Netlist and System Partitioning

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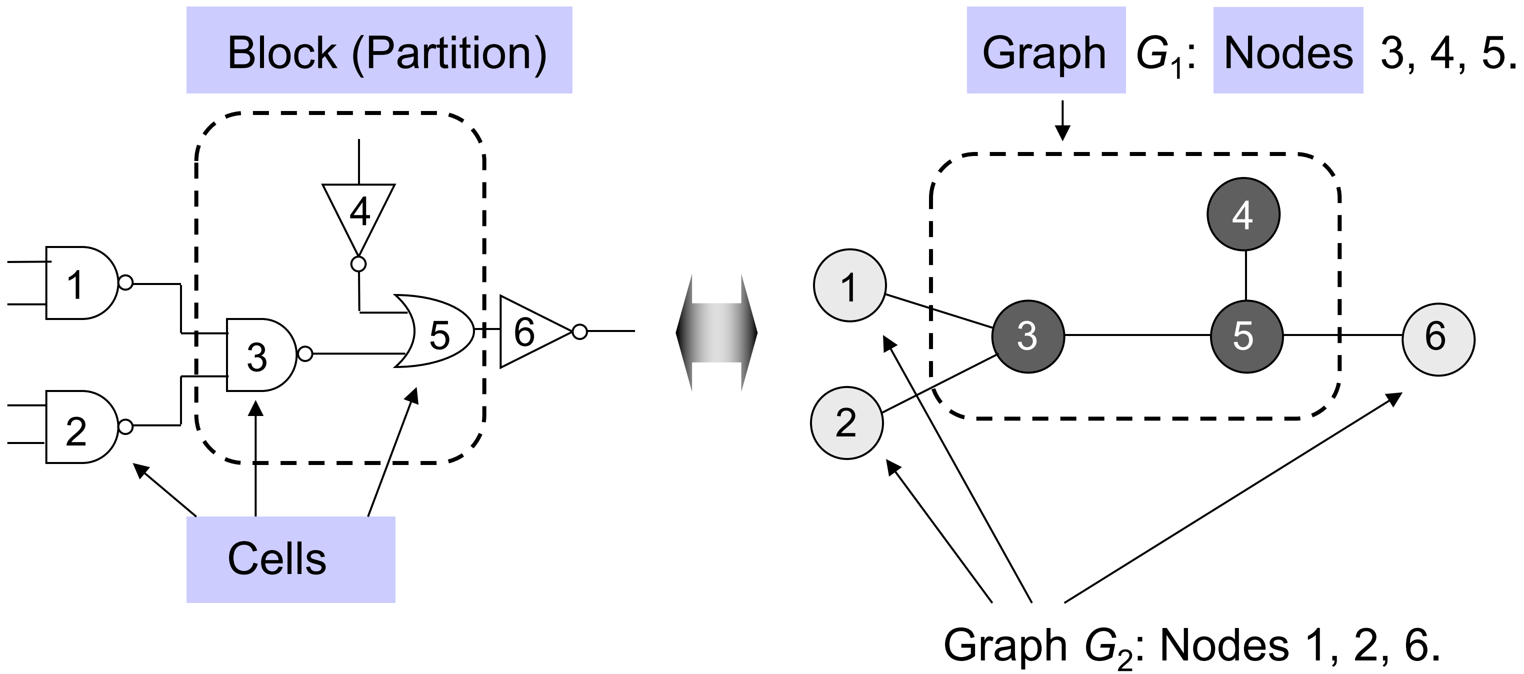
Netlist and System Partitioning

- Introduction
- Optimization Goals
- Partitioning Algorithms
  - Kernighan-Lin (KL) Algorithm
  - Extensions of the Kernighan-Lin Algorithm
  - Fiduccia-Mattheyses (FM) Algorithm
- Framework for Multilevel Partitioning
  - Clustering
  - Multilevel Partitioning

Chip partitioning



Collection of cut edges

Cut set: (1,3), (2,3), (5,6),

Optimization Goals

- Given a graph $G(V,E)$ with $|V|$ nodes and $|E|$ edges where each node $v \in V$ and each edge $e \in E$.

- Each node has area $s(v)$ and each edge has cost or weight $w(e)$.

- The objective is to divide the graph G into k disjoint subgraphs such that all optimization goals are achieved and all original edge relations are respected.

Optimization Goals

- In detail, what are the optimization goals?
  - Number of connections between partitions is minimized
  - Each partition meets all design constraints (size, number of external connections..)
  - Balance every partition as well as possible

- How can we meet these goals?
  - Unfortunately, this problem is NP-hard
  - Efficient heuristics are developed in the 1970s and 1980s. They are high quality and in low-order polynomial time.

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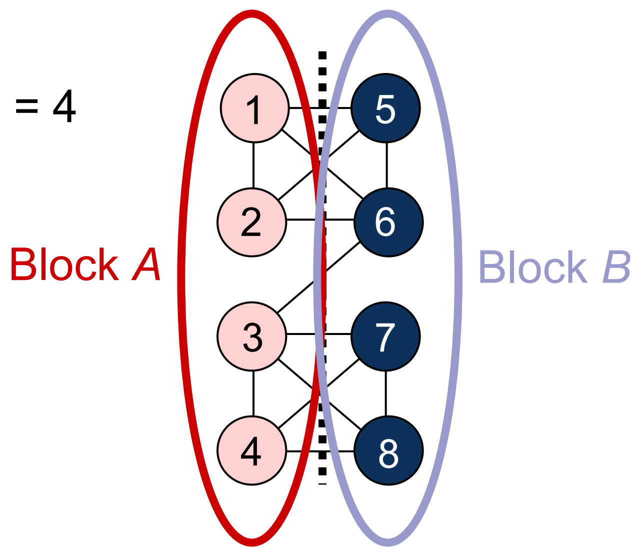
Kernighan-Lin (KL) Algorithm

Given: A graph with $2n$ nodes where each node has the same weight.

Goal: A partition (division) of the graph into two disjoint subsets A and B with minimum cut cost and $|A| = |B| = n$.

Example: $n = 4$



Kernighan-Lin (KL) Algorithm – Terminology

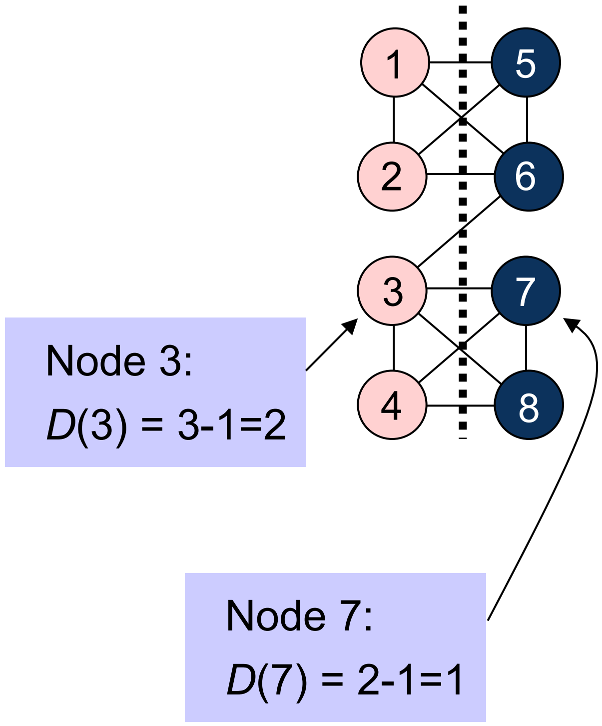
Cost $D(v)$ of moving a node v

$$D(v) = |E_c(v)| - |E_{nc}(v)| \ ,$$

where

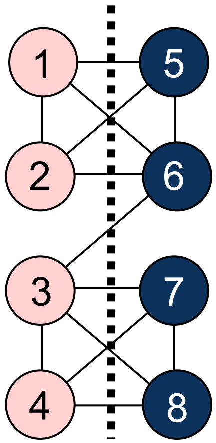
$E_c(v)$ is the set of v's incident edges that are cut by the cut line, and

$E_{nc}(v)$ is the set of v's incident edges that are not cut by the cut line.



High costs ($D > 0$) indicate that the node should move, while low costs ($D < 0$) indicate that the node should stay within the same partition.

Kernighan-Lin (KL) Algorithm – Terminology



Gain of swapping a pair of nodes *a* und *b*

$\Delta g = D(a) + D(b) - 2 * c(a,b),$

where

- *D*(*a*), *D*(*b*) are the respective costs of nodes *a*, *b*
- *c*(*a*,*b*) is the connection weight between *a* and *b*:
  If an edge exists between *a* and *b*,
  then *c*(*a*,*b*) = edge weight (here 1),
  otherwise, *c*(*a*,*b*) = 0.

The gain Δg indicates how useful the swap between two nodes will be

The larger Δg, the more the total cut cost will be reduced

Kernighan-Lin (KL) Algorithm – Terminology

Gain of swapping a pair of nodes *a* und *b*

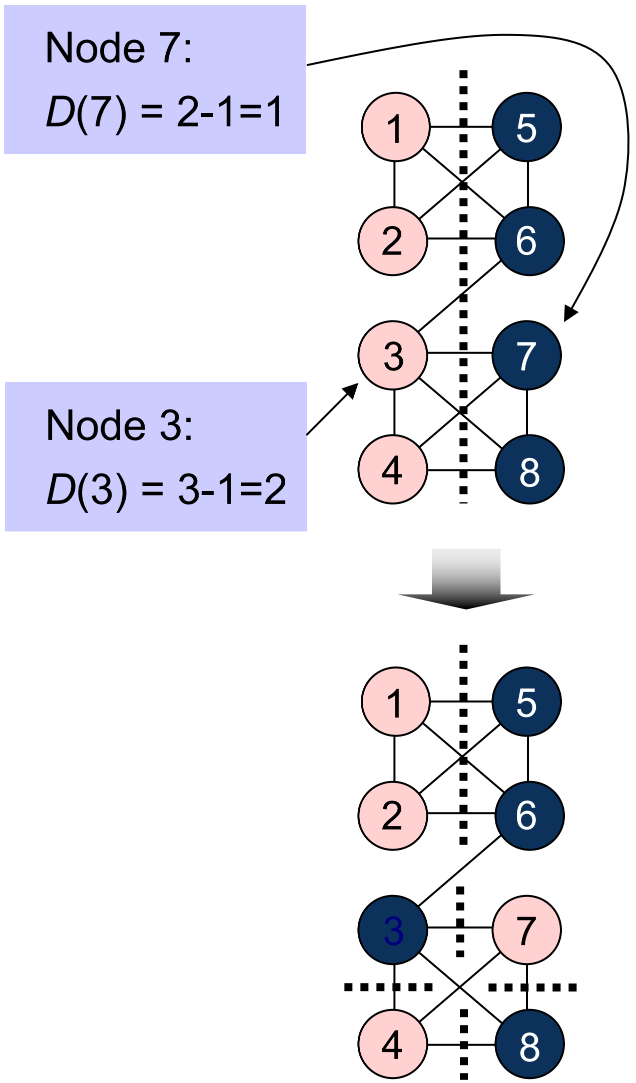
$\Delta g = D(a) + D(b) - 2 * c(a,b)$,

where

- $D(a)$, $D(b)$ are the respective costs of nodes *a*, *b*
- $c(a,b)$ is the connection weight between *a* and *b*:
  If an edge exists between *a* and *b*,
  then $c(a,b)$ = edge weight (here 1),
  otherwise, $c(a,b) = 0$.

$\Delta g(3,7) = D(3) + D(7) - 2 * c(a,b) = 2 + 1 – 2 = 1$

=> Swapping nodes 3 and 7 would reduce the cut size by 1



Kernighan-Lin (KL) Algorithm – Terminology

Gain of swapping a pair of nodes *a* und *b*

$\Delta g = D(a) + D(b) - 2 * c(a,b),$

where

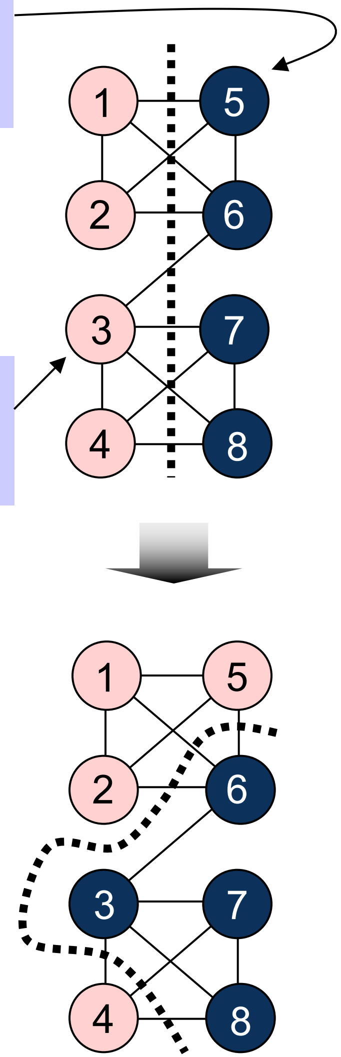
- $D(a)$, $D(b)$ are the respective costs of nodes a, b
- $c(a,b)$ is the connection weight between a and b:
  If an edge exists between a and b,
  then $c(a,b)$ = edge weight (here 1),
  otherwise, $c(a,b) = 0$.

$\Delta g\,(3,5) = D(3) + D(5) - 2 * c(a,b) = 2 + 1 – 0 = 3$

=> Swapping nodes 3 and 5 would reduce the cut size by 3

Node 5:
$D(5)$ = 2-1=1

Node 3:
$D(3)$ = 3-1=2



Kernighan-Lin (KL) Algorithm – Terminology

Gain of swapping a pair of nodes *a* and *b*

The goal is to find a pair of nodes *a* and *b* to exchange such that Δg is maximized and swap them.

Kernighan-Lin (KL) Algorithm – Terminology

Maximum positive gain G_m of a pass

The maximum positive gain G_m corresponds to the best prefix of *m* swaps within the swap sequence of a given pass.

These *m* swaps lead to the partition with the minimum cut cost encountered during the pass.

G_m is computed as the sum of Δg values over the first *m* swaps of the pass, with *m* chosen such that G_m is maximized.

$$G_m = \sum_{i=1}^{m} \Delta g_i$$

Kernighan-Lin (KL) Algorithm – One pass

Step 0:

- $V = 2n$ nodes
- $\{A, B\}$ is an initial arbitrary partitioning

Step 1:

- $i = 1$
- Compute $D(v)$ for all nodes $v \in V$

Step 2:

- Choose a_i and b_i such that $\Delta g_i = D(a_i) + D(b_i) - 2 * c(a_i b_i)$ is maximized
- Swap and fix a_i and b_i

Step 3:

- If all nodes are fixed, go to Step 4. Otherwise
- Compute and update D values for all nodes that are not connected to a_i and b_i and are not fixed.
- $i = i + 1$
- Go to Step 2

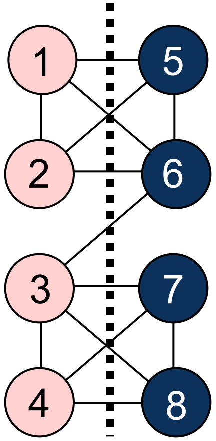
Step 4:

- Find the move sequence 1...m ($1 \le m \le i$), such that $G_m = \sum_{i=1}^{m} \Delta g_i$ is maximized
- If $G_m > 0$, go to Step 5. Otherwise, END

Step 5:

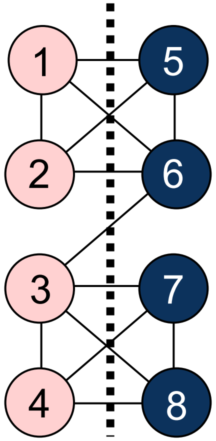
- Execute m swaps, reset remaining nodes

Kernighan-Lin (KL) Algorithm – Example



Cut cost: 9
Not fixed:
1,2,3,4,5,6,7,8

Kernighan-Lin (KL) Algorithm – Example



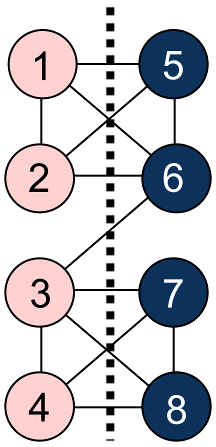
Cut cost: 9
Not fixed:
1,2,3,4,5,6,7,8

Costs $D(v)$ of each node:

$D(1) = 1$	$D(5) = 1$
$D(2) = 1$	$D(6) = 2$
$D(3) = 2$	$D(7) = 1$
$D(4) = 1$	$D(8) = 1$

Nodes that lead to maximum gain

Kernighan-Lin (KL) Algorithm – Example



Cut cost: 9
Not fixed:
1,2,3,4,5,6,7,8

Costs $D(v)$ of each node:

$D(1) = 1$ **$D(5) = 1$**
$D(2) = 1$ $D(6) = 2$
$D(3) = 2$ ~~$D(7) = 1$~~
$D(4) = 1$ $D(8) = 1$

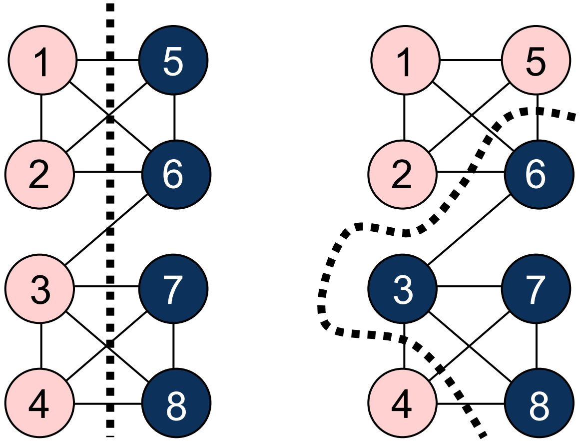
Nodes that lead to maximum gain

$\Delta g_1 = 2+1-0 = 3$ ← Gain after node swapping

Swap (3,5)

$G_1 = \Delta g_1 = 3$ ← Gain in the current pass

Kernighan-Lin (KL) Algorithm – Example



Cut cost: 9
Not fixed:
1,2,3,4,5,6,7,8

$D(1) = 1$	$\mathbf{D(5) = 1}$
$D(2) = 1$	$D(6) = 2$
$\mathbf{D(3) = 2}$	$D(7) = 1$
$D(4) = 1$	$D(8) = 1$

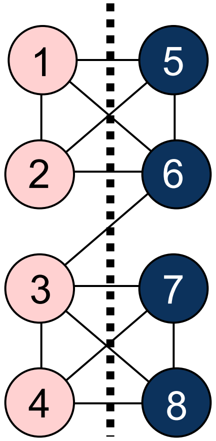
Nodes that lead to maximum gain

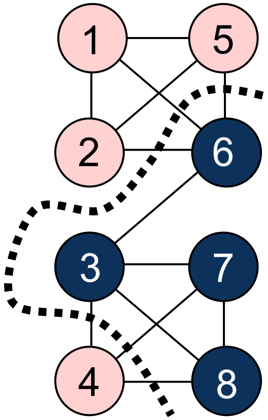
$\Delta g_1 = 2+1-0 = 3$ ← Gain after node swapping

Swap (3,5)

$G_1 = \Delta g_1 = 3$ ← Gain in the current pass

Kernighan-Lin (KL) Algorithm – Example





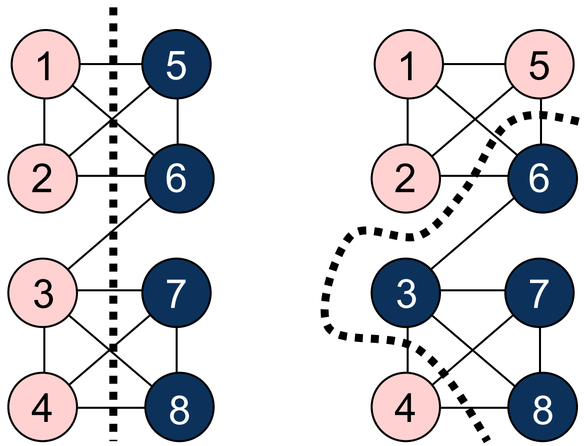
Cut cost: 9
Not fixed:
1,2,3,4,5,6,7,8

Cut cost: 6
Not fixed:
1,2,4,6,7,8

$D(1) = 1$	$\mathbf{D(5) = 1}$
$D(2) = 1$	$D(6) = 2$
$\mathbf{D(3) = 2}$	$D(7) = 1$
$D(4) = 1$	$D(8) = 1$

$\Delta g_1 = 2+1-0 = 3$
Swap (3,5)
$G_1 = \Delta g_1 = 3$

Kernighan-Lin (KL) Algorithm – Example



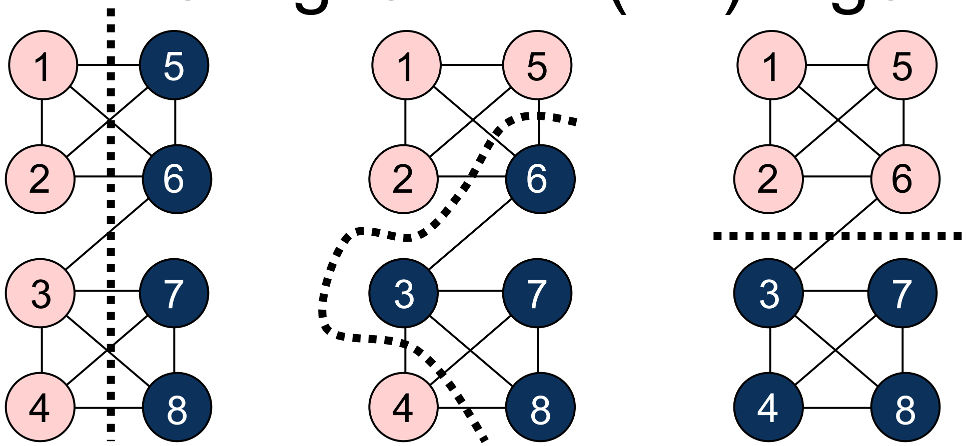
Cut cost: 9 Not fixed: 1,2,3,4,5,6,7,8	Cut cost: 6 Not fixed: 1,2,4,6,7,8

$D(1) = 1$ **$D(5) = 1$**
$D(2) = 1$ $D(6) = 2$
$D(3) = 2$ $D(7) = 1$
$D(4) = 1$ $D(8) = 1$

$\Delta g_1 = 2+1-0 = 3$
Swap (3,5)
$G_1 = \Delta g_1 = 3$

$D(1) = -1$ **$D(6) = 2$**
$D(2) = -1$ $D(7) = -1$
$D(4) = 3$ $D(8) = -1$

Kernighan-Lin (KL) Algorithm – Example



Cut cost: 9
Not fixed:
1,2,3,4,5,6,7,8

Cut cost: 6
Not fixed:
1,2,4,6,7,8

$D(1) = 1$	**$D(5) = 1$**
$D(2) = 1$	$D(6) = 2$
$D(3) = 2$	$D(7) = 1$
$D(4) = 1$	$D(8) = 1$

$\Delta g_1 = 2+1-0 = 3$
Swap (3,5)
$G_1 = \Delta g_1 = 3$

$D(1) = -1$	**$D(6) = 2$**
$D(2) = -1$	$D(7)=-1$
$D(4) = 3$	$D(8)=-1$

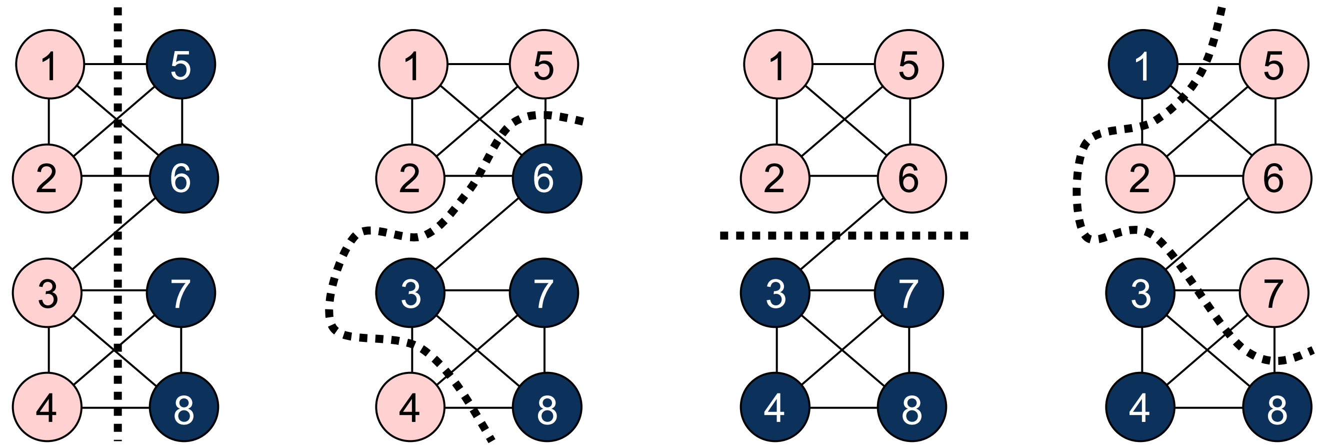
$\Delta g_2 = 3+2-0 = 5$
Swap (4,6)
$G_2 = G_1+\Delta g_2 = 8$

Nodes that lead to maximum gain

Gain after node swapping

Gain in the current pass

Kernighan-Lin (KL) Algorithm – Example



Cut cost: 9
Not fixed:
1,2,3,4,5,6,7,8

Cut cost: 6
Not fixed:
1,2,4,6,7,8

Cut cost: 1
Not fixed:
1,2,7,8

Cut cost: 7
Not fixed:
2,8

$D(1) = 1$ **$D(5) = 1$**
$D(2) = 1$ $D(6) = 2$
$D(3) = 2$ $D(7) = 1$
$D(4) = 1$ $D(8) = 1$

$\Delta g_1 = 2+1-0 = 3$
Swap (3,5)
$G_1 = \Delta g_1 = 3$

$D(1) = -1$ **$D(6) = 2$**
$D(2) = -1$ $D(7)=-1$
$D(4) = 3$ $D(8)=-1$

$\Delta g_2 = 3+2-0 = 5$
Swap (4,6)
$G_2 = G_1+\Delta g_2 = 8$

$D(1) = -3$ **$D(7)=-3$**
$D(2) = -3$ $D(8)=-3$

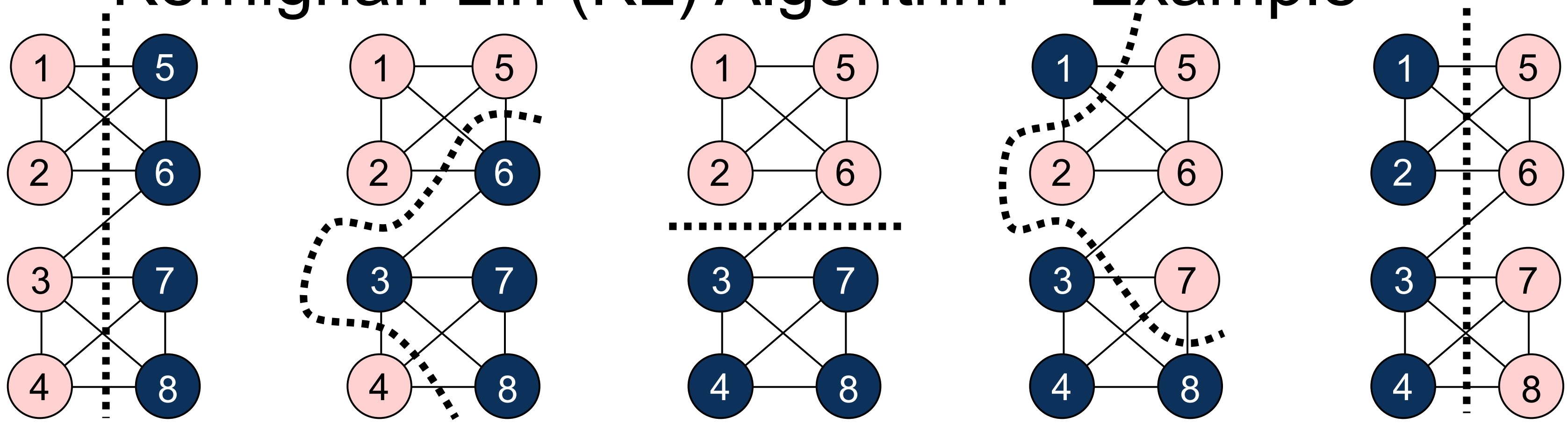
$\Delta g_3 = -3-3-0 = -6$
Swap (1,7)
$G_3 = G_2 + \Delta g_3 = 2$

Nodes that lead to maximum gain

Gain after node swapping

Gain in the current pass

Kernighan-Lin (KL) Algorithm – Example



Cut cost: 9 Not fixed: 1,2,3,4,5,6,7,8	Cut cost: 6 Not fixed: 1,2,4,6,7,8	Cut cost: 1 Not fixed: 1,2,7,8	Cut cost: 7 Not fixed: 2,8	Cut cost: 9 Not fixed: –

$D(1) = 1$ **$D(5) = 1$**
$D(2) = 1$ $D(6) = 2$
$D(3) = 2$ $D(7) = 1$
$D(4) = 1$ $D(8) = 1$

$\Delta g_1 = 2+1-0 = 3$
Swap (3,5)
$G_1 = \Delta g_1 = 3$

$D(1) = -1$ **$D(6) = 2$**
$D(2) = -1$ $D(7) = -1$
$D(4) = 3$ $D(8) = -1$

$\Delta g_2 = 3+2-0 = 5$
Swap (4,6)
$G_2 = G_1 + \Delta g_2 = 8$

$D(1) = -3$ **$D(7) = -3$**
$D(2) = -3$ $D(8) = -3$

$\Delta g_3 = -3-3-0 = -6$
Swap (1,7)
$G_3 = G_2 + \Delta g_3 = 2$

$D(2) = -1$ **$D(8) = -1$**

$\Delta g_4 = -1-1-0 = -2$
Swap (2,8)
$G_4 = G_3 + \Delta g_4 = 0$

Kernighan-Lin (KL) Algorithm – Example

$D(1) = 1$ **$D(5) = 1$** $D(2) = 1$ $D(6) = 2$ **$D(3) = 2$** $D(7) = 1$ $D(4) = 1$ $D(8) = 1$ $\Delta g_1 = 2+1-0 = 3$ **Swap (3,5)** $G_1 = \Delta g_1 = 3$	$D(1) = -1$ **$D(6) = 2$** $D(2) = -1$ $D(7)=-1$ **$D(4) = 3$** $D(8)=-1$ $\Delta g_2 = 3+2-0 = 5$ **Swap (4,6)** $G_2 = G_1+\Delta g_2 = 8$	**$D(1) = -3$** **$D(7)=-3$** $D(2) = -3$ $D(8)=-3$ $\Delta g_3 = -3-3-0 = -6$ **Swap (1,7)** $G_3 = G_2 + \Delta g_3 = 2$	**$D(2) = -1$** **$D(8)=-1$** $\Delta g_4 = -1-1-0 = -2$ **Swap (2,8)** $G_4 = G_3 + \Delta g_4 = 0$

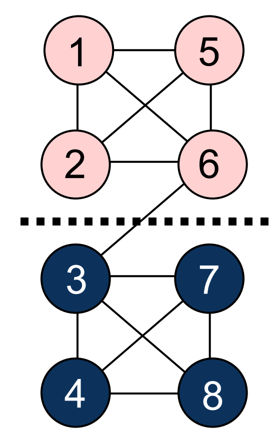
Maximum positive gain $G_m = 8$ with $m = 2$.

Kernighan-Lin (KL) Algorithm – Example

$D(1) = 1$ $D(2) = 1$ $\mathbf{D(3) = 2}$ $D(4) = 1$	$\mathbf{D(5) = 1}$ $D(6) = 2$ $D(7) = 1$ $D(8) = 1$	$D(1) = -1$ $D(2) = -1$ $\mathbf{D(4) = 3}$	$\mathbf{D(6) = 2}$ $D(7) = -1$ $D(8) = -1$
$\Delta g_1 = 2+1-0 = 3$ **Swap (3,5)** $G_1 = \Delta g_1 = 3$		$\Delta g_2 = 3+2-0 = 5$ **Swap (4,6)** $G_2 = G_1 + \Delta g_2 = 8$	
$\mathbf{D(1) = -3}$ $D(2) = -3$	$\mathbf{D(7) = -3}$ $D(8) = -3$	$\mathbf{D(2) = -1}$	$\mathbf{D(8) = -1}$
$\Delta g_3 = -3-3-0 = -6$ **Swap (1,7)** $G_3 = G_2 + \Delta g_3 = 2$		$\Delta g_4 = -1-1-0 = -2$ **Swap (2,8)** $G_4 = G_3 + \Delta g_4 = 0$	

Maximum positive gain $G_m = 8$ with $m = 2$.

Since $G_m > 0$, the first $m = 2$ swaps (3,5) and (4,6) are executed.



Since $G_m > 0$, more passes are needed until $G_m \leq 0$.

Extensions of the Kernighan-Lin (KL) Algorithm

- Unequal partition sizes
  - Apply the KL algorithm with only min(|*A*|,|*B*|) pairs swapped
- Unequal node weights
  - Try to rescale weights to integers, e.g., as multiples of *the greatest common divisor* of all node weights
  - Maintain area balance or allow a *one-move deviation* from balance
- *k*-way partitioning (generating *k* partitions)
  - Apply the KL two-way partitioning algorithm to all possible pairs of partitions
  - Recursive partitioning (convenient when k is a power of two)
  - Direct k-way extensions exist

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Fiduccia-Mattheyses (FM) Algorithm

- Single cells are moved independently instead of swapping pairs of cells --- cannot and do not need to maintain exact partition balance
  - The area of each individual cell is taken into account
  - Applicable to partitions of unequal size and in the presence of initially fixed cells
- Cut costs are extended to include hypergraphs
  - nets with 2+ pins
- While the KL algorithm aims to minimize cut costs based on edges, the FM algorithm minimizes cut costs based on nets
- Nodes and subgraphs are referred to as *cells* and *blocks*, respectively

Fiduccia-Mattheyses (FM) Algorithm

Given: a hypergraph $G(V,H)$ with nodes and *weighted* hyperedges
partition size constraints

Goal: to assign all nodes to disjoint partitions,
so as to minimize the total cost (weight) of all cut nets
while satisfying *partition size constraints*

Fiduccia-Mattheyses (FM) Algorithm

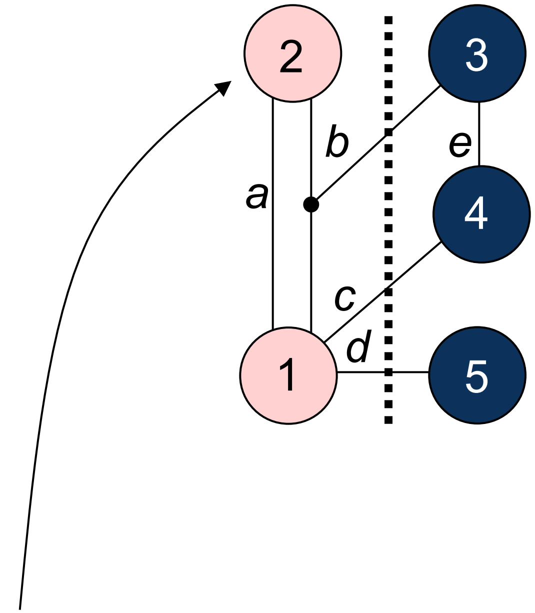
Gain $\Delta g(c)$ for cell c

$$\Delta g(c) = FS(c) - TE(c) ,$$

where

the “moving force“ $FS(c)$ is the number of nets connected to c but not connected to any other cells within c’s partition, i.e., cut nets that connect only to c, and

the “retention force“ $TE(c)$ is the number of *uncut* nets connected to c.



Cell 2: $FS(2) = 0$ $TE(2) = 1$ $\Delta g(2) = -1$

The higher the gain $\Delta g(c)$, the higher is the priority to move the cell c to the other partition.

Fiduccia-Mattheyses (FM) Algorithm–Terminology

Gain $\Delta g(c)$ for cell c

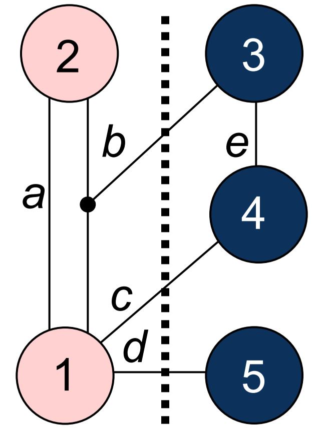
$$\Delta g(c) = FS(c) - TE(c),$$

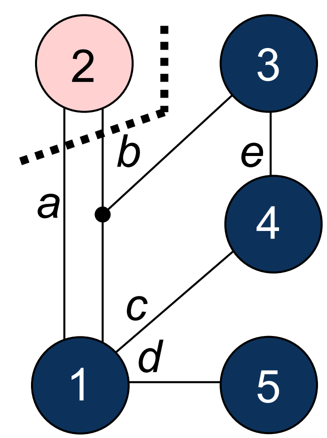
where

the "moving force" $FS(c)$ is the number of nets connected to c but not connected to any other cells within c's partition, i.e., cut nets that connect only to c, and

the "retention force" $TE(c)$ is the number of *uncut* nets connected to c.

Cell 1:	$FS(1) = 2$	$TE(1) = 1$	$\Delta g(1) = 1$
Cell 2:	$FS(2) = 0$	$TE(2) = 1$	$\Delta g(2) = -1$
Cell 3:	$FS(3) = 1$	$TE(3) = 1$	$\Delta g(3) = 0$
Cell 4:	$FS(4) = 1$	$TE(4) = 1$	$\Delta g(4) = 0$
Cell 5:	$FS(5) = 1$	$TE(5) = 0$	$\Delta g(5) = 1$





Fiduccia-Mattheyses (FM) Algorithm–Terminology

Maximum positive gain G_m of a pass

The maximum positive gain G_m is the cumulative cell gain of m moves that produce a minimum cut cost.

G_m is determined by the maximum sum of cell gains Δg over a prefix of m moves in a pass

$$G_m = \sum_{i=1}^{m} \Delta g_i$$

Fiduccia-Mattheyses (FM) Algorithm-Terminology

Ratio factor

The *ratio factor* is the relative balance between the two partitions with respect to cell area

It is used to prevent all cells from clustering into one partition.

The ratio factor r is defined as

$$r = \frac{area(A)}{area(A) + area(B)}$$

where $area(A)$ and $area(B)$ are the total respective areas of partitions A and B

Fiduccia-Mattheyses (FM) Algorithm- Terminology

Balance criterion

The balance criterion enforces the ratio factor.

To ensure feasibility, the maximum cell area $area_{max}(V)$ must be taken into account.

A partitioning of V into two partitions A and B is said to be balanced if

$$[\ r \cdot area(V) - area_{max}(V)\ ] \leq area(A) \leq [\ r \cdot area(V) + area_{max}(V)\ ]$$

Fiduccia-Mattheyses (FM) Algorithm-Terminology

Base cell

A base cell is a cell *c* that has the greatest cell gain $\Delta g(c)$ among all free cells, and whose move does not violate the balance criterion.

Base cell

Cell 1:	$FS(1) = 2$	$TE(1) = 1$	$\Delta g(1) = 1$
Cell 2:	$FS(2) = 0$	$TE(2) = 1$	$\Delta g(2) = -1$
Cell 3:	$FS(3) = 1$	$TE(3) = 1$	$\Delta g(3) = 0$
Cell 4:	$FS(4) = 1$	$TE(4) = 1$	$\Delta g(4) = 0$

Fiduccia-Mattheyses (FM) Algorithm - One pass

Step 0: Compute the balance criterion

Step 1: Compute the cell gain Δg_1 of each cell

Step 2: $i = 1$

- Choose base cell c_1 that has maximal gain Δg_1 , move this cell

Step 3:

- Fix the base cell c_i
- Update all cells' gains that are connected to critical nets via the base cell c_i

Step 4:

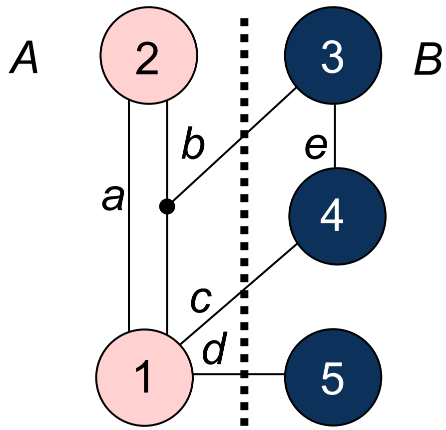
- If all cells are fixed, go to Step 5. If not:
- Choose next base cell c_i with maximal gain Δg_i and move this cell
- $i = i + 1$, go to Step 3

Step 5:

- Determine the best move sequence $c_1, c_2, .., c_m$ $(1 \leq m \leq i)$, so that $G_m = \sum_{i=1}^{m} \Delta g_i$ is maximized
- If $G_m > 0$, go to Step 6. Otherwise, END

Step 6:

Fiduccia-Mattheyses (FM) Algorithm – Example



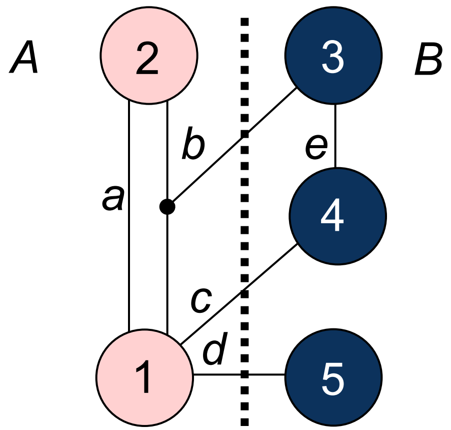
Given:
Ratio factor $r = 0.375$
area(Cell_1) = 2
area(Cell_2) = 4
area(Cell_3) = 1
area(Cell_4) = 4
area(Cell_5) = 5.

Step 0: Compute the balance criterion

$$[\, r \cdot area(V) - area_{max}(V)\,] \leq area(A) \leq [\, r \cdot area(V) + area_{max}(V)\,]$$

$$0{,}375 * 16 - 5 = 1 \leq area(A) \leq 11 = 0{,}375 * 16 + 5.$$

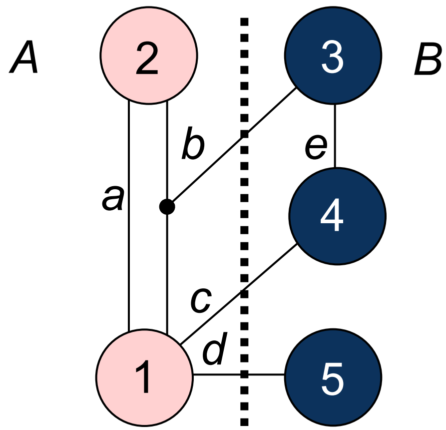
Fiduccia-Mattheyses (FM) Algorithm – Example



Step 1: Compute the gains of each cell

Cell 1:	*FS*(Cell_1) = 2	*TE*(Cell_1) = 1	Δg(Cell_1) = 1
Cell 2:	*FS*(Cell_2) = 0	*TE*(Cell_2) = 1	Δg(Cell_2) = -1
Cell 3:	*FS*(Cell_3) = 1	*TE*(Cell_3) = 1	Δg(Cell_3) = 0
Cell 4:	*FS*(Cell_4) = 1	*TE*(Cell_4) = 1	Δg(Cell_4) = 0
Cell 5:	*FS*(Cell_5) = 1	*TE*(Cell_5) = 0	Δg(Cell_5) = 1

Fiduccia-Mattheyses (FM) Algorithm – Example



Cell1:	*FS*(Cell_1) = 2	*TE*(Cell_1) = 1	Δg(Cell_1) = 1
Cell 2:	*FS*(Cell_2) = 0	*TE*(Cell_2) = 1	Δg(Cell_2) = -1
Cell 3:	*FS*(Cell_3) = 1	*TE*(Cell_3) = 1	Δg(Cell_3) = 0
Cell 4:	*FS*(Cell_4) = 1	*TE*(Cell_4) = 1	Δg(Cell_4) = 0
Cell 5:	*FS*(Cell_5) = 1	*TE*(Cell_5) = 0	Δg(Cell_5) = 1

Step 2: Select the base cell

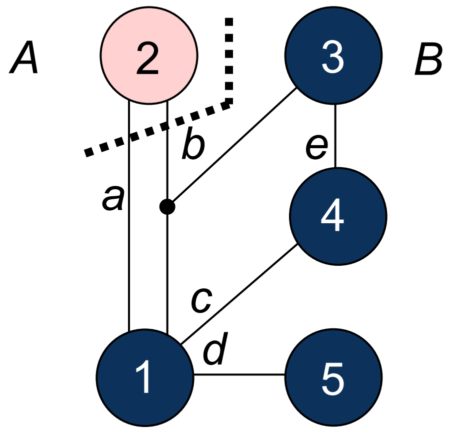
Possible base cells are Cell 1 and Cell 5

Balance criterion after moving Cell 1: *area*(*A*) = *area*(Cell_2) = 4

Balance criterion after moving Cell 5: *area*(*A*) = *area*(*Cell_1*) + *area*(*Cell_2*) + *area*(*Cell_5*) = 11

Both moves respect the balance criterion, but Cell 1 is selected, moved, and fixed as a result of the tie-breaking criterion.

Fiduccia-Mattheyses (FM) Algorithm – Example

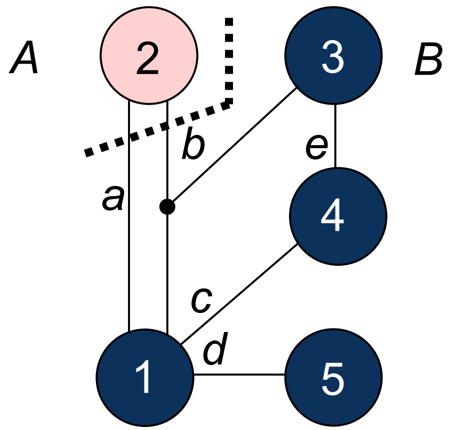


Step 3: Fix base cell, update Δg values

Cell 2:	FS(Cell_2) = 2	TE(Cell_2) = 0	Δg(Cell_2) = 2
Cell 3:	FS(Cell_3) = 0	TE(Cell_3) = 1	Δg(Cell_3) = -1
Cell 4:	FS(Cell_4) = 0	TE(Cell_4) = 2	Δg(Cell_4) = -2
Cell 5:	FS(Cell_5) = 0	TE(Cell_5) = 1	Δg(Cell_5) = -1

After Iteration i = 1: Partition $A_1 = \{2\}$, Partition $B_1 = \{1,3,4,5\}$, with fixed cell $\{1\}$.

Fiduccia-Mattheyses (FM) Algorithm – Example



Iteration i = 1

Cell 2:	*FS*(Cell_2) = 2	*TE*(Cell_2) = 0	Δg(Cell_2) = 2
Cell 3:	*FS*(Cell_3) = 0	*TE*(Cell_3) = 1	Δg(Cell_3) = -1
Cell 4:	*FS*(Cell_4) = 0	*TE*(Cell_4) = 2	Δg(Cell_4) = -2
Cell 5:	*FS*(Cell_5) = 0	*TE*(Cell_5) = 1	Δg(Cell_5) = -1

Iteration i = 2

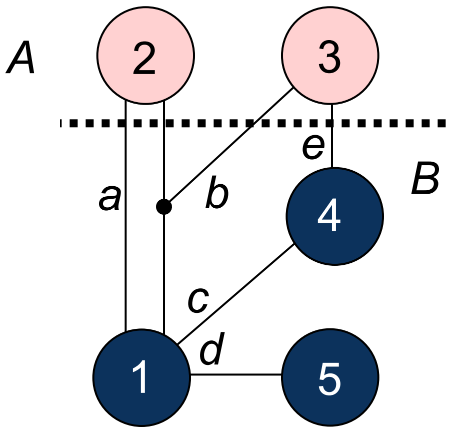
Cell *2* has maximum gain $\Delta g_2 = 2$, *area*(A) = 0, balance criterion is violated.

Cell *3* has next maximum gain $\Delta g_2 = -1$, *area*(A) = 5, balance criterion is met.

Cell *5* has next maximum gain $\Delta g_2 = -1$, *area*(A) = 9, balance criterion is met.

Move cell *3*, updated partitions: A_2 = {2,3}, B_2 = {1,4,5}, with fixed cells {1,3}

Fiduccia-Mattheyses (FM) Algorithm – Example



Iteration i = 2

Cell 2: $\Delta g(\text{Cell\_2}) = 1$

Cell 4: $\Delta g(\text{Cell\_4}) = 0$

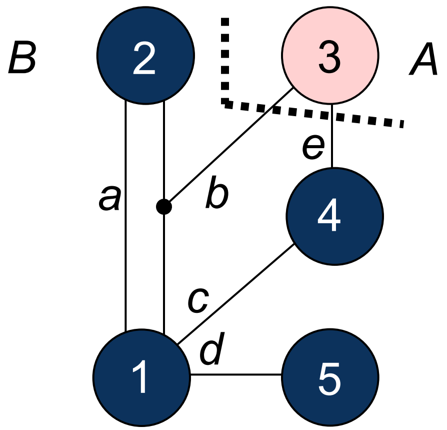
Cell 5: $\Delta g(\text{Cell\_5}) = -1$

Iteration i = 3

Cell *2* has maximum gain $\Delta g_3 = 1$, $area(A) = 1$, balance criterion is met.

Move cell 2, updated partitions: $A_3 = \{3\}$, $B_3 = \{1,2,4,5\}$, with fixed cells {1,2,3}

Fiduccia-Mattheyses (FM) Algorithm – Example



Iteration i = 3

Cell 4: Δg(Cell_4) = 0

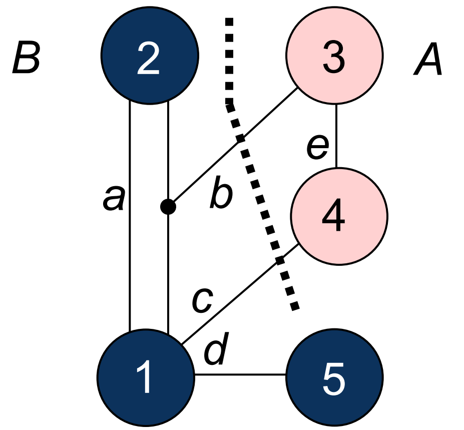
Cell 5: Δg(Cell_5) = -1

Iteration i = 4

Cell 4 has maximum gain $\Delta g_4 = 0$, $area(A) = 5$, balance criterion is met.

Move cell 4, updated partitions: $A_4 = \{3,4\}$, $B_3 = \{1,2,5\}$, with fixed cells {1,2,3,4}

Fiduccia-Mattheyses (FM) Algorithm – Example



Iteration i = 4

Cell 5: Δg(Cell_5) = -1

Iteration i = 5

Cell 5 has maximum gain Δg_5 = -1, *area*(A) = 10, balance criterion is met.

Move cell 5, updated partitions: A_4 = {3,4,5}, B_3 = {1,2}, all cells {1,2,3,4,5} fixed.

Fiduccia-Mattheyses (FM) Algorithm – Example

Step 5: Find best move sequence $c_1 \ldots c_m$

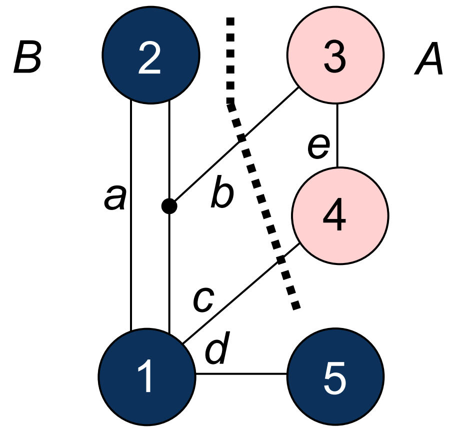
$G_1 = \Delta g_1 = 1$

$G_2 = \Delta g_1 + \Delta g_2 = 0$

$G_3 = \Delta g_1 + \Delta g_2 + \Delta g_3 = 1$

$G_4 = \Delta g_1 + \Delta g_2 + \Delta g_3 + \Delta g_4 = 1$

$G_5 = \Delta g_1 + \Delta g_2 + \Delta g_3 + \Delta g_4 + \Delta g_5 = 0.$



Maximum positive cumulative gain $G_m = \sum_{i=1}^{m} \Delta g_i = 1$

found in iterations 1, 3 and 4.

The move prefix *m* = 4 is selected due to the better balance ratio (*area*(*A*) = 5); the four cells 1, 2, 3 and 4 are then moved.

Result of Pass 1: Current partitions: *A* = {3,4}, *B* = {*1,2,5*}, cut cost reduced from 3 to 2.

Runtime difference between KL & FM

- Runtime of partitioning algorithms
  - KL is sensitive to the number of nodes and edges
  - FM is sensitive to the number of nodes and nets (hyperedges)
- Asymptotic complexity of partitioning algorithms
  - KL has cubic time complexity *per pass*
  - FM has linear time complexity *per pass*

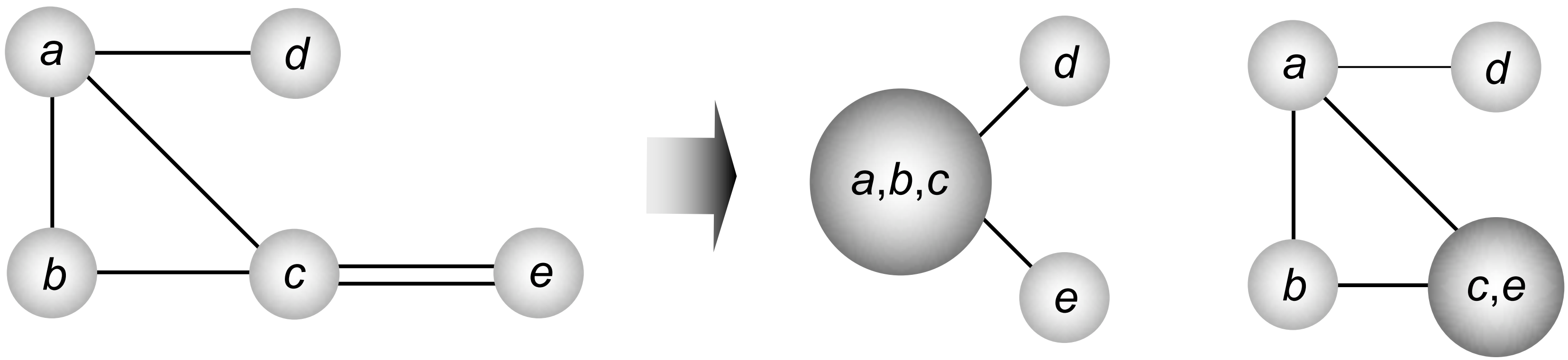
Clustering

- To simplify the problem, groups of tightly-connected nodes can be clustered, absorbing connections between these nodes

- Size of each cluster is often limited so as to prevent degenerate clustering, i.e. a single large cluster dominates other clusters

- Refinement should satisfy balance criteria

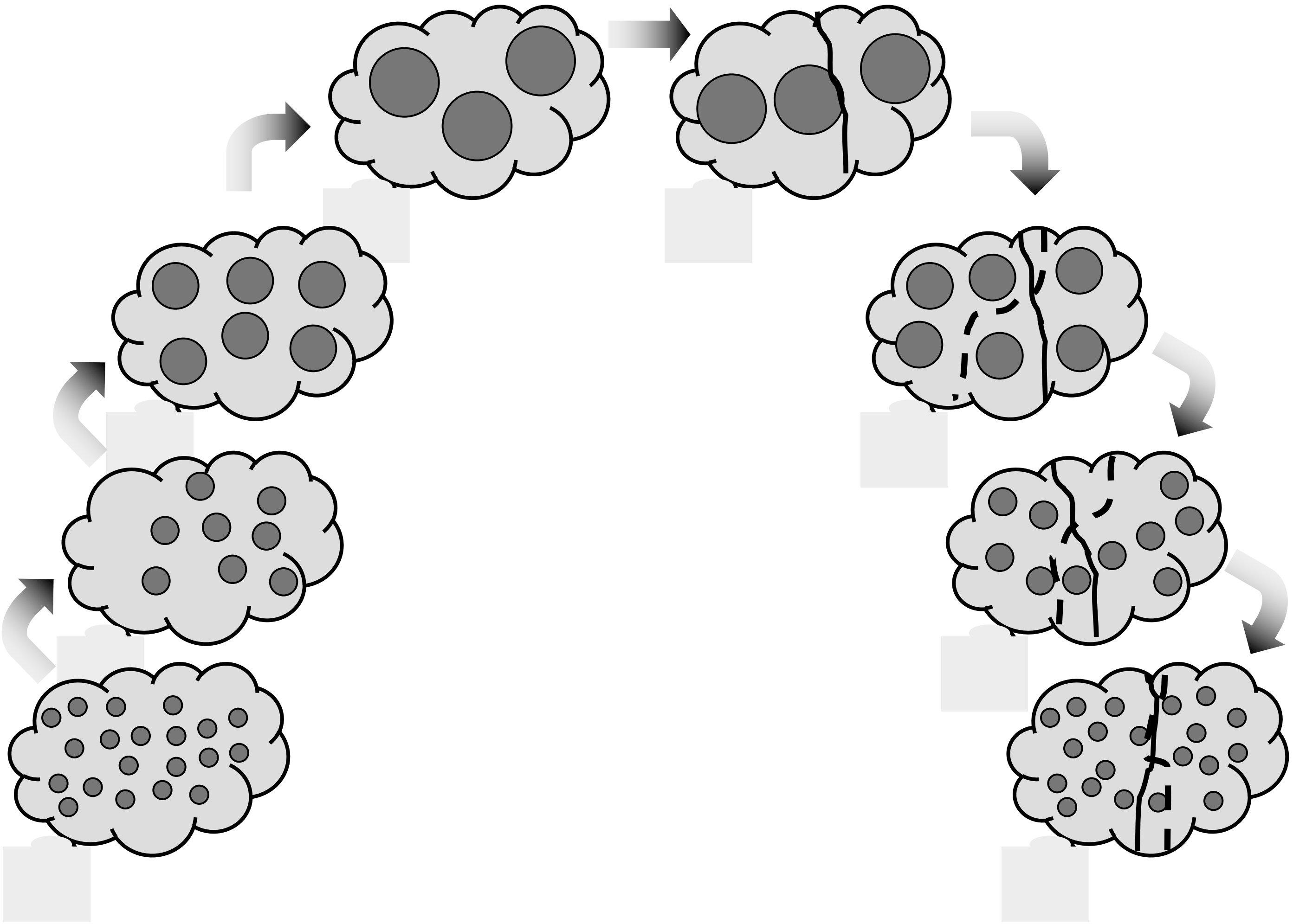
Clustering



Initital graph

Possible clustering hierarchies of the graph

Multilevel Partitioning



Summary

- Circuit netlists can be represented by graphs
- Partitioning a graph means assigning nodes to disjoint partitions
  - Total size of each partition (number/area of nodes) is limited
  - Objective: minimize the number connections between partitions
- Basic partitioning algorithms
  - Move-based, move are organized into passes
  - KL swaps pairs of nodes from different partitions
  - FM re-assigns one node at a time
  - FM is faster, usually more successful
- Multilevel partitioning
  - Clustering
  - FM partitioning
  - Refinement (also uses FM partitioning)