Department of Electrical Engineering
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EE 5140: Digital Modulation & Coding

September 2017 **Tutorial #2** KG/IITM

1. Consider a band-pass signal $s(t) = I(k)g(t)Cos(2\pi f_c t)$ for $kT \le t \le (k+1)T$ where the pulse shape $g(t)$=sqrt(2/T) for $0 \le t \le T$. Here, message symbol $I(k) \in \{+3d, +d, -d, -3d\}$, and the received sample at the output of the matched filter can be written as $z(k) = \alpha I(k) + v(k)$ where $v(k)$ is WGN with variance No/2, and the real scalar α accounts for any possible gain(scaling) error encountered in the AGC-ADC operations.

(a) If the average energy E_a for this signal set is 4 Joules, what is d? *Hint:* Also, relate this E_a to the distance 2d between the neighbouring points in the constellation in order to answer part-(c).

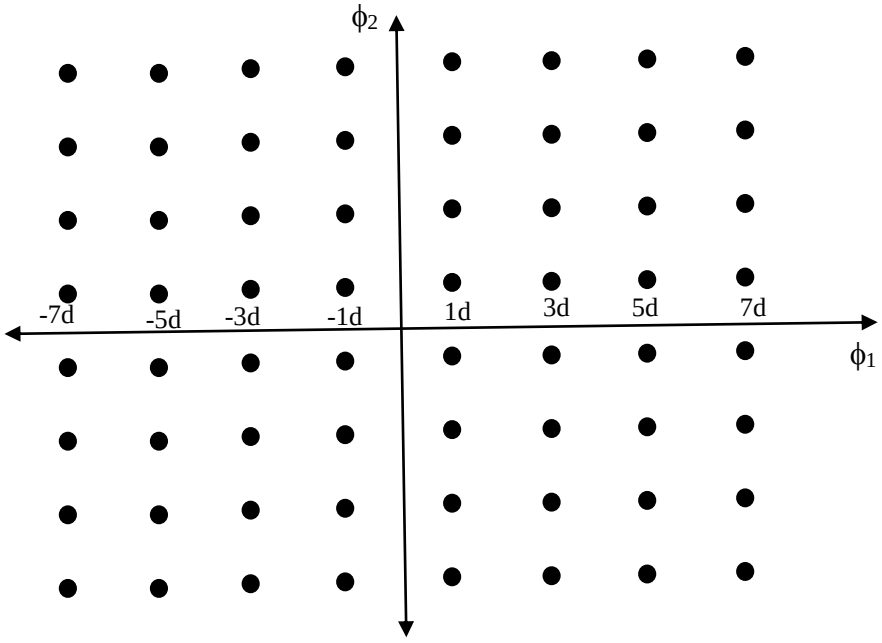
(b) For α=1, find the exact expression for the average probability of symbol error P_e in the above AWGN channel. Express your answer in terms of $q(d) = Q\left(\frac{d}{\sqrt{N_0/2}}\right)$ with 2d as in part (a).

(c) Now, if α=1.5 and this knowledge is not known at the receiver, find the new expression for P_e when the same decoder (decision regions) as in part (b) is used.

2. Derive the average probability of symbol error P(e) for the square 64-QAM constellation shown below in terms of $q(d)$.

(a) Assume instead that the union bound is used only on the "nearest neighbor" symbols. Use this to compute bound on $P_{UB}(e)$. What is this expression?

(b) How does this compare with the true P(e)? Numerically evaluate both of them for Eb/No = 10dB.



3. Consider a band-pass signal $s(t)=I_1(k)g(t)\text{Cos}(2\pi f_c t)+I_2(k)g(t)\text{Sin}(2\pi f_c t)$, for $kT\le t\le(k+1)T$, where the pulse shape $g(t)=\text{sqrt}(2/T)$ for $0\le t\le T$. If $I_1(k)\in\{+1,-1\}$, while $I_2(k)\in\{+3,+1,-1,-3\}$, determine the following:

(a) What is the ortho-normal basis set and plot the corresponding signal constellation.

(b) What is the average energy E_a for this signal set? *Hint:* Also, relate this E_a to the distance 2d between the neighbouring points in the constellation in order to answer part-(c).

(c) Find the exact expression for the probability of symbol error in an AWGN channel with PSD No/2. Express your answer in terms of q where $q(d)=Q(\ d/\text{sqrt}(No/2)\ )$ with 2d as in part (b).

(d) Perform Gray coding for the constellation. Using this, provide the expression for the <u>bit</u> error probability (i.e., bit error rate) for the above measurement model.

4. Consider the "square" 4-QAM (set x), 16-QAM (set y), and 64-QAM (set z) signal sets, discussed in class and/or tutorial.

(a) How many bits per symbol are carried by each of the 3 sets?

(b) Plot the 16-QAM signal constellation with "Gray Coding" to ensure that all the nearest neighbor symbols differ only by 1-bit labels.

(c) For the *same* average energy per bit, E_b, find the minimum distances d_x of 4-QAM and d_y of 16-QAM in terms of the minimum distance d_z of the 64-QAM constellation.

5. Consider the signal constellation in Fig. 2 with minimum distance 2*d*. When this signal is sent through an ideal channel and corrupted by additive white Gaussian noise with variance No/2, and after matched filtering and sampling, the received samples are given by $r(k)=s_i(k)+n(k)$ where $i\in\{1,2,...,6\}$.

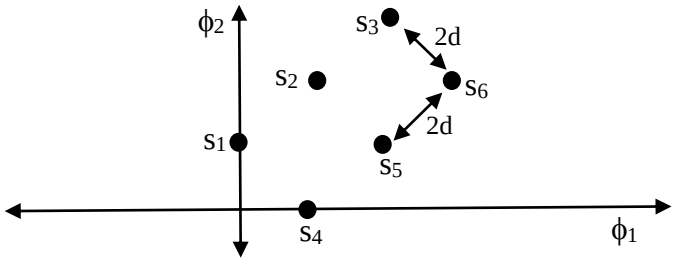


Fig. 2

(a) Assuming that all the symbols are equi-probable, find the exact expression for the average probability of symbol error P_e in the above AWGN channel. Express your answer in terms of $q(d)=Q\left(\frac{d}{\sqrt{N_0/2}}\right)$ with 2*d* as the minimum distance.

(b) Now instead, if the probability of occurrence of symbols s_2 and s_5 is 1/3, while that of the remaining 4 symbols is 1/12 each, find the new expression for P_e. Make a rough plot of the new decision regions, if any.

6. Consider a band-pass signal $s_{i,j}(t)=d_i(k)\varphi_j(t)$, for $kT \le t \le (k+1)T$, where $\varphi_j(t)$, j=1,2,..,N, are orthonormal basis functions defined between $0 \le t \le T$, and $d_i(k)$ is uniformly drawn from $\{+3d, +1d, -1d, -3d\}$. Given that the total number of signals in the constellation is therefore given by $i \times j = 4 \times N$, answer the following:

(a) For N=2, plot the signal constellation.

(b) For N=2, what is the exact expression for average probability of symbol error P_e in AWGN channels in terms of d and noise variance No/2?

(c) Develop an approximate expression for P_e for N=2 by considering the Union Bound with only the nearest neighbours.

7. Do the following problems from the 7th chapter in the text-book (Proakis and Salehi), starting with page. 453 in the E-version. The problems marked with "*" are a little bit harder, since they were not discussed in the class (as yet).

- ➔ All problems from **7.10 to 7.35** excepting 7.17. The possibly hard-ones are 7.23*, 7.28*, 7.31*, 7.32*, and 7.35*
- ➔ All problems from **7.42 to 7.46**.