Advanced Topics in VLSI EE-6361

Assignment 2 - Due on 6 March 2017

February 27, 2017

1. The delay of a logic gate can be written as

$$D = 30 + 0.5Z$$

where Z is a standard normal random variable. In MATLAB/ OCTAVE use the "randn"function to generate N samples of X. $P(D > \alpha)$ can be evaluated as the fraction of values of D that exceed α. Now evaluate the following by varying N from 10^3 to 10^7 in geometrically progressing steps of $2X$

Algorithm 1 Standard Monte Carlo

procedure MC(N, α) ▷ N: Number of samples. α: Threshold of delay
    $Z \leftarrow randn(N, 1)$ ▷ Generate N random samples using the *randn* function and evaluate D
    $D \leftarrow 30 + 0.5Z$
    $sum \leftarrow 0$
    for $i = 1 : N$ **do**
        if $D(i) \geq \alpha$ **then**
            $sum \leftarrow sum + 1$
        end if
        $i \leftarrow i + 1$
    end for
    $prob = sum/N$
end procedure

- $P(D > 30.5) = P(Z > 1)$
- $P(D > 31.5) = P(Z > 3)$
- $P(D > 33) = P(Z > 6)$

Repeat the experiment five times and plot the above probabilities vs N. Can you comment on the variance of the sample estimate?

2. Consider the third case in the previous problem. Since $P(D > 33) = P(Z > 6)$ we will now deal, instead, with the standard normal RV Z whose PDF is given by $f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(z)^2}{2}}$. Implement the following pseudo-code in MATLAB and show how Importance sampling helps reduce the number of samples by orders of magnitude when trying to evaluate $P(Z > 6)$

Algorithm 2 Importance Sampling

procedure IMPSAMPLING(N, α)
    $Z \leftarrow randn(N, 1)$ ▷ Generate N random samples using the *randn* function and evaluate $Z1$
    $Z_1 \leftarrow Z + 2$ ▷ A standard normal PDF shifted by 2σ - $g_{Z_1}(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(z-2)^2}{2}}$
    $sum \leftarrow 0$
    for $i = 1 : N$ **do**
        if $Z1(i) \geq \alpha$ **then**
            $sum \leftarrow sum + f_Z(Z_1(i))/g_{Z_1}(Z_1(i))$
        end if
        $i \leftarrow i + 1$
    end for
    $prob = sum/N$
end procedure

Repeat the experiment five times and plot the above probabilities vs N.