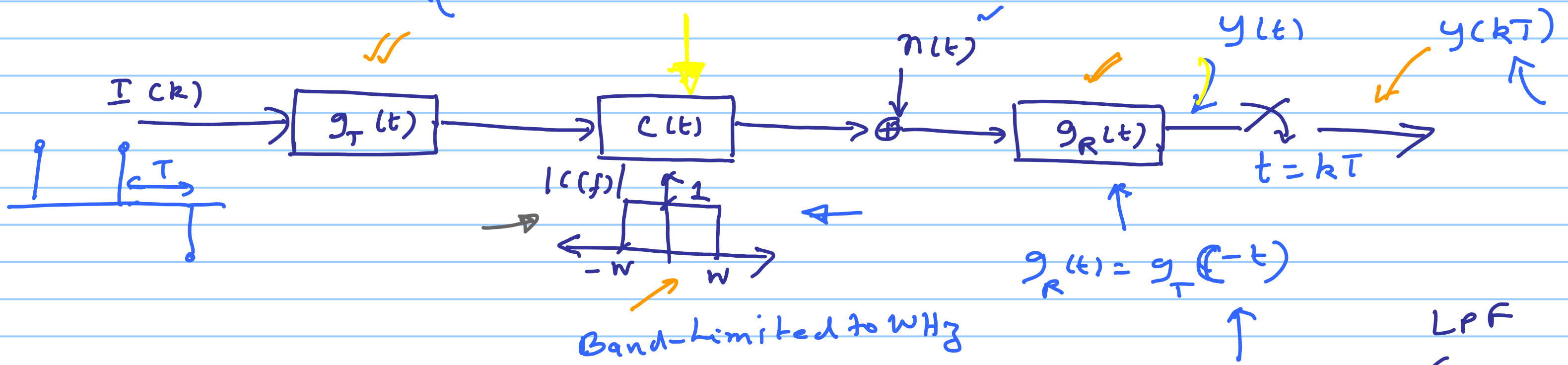
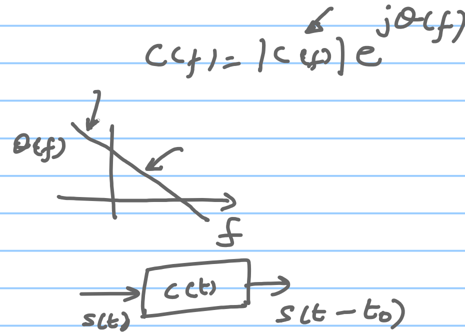
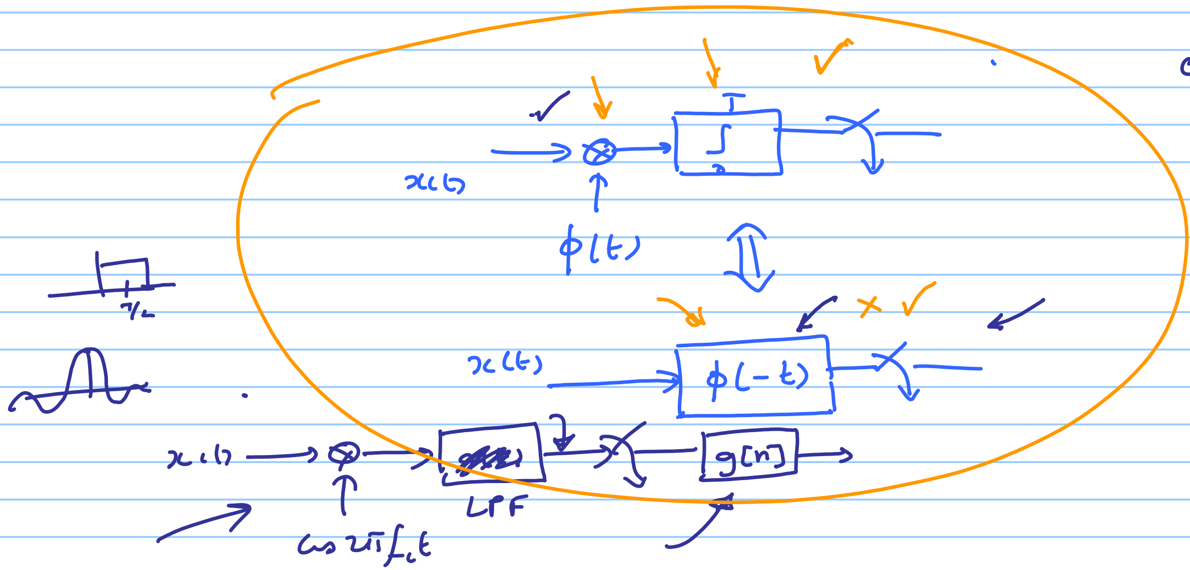
Lesson 3 -- Digital communications thro band-limited “flat” channels – Symbol-by-symbol modulation based signal design for band-limited channels (Nyquist criterion), partial response signaling





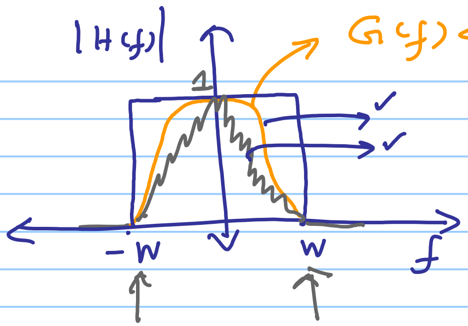
LPF

$$\phi_1(t) = g(t)\cos(2\pi f_c t)$$
$$\phi_2(t) = g(t)\sin(2\pi f_c t)$$



$$\boxed{\text{Rate} < W \cdot \log_2(1 + \text{SNR})}$$

$h(t) \rightleftharpoons$



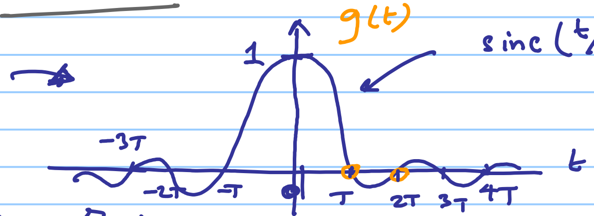
$G(f) \leftarrow g(t)$

$\delta(t) \rightleftharpoons 1$

$$\boxed{T \longrightarrow \frac{1}{2T} \le W}$$

then the signal goes thro' the channel without any distortion

Recall

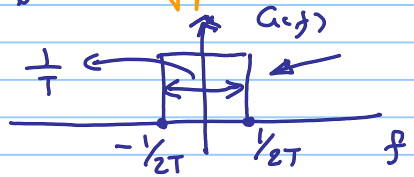


zero-crossing every $t = T$ seconds except $t = 0$

$$\mathrm{sinc}(t/T) \rightarrow \frac{\sin(\pi t/T)}{(\pi t/T)} \qquad t = 0$$

$$t = nT, \; n \rightarrow \text{integer}$$

Fourier Transform



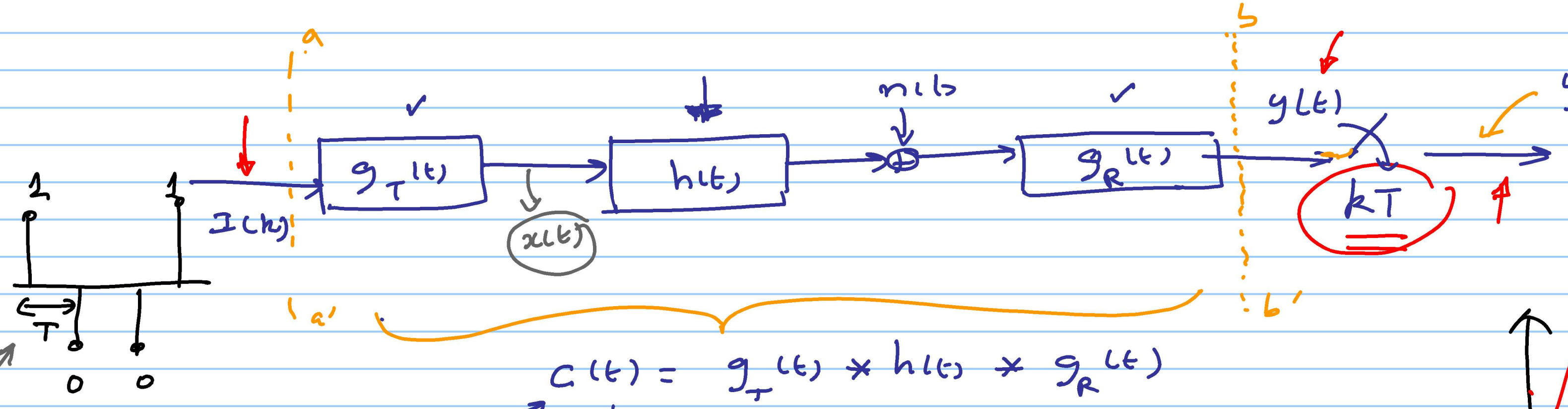
$$g(t) = \int_{-1/2T}^{1/2T} \underbrace{G(f)}_{1} e^{j2\pi f t} \, df$$

$$1 \cdot \frac{e^{j2\pi f t}}{j2\pi t} \Bigg|_{-1/2T}^{+1/2T} = \frac{\sin(\pi t/T)}{(\pi t/T)}$$

Nyquist Pulse Shaping

$g(t) \longrightarrow$ Sinc Pulse ✓

$\quad \longrightarrow$ Any Nyquist Pulse $\longrightarrow$ Square Root Raised Cosine

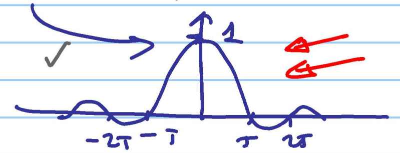


$y(k) = I(k-\Delta)$ ✓

group delay is the slope of the phase response

$\Delta = 3T$

$$c(t) = g_T(t) * h(t) * g_R(t)$$

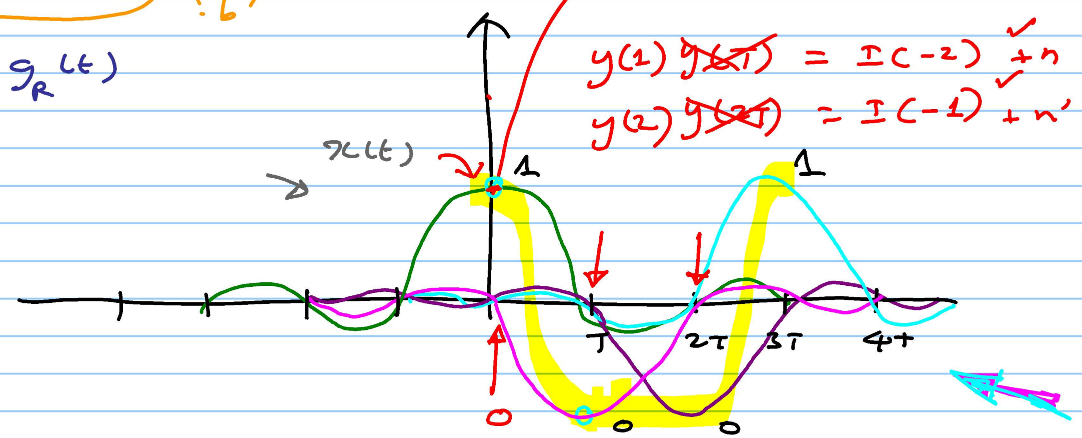


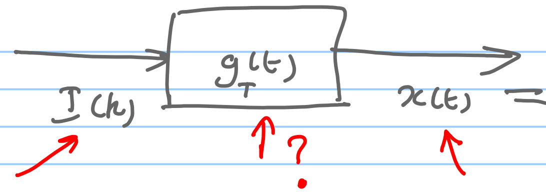
$$y(t) = c(t) * \{I(k)\}$$

$y(0) = I(-3)$

$y(1)\ \cancel{y(T)} = I(-2) + n$

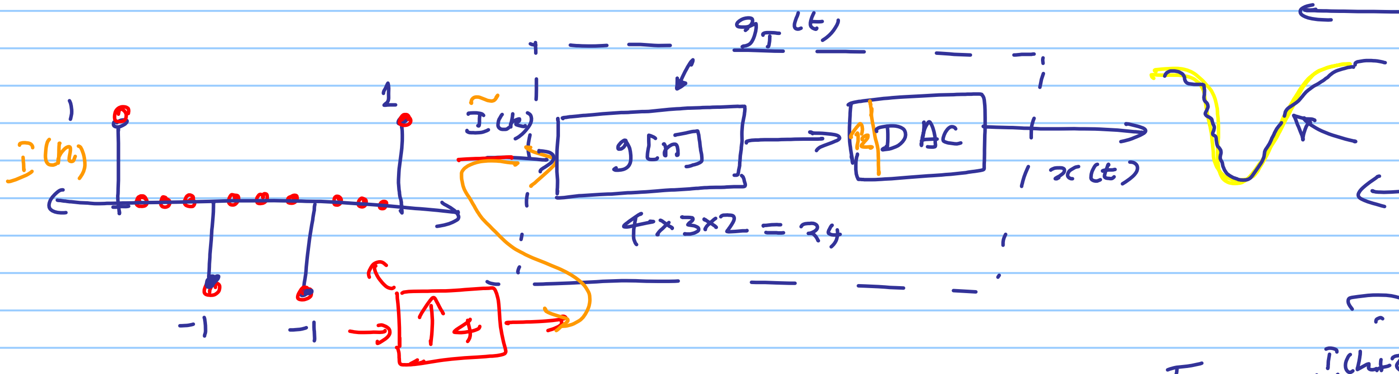
$y(2)\ \cancel{y(2T)} = I(-1) + n'$





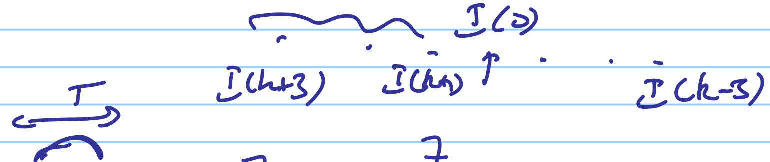
$$x(t) = \sum_{k=-\infty}^{+\infty} I(k)\, g_T(t-kT)$$

truncating the g(t) → g[n]

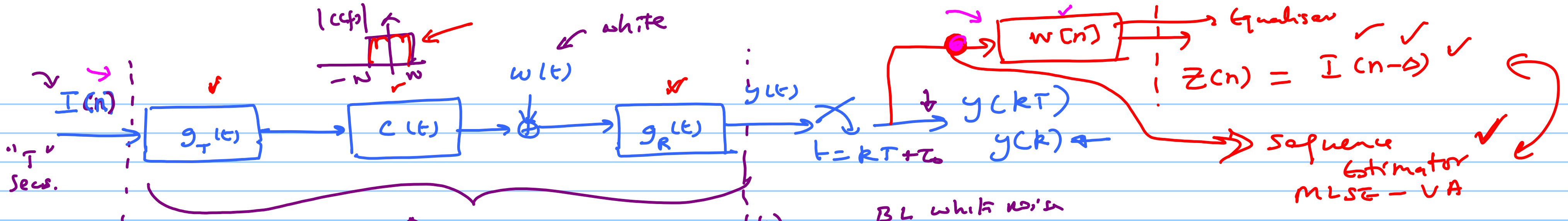


4×3×2 = 24

7 values of I(k)



$7 \to 2^7$



$$x(t) \triangleq g_T(t) * c(t) * g_R(t)$$

BL white noise

$w(t) * g_R(t)$

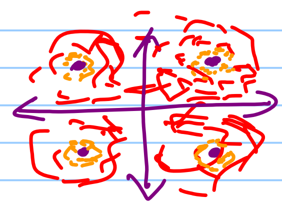
$$y(t) = \sum_{n=-\infty}^{+\infty} I(n)\, x(t - nT) + n(t)$$

every $t = kT + \tau_0$ timing offset

$\tau_0 = 0$

$kT \to k$

$$y(kT + \tau_0) = \sum I(n)\, \underbrace{x(kT - nT + \tau_0)}_{x(k-n)} + n(kT + \tau_0)$$



$$y(k) = \sum_{n=-\infty}^{+\infty} I(n)\, x(k-n) + n(k)$$

$$= \underbrace{I(k)\, x(0)}_{\text{desired symbol}} + \underbrace{\sum_{\substack{n=-\infty \\ n \neq k}}^{+\infty} I(n)\, x(k-n)}_{\text{Inter-Symbol Interference}} + \underbrace{n(k)}_{\text{noise}}$$

✓ $x(0) = 1$

(ISI) ✗

$I(n)\, x(0) + I(n-1)\, x(1) + n(k)$

Nyquist Condition for zero ISI

Given $g_R(t)$, $g_T(t)$ and band-limited $c(t)$ (with unit gain between $-W$ & W Hz), choose

$$x(n) = \begin{cases} 1, & n=0 \\ 0, & \text{otherwise} \end{cases} \longrightarrow (1)$$

$$\boxed{x(t) \rightleftharpoons X(f)}$$

Theorem: The necessary & sufficient condition for $x(t)$ to satisfy (1) is that $X(f)$ must satisfy

$$\rightarrow \sum_{m=-\infty}^{+\infty} X\left(f + \frac{m}{T}\right) = \underbrace{T}_{\text{constant}} \longrightarrow (2)$$

$\Leftarrow$ "Folded Spectrums" is constant (or flat)

Proof:

$$x(t) = \int_{-\infty}^{+\infty} X(f)\, e^{j2\pi f t}\, df$$

$$\Rightarrow x(nT) = \int_{-\infty}^{+\infty} X(f)\, e^{j2\pi f nT}\, df = \sum_{m=-\infty}^{+\infty} \int_{\frac{2m-1}{2T}}^{\frac{2m+1}{2T}} X\left(f+\frac{m}{T}\right) \cdot e^{j2\pi f nT}\, df$$

$$= \int_{-1/2T}^{+1/2T} B(f)\, e^{j2\pi f nT}\, df \longrightarrow (3) \quad \text{where} \quad B(f) = \sum_{m=-\infty}^{+\infty} X\left(f+\frac{m}{T}\right);$$

Since $B(f)$ is periodic with $\frac{1}{T}$, its Fourier series expansion is given by

$$B(f) = \sum_{n=-\infty}^{+\infty} b_n e^{j2\pi n f T} \longrightarrow (4)$$

where $$b_n = T \cdot \int_{-1/2T}^{+1/2T} B(f) e^{-j2\pi n f T} \, df \longrightarrow (5)$$

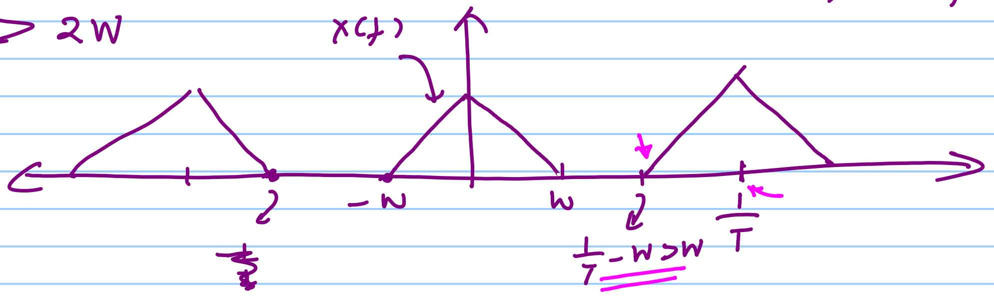
Comparing (3) & (5) $\quad b_{-n} = T\, x(n) \Rightarrow b_n = \begin{cases} T, & n = 0 \\ 0, & n \neq 0 \end{cases}$

using this result in (4) $\quad \boxed{B(f) = \sum x\left(f + \frac{m}{T}\right) = T} \qquad x(f) \rightleftharpoons X(f)$

$\longrightarrow$ signalling Rate $= \frac{1}{T}$ symbols/sec $\qquad T \rightarrow$ symbol duration

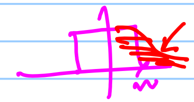
$x(t) \rightleftharpoons X(f)$

Case (i) : $\frac{1}{T} > 2W$

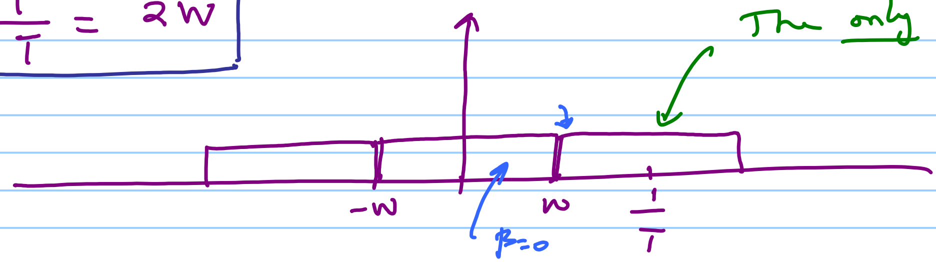


$B(f) \neq$ Const. (T)

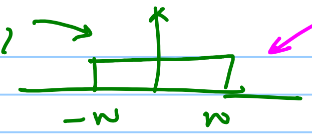
$\therefore$ ISI $\neq 0$



Case ii : $\boxed{\frac{1}{T} = 2W}$



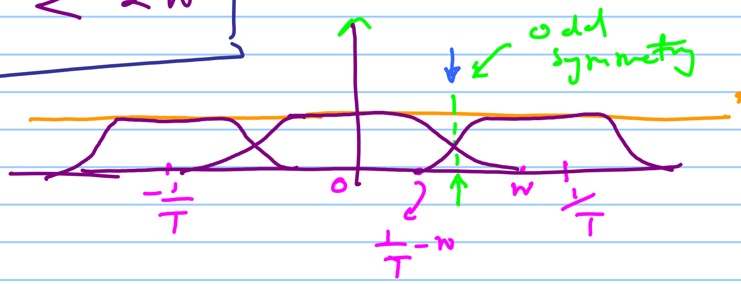
The only choice for $X(f)$ →

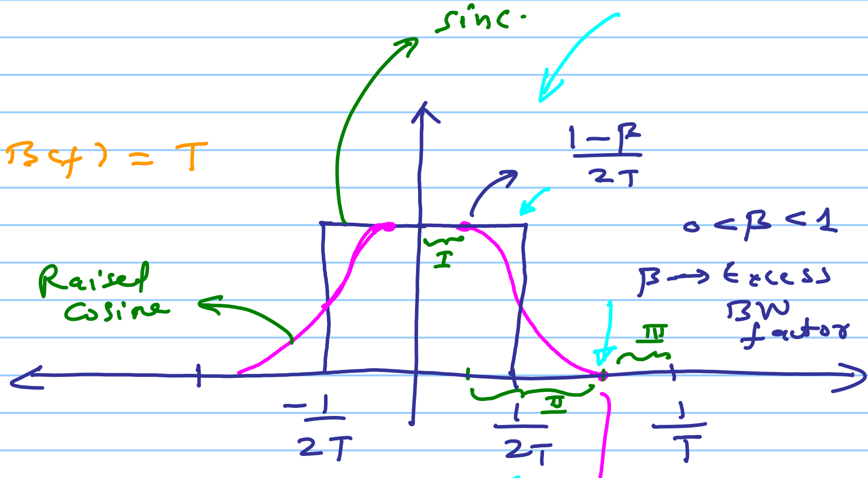


Case iii : $\boxed{\frac{1}{T} < 2W}$

$W > \frac{1}{2T}$

$\frac{1+\beta}{2T}$





$0 < \beta < 1$

$\beta \rightarrow$ Excess BW factor

Raised Cosine Family

$$X_{rc}(f) = \begin{cases} T, & 0 \le |f| \le \frac{1-\beta}{2T} \\ \frac{T}{2}\left[1 + \cos\frac{\pi T}{\beta}\left(|f| - \frac{1-\beta}{2T}\right)\right], & \frac{1-\beta}{2T} \le |f| \le \frac{1+\beta}{2T} \\ 0, & |f| > \frac{1+\beta}{2T} \end{cases}$$

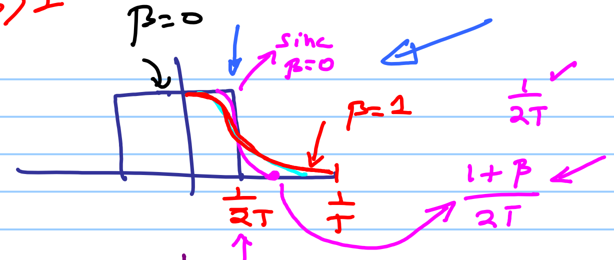
$\frac{1+\beta}{2T}$

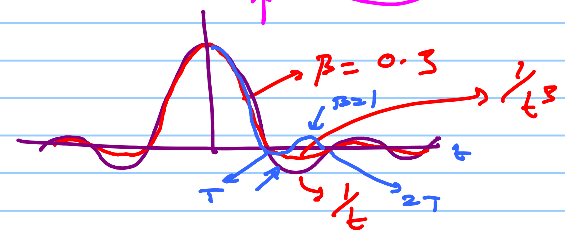
$x_T(t)$

$$x(t) = \text{sinc}\left(\frac{\pi t}{T}\right) \cdot \left\{ \frac{\cos\left(\pi \beta t / T\right)}{1 - \left(4\beta^2 t^2 / T^2\right)} \right\}$$

$\frac{\sin \pi t/T}{\pi t/T}$

$\frac{4\beta^2 t^2}{T^2} >> 1$





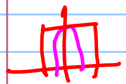
Choose $X_{RC}(f) = G_T(f) \cdot C(f) \cdot G_R(f)$ ← 1

$$\Rightarrow \quad G_T(f) = \sqrt{|X_{RC}(f)|}\; e^{-j2\pi f t_0}$$

Matched Filter → $G_R(f) = \sqrt{|X_{RC}(f)|}\; e^{j2\pi f t_0}$ $\qquad G_R(f) = G_T^*(f)$

zero ISI

Rate ⟷ Realisable Filter



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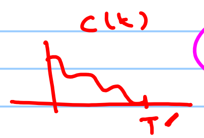


$T \Rightarrow T' \approx$

T → symbol duration

$2W = \frac{1}{T}$

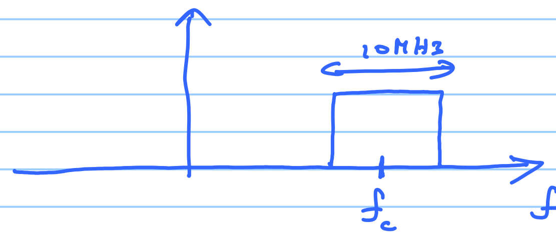
$\cancel{\frac{1}{T} 2W}$

→ Duo-Binary Signalling | Correlative Coding | Controlled ISI

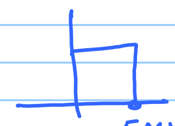
Question: What is the bit-rate if 16-QAM symbols are used over a 10 MHz (RF) channel?

(a) For $\beta = 0$

(b) For $\beta = 0.5$



$\rightarrow$ one sided BW $W = 5$ MHz



$5\text{MHz} = \frac{1}{2T}$

(a) $\Rightarrow \frac{1}{T} = 10$ MHz

$\Rightarrow$ 10 M symbols/sec. ✓ $\leftarrow$ symbol rate

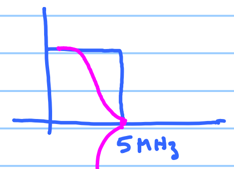
$T \quad 1$ symbol
1

$\Rightarrow$ For 16-QAM (1 symbol $\rightarrow$ 4 bits) $\Rightarrow$ $10 \times 4 \Rightarrow 40$ Mbits/sec ✓

$\beta \rightarrow$ excess BW factor

$0 \le \beta \le 1$

(b)



$\frac{1+\beta}{2T}$

$\frac{1+\beta}{2T} = 5$ MHz

$\Rightarrow \frac{1.5}{2T} = 5$ MHz

$\Rightarrow \frac{1}{T} = \frac{2 \times 5 \times 2}{3} = \frac{20}{3} = 6.66$ MHz

$\Rightarrow$ Symbol Rate $= 6.66$ M symbols/sec

$\Rightarrow$ bit Rate $6.66 \times 4 \approx 26$ Mbits/sec

1G $\rightarrow$ AMPS

2G $\rightarrow$ D-AMPS (ANSI-136) $\rightarrow$ $\beta = 0.33$

V. fast
V.32 bis

Allow controlled ISI

But full-Rate

$W \rightarrow 2W$

and realizable filters

Zero-ISI

theoretically, for best-rate use Sync. fn. $(\beta = 0)$

$W \rightarrow 2W$ symbols/sec
Hz

practically, reduce the rate and use RC fns. $0 < \beta \le 1$

$W \rightarrow \dfrac{2W}{(1+\beta)}$ symbols/secs
Hz

Controlled ISI

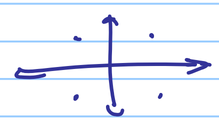
Partial-Response Signalling

Duo-Binary Signalling

eg

$$\boxed{y(k) = x_0 I(k) + x_1 I(k-1) + n(k)}$$

$$y(k+1) = x_0 I(k+1) + x_1 I(k) + n(k+1)$$



$$y(k) = x_0 I(k) + n(k)$$

$$z(k) = \frac{y(k)}{x_0} \approx I(k)$$

Decision Feedback

$$A \xrightarrow{I(1)} B$$

Tx Rx x_0 x_1

$$y(2) = \underset{=1}{x_0} I(2) + \underset{=1}{x_1} I(1) + n(2)$$

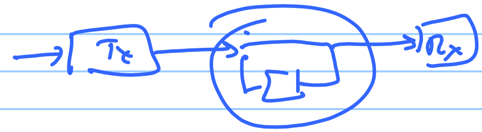
$$\left\{ \begin{array}{l} y(2) - x_1 I(1) = \tilde{z}(2) \\ \tilde{z}(2) / x_0 = z(2) \end{array} \right.$$

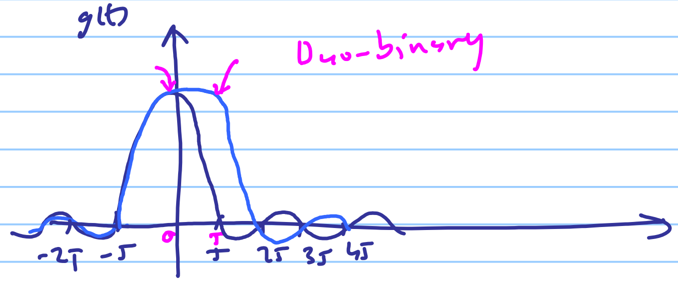
$\hat{I}(2)$

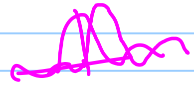
$$y(3) = x_0\, I(3) + x_1\, I(2) + n(3) \quad \rightarrow \quad y(3) - x_1\, \hat{I}(2)$$

$$\hat{I}(2) \cong I(2)$$

Decision Feedback $\longrightarrow$ to the transmitter !! using a Precoder







$$\boxed{y(k) = \sum_{l=0}^{L-1} f_l\, I(k-l) + n(k)}$$

ISI (FIR) Model

Sequence Estimation