall k \in \mathbb{N}\) and \(\gamma_1(t_1) = \gamma_1(t_1 + 1) = \gamma\), then
\[
p_i(t + 1) = \frac{\psi_i(t) \prod_{j=t+1}^{t} \{P(\omega_i(j + 1)|z_i(j), y_i(j))\}}{\sum_{k=1}^{L_S(t+1)} \psi_k(t) \prod_{j=t+1}^{t} \{P(\omega_k(j + 1)|z_k(j), y_k(j))\}}, \forall i \in S(t);
\]

2) if \(t_1 + 1 \neq kM, \forall k \in \mathbb{N}\) and \(\gamma_1(t_1) > \gamma_1(t_1 + 1) = \gamma\), then \(p_i(t + 1) = p_{2i-1}(t) + p_{2i}(t)\);
3) if \( t_1 + 1 = kM \), for some \( k \in \mathbb{N} \), then

\[
p_{2i}(t + 1) = \frac{f(\gamma_1)\mathbb{P}(S_i(t), 1|\bar{\gamma}(t))}{\sum_{P'} f(\gamma|P') (\sum_{k=1}^{L_S(t)} \mathbb{P}(S_k(t), P'|\bar{\gamma}(t)))}, \quad p_{2i-1}(t + 1) = p_i(t) - p_{2i}(t + 1).
\]

For \( P' \in \{0, 1\} \), \( \mathbb{P}(S_i(t), P'|\bar{\gamma}(t)) = \mathbb{P}(S_i(t), P_1(t - \delta + 1) = P'|\bar{\gamma}(t)) \).

**Proof:** For the first case, we have

\[
p_i(t + 1) = \mathbb{P}(S(t + 1)|\bar{\gamma}(t))
\]

\[
= \mathbb{P}(S(t)|\bar{\gamma}(t), \gamma(t - \delta + 1) = \gamma(t - \delta))
\]

\[
= \mathbb{P}(S(\tau(t))|\bar{\gamma}(\tau(t) + \delta), \{\gamma(j) = \gamma, \forall j \geq \tau(t)\}) \tag{49}
\]

\[
= \psi_i(t) \prod_{j=\tau(t)}^{t} \{\mathbb{P}(\omega_i(j + 1)|z_i(j), y_i(j))\}
\]

\[
= \frac{\psi_i(t) \prod_{j=\tau(t)}^{t} \{\mathbb{P}(\omega_i(j + 1)|z_i(j), y_i(j))\}}{\sum_{k=1}^{L_S(t+1)} \psi_k(t) \prod_{j=\tau(t)}^{t} \{\mathbb{P}(\omega_k(j + 1)|z_k(j), y_k(j))\}} \tag{50}
\]

Here, (49) follows from the fact that state \( S(t + 1) \) is deterministically obtained, given that there is no change in the SINR observation. Using induction we can see that (50) follows from (49). Finally (51) follows from Bayes’ rule and the statistics of the ON/OFF process.

Next, we note that the observation of an OFF\( \rightarrow \)ON transition implies that the interferer was OFF from the most recent fade block boundary up to the transition. Thus the probability of the new states is the probability at the fade boundary of each branch. Thus the result follows.

Finally at the fade boundary, we use the Bayes’ rule and fading and interference statistics. \( \blacksquare \)

We now extend the algorithm to make use of the data of all \( K \) users in the network. Given observations of a user in \( D_1 \), observations from other users are deterministic at all \( t \) within a fading block as multiple reports of the same information do not affect the belief update.

On the other hand, observations made by multiple users at the fade boundaries provides more information on state beliefs. Thus, at \( t + 1 = kM - \delta_{\text{min}} \) for some \( k \in \mathbb{N} \), we make use of the observations made by all users of the minimum delay class, \( D_1 \) to update the state beliefs as

\[
p_{2i}(t + 1) = \frac{f(\gamma^{n_1}|1)\mathbb{P}(S_i(t), 1|\bar{\gamma}^{n_1}(t))}{\sum_{P'} f(\gamma^{n_1}|P') (\sum_{k=1}^{L_S(t)} \mathbb{P}(S_k(t), P'|\bar{\gamma}^{n_1}(t)))}
\]

where \( |D_1| = n_1 \), \( \gamma^{n_1} = (\gamma_1, \ldots, \gamma_{n_1}) \) is the vector of observations made by the minimum delay class. Similarly, \( \bar{\gamma}^{n_1}(t) = \{\bar{\gamma}_1(t), \ldots, \bar{\gamma}_{n_1}(t)\} \), is the corresponding history of observations. Owing to the independence of fading across users, we get, \( f(\gamma^{n_1}|P') = \prod_{i=1}^{n_1} f(\gamma_i|P'), \forall P' \in \{0, 1\} \).

As was computed earlier, \( p_{2i-1}(t + 1) = p_i(t) - p_{2i}(t + 1) \).

Additionally, observations of UEs of other classes are significant only at fade boundaries that are the most recent observed change points. This is owing to the fact that the observation of
a change following the fade boundary indicates either a renewal or an OFF→ON transition. In either case, the state of the interferer at the fade boundary can be determined and thus the observations made subsequent to the observation of the change do not affect state beliefs.

Lemma 9: Assume that \( t_1 \) is in the \((k+1)^{th}\) fade block and \( t_1 = \tau(t_1) = kM + 1 \), for some \( l \geq 2 \). If the initial computation of the algorithm, ignoring the delay class \( D_i \) gives state \( i \) the probability \( p_i \) and change point probability of \( \psi_i \), then the necessary updates are

\[
p_i(t) = \begin{cases} \frac{(1+\eta)f_0\psi_i}{f_0+\eta f_1} & i \text{ is odd} \\ \frac{(1+\eta)f_1\psi_i}{f_0+\eta f_1} & i \text{ is even} \end{cases} \quad \psi_i(t) = \begin{cases} \frac{f_0\psi_i}{\sum_{k \in \text{odd}} f_0\psi_k + \sum_{k \in \text{even}} f_1\psi_k} & i \text{ is odd} \\ \frac{f_1\psi_i}{\sum_{k \in \text{odd}} f_0\psi_k + \sum_{k \in \text{even}} f_1\psi_k} & i \text{ is even} \end{cases}
\]  

(52)

where \( f_j = \prod_{k \in D_i} f(\gamma_k(t_i)|P_i(t_i) = j) \forall j \in \{0,1\} \) and \( \eta = \frac{\sum_{j \leq L_S(t)/2} P_{2j}(t)}{\sum_{j \leq L_S(t)/2} P_{2j-1}(t)} \).

Proof: Let us consider just two delay classes for convenience. The argument can be easily extended to the case of multiple delay classes.

\[
p_i(t) = \mathbb{P}(S_1(t)|\bar{\gamma}_{D_1}(t), \bar{\gamma}_{D_1}(t)) = \frac{q'_i}{\sum_{k=1}^{L_S(t)} q_k} \quad \text{(53)}
\]

where

\[
q'_i = \psi_i f(\gamma_{D_i}(t_i)|S_i(\tau(t_i) + \delta_1)) \left( \prod_{j=\tau(t_i)}^{t_1} \mathbb{P}(\omega_i(j+1)|z_i(j), y_i(j)) \right)
\]

This follows as from the independence of fading across users. Further, we make use of the fact that given \( \bar{\gamma}_{D_1}(t) \), \( \bar{\gamma}_{D_1}(t) \) can be deterministically obtained at all points other than at the fade boundary. This in turn proves the claim.

The update for change point probabilities follows from similar arguments. □

REFERENCES


