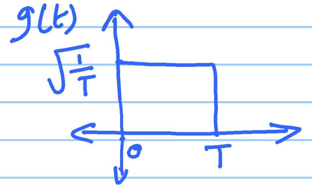
Lesson 1 -- Single Shot Communication, Intro to Matched Filter Rx, Basis Function, Signal Representation using Vectors, and Signal Constellation

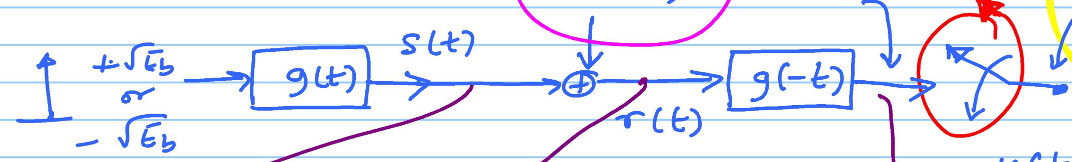
Note Title — 08/08/2017 — 17^{th} Aug '17



$\int_0^T g^2(t)\,dt = 1$;

"white" noise

$\sigma_n^2 \to \infty$



noise

$y(k) = \pm\sqrt{E_b} + V(k)$

signal

$r(t) = s(t) + n(t)$

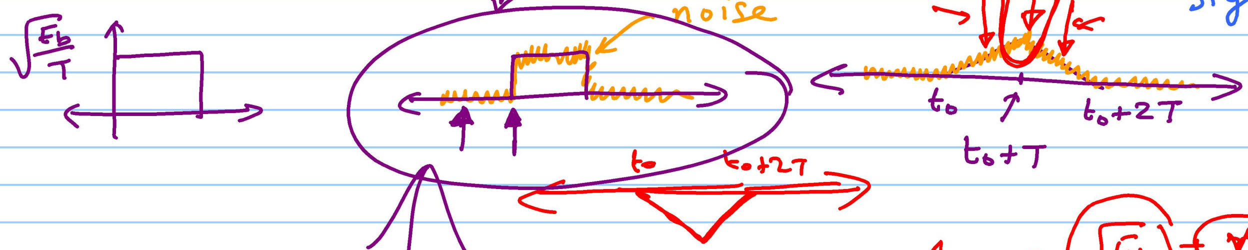
Rx Signal energy $= E_b$ Joules

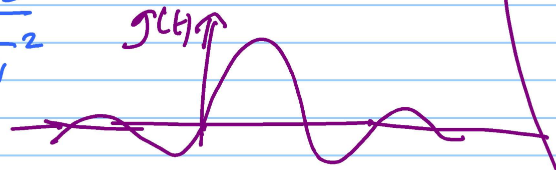
$V(k) \longrightarrow \sigma_v^2 \leftarrow$ variance

$E_b > \sigma_v^2$

Band Limited White Noise

$\frac{E_b}{\sigma_v^2}$





$\to 1 \longrightarrow \sqrt{E_b} + V_1$

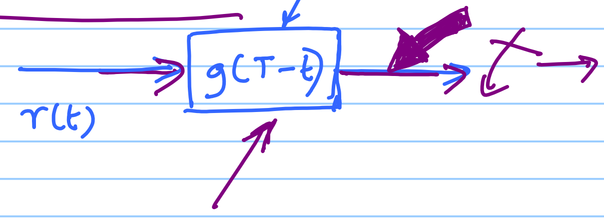
$\to 0 \longrightarrow \alpha\sqrt{E_b} + V_0$

$\to 2 \longrightarrow \beta\sqrt{E_b} + V_2$

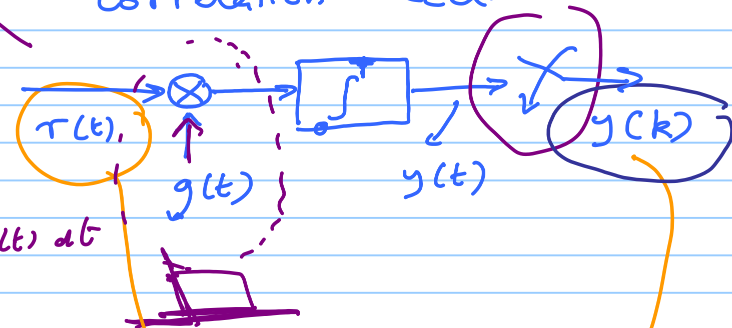
$(1+\alpha+\beta)^2 E_b \leftarrow (1+\alpha+\beta)\sqrt{E_b} +$

$3\sigma_v^2 \leftarrow V_1 + V_0 + V_2$

Matched filter Receiver



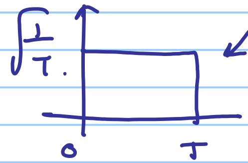
Correlation Receiver



$$\int_{t_0}^{t_0+T} r(t)\, g(t)\, dt$$

$s(t) + \cancel{n(t)}$

Basis function



$\phi(t)$ Basis function

$$\int_0^T \phi^2(t)\, dt = 1$$

$$y(k) = \pm\sqrt{E_b} + v(k)$$

$s(t)$ → $s_1(t)$: $+\sqrt{E_b}\ \phi(t)$ "1"

$s(t)$ → $s_2(t)$: $-\sqrt{E_b}\ \phi(t)$ "0"

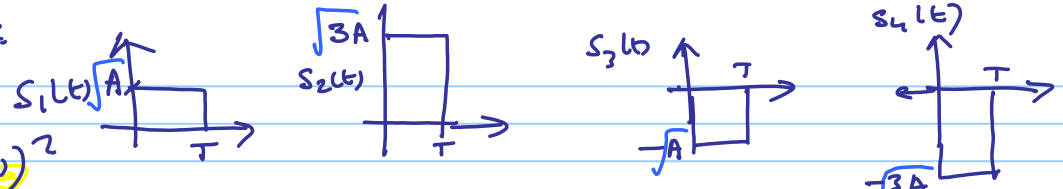
$s(t) \longleftrightarrow s_i$

waveform scalar

$s_1 = \sqrt{E_b}$

$s_2 = -\sqrt{E_b}$

Example 1:

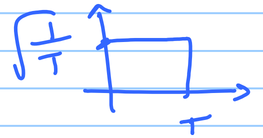


Question: Find the "minimal" basis set of functions $\{\phi_1(t), \phi_2(t) \dots\}$ whose linear combination should give $\{s_1(t), s_2(t), s_3(t), s_4(t)\}$

$$\int_0^T (s_1(t) - s_2(t))^2 \quad ✓$$

$$(v_1 - v_2)^T (v_1 - v_2) \quad ✓$$

$\phi_1(t) =$



$s_1(t) = \sqrt{A}\, \phi_1(t)$; $s_4(t) = -\sqrt{3A}\, \phi_1(t)$

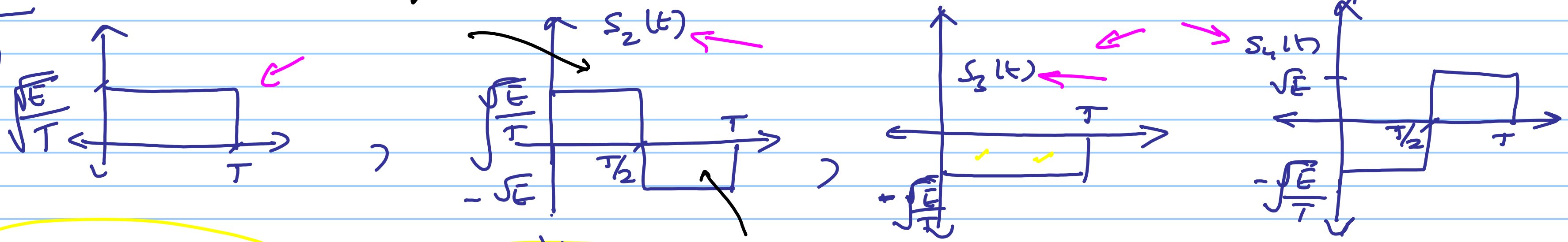
$s_1 = \sqrt{A}$; $s_4 = -\sqrt{3A}$

Example 2:

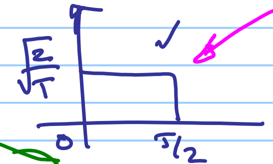
$$s_2(t) = \sqrt{\frac{E}{2}} \phi_1(t) - \sqrt{\frac{E}{2}} \phi_2(t)$$

$s_1(t)$

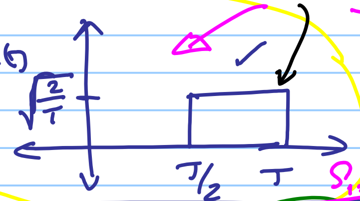
E Joules



$\phi_1(t)$ ✓ &



$\phi_2(t)$ ✓



$$\boxed{\int_0^T \phi_1(t)\, \phi_2(t)\, dt = 0}$$

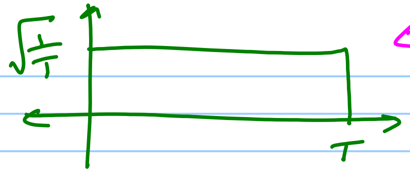
"orthogonality"

$$s_1(t) \rightarrow \left(\sqrt{\frac{E}{2}}, \sqrt{\frac{E}{2}}\right)$$

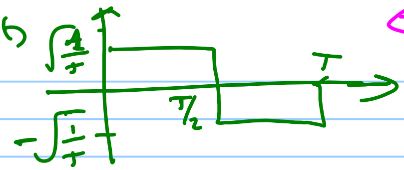
$$\boxed{s_1(t) = \underbrace{\sqrt{\frac{E}{2}}}_{s_{11}} \times \phi_1(t) + \underbrace{\sqrt{\frac{E}{2}}}_{s_{12}} \times \phi_2(t)}$$

$$\int_0^T s_1^2(t)\, dt = \frac{E}{2} + \frac{E}{2} = E \quad ✓$$

- Use instead $\phi_1'(t)$



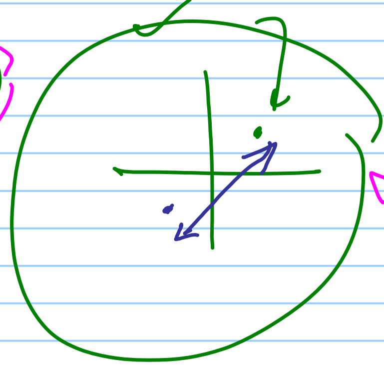
& $\phi_2'(t)$



$s_1(t) \rightarrow \begin{bmatrix} s_{11} \\ s_{12} \end{bmatrix}$ $\quad s_3(t) \rightarrow \begin{bmatrix} s_{31} \\ s_{32} \end{bmatrix}$

$s_1(t) \longrightarrow (s_{11}, s_{12}) \longrightarrow \begin{bmatrix} s_{11} \\ s_{12} \end{bmatrix} \longrightarrow (s_{11} + j\, s_{12})$

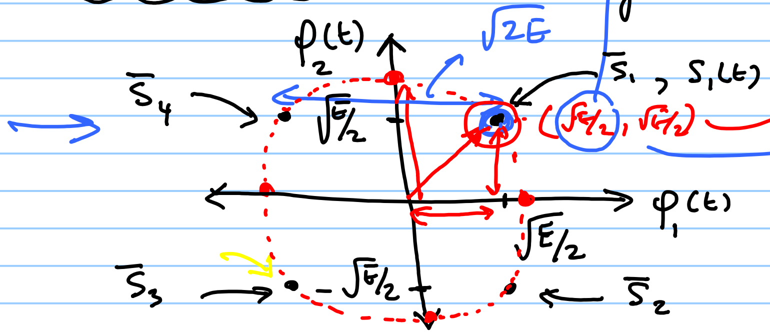
signal $\quad$ doublet $\quad$ 1x2 $\quad \bar{s}_1$ $\quad$ vector $\quad$ complex #



signal constellation

Lesson #1 contd.

Signal Constellation for $s(t)$ $\qquad$ 22 Aug, 2017



$\sqrt{\frac{E}{2}}$

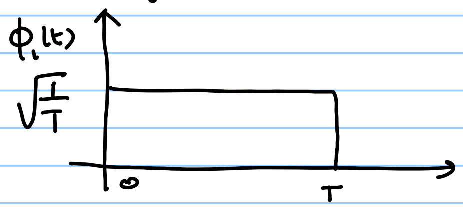
$\begin{bmatrix} \sqrt{E/2} \\ \sqrt{E/2} \end{bmatrix} \leftarrow \bar{s}_1$

$\longrightarrow \quad I$

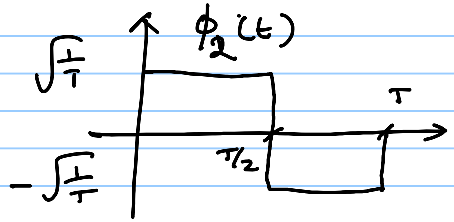
$(\sqrt{E/2} - 0)^2 + (\sqrt{E/2} - 0)^2 = E$

$\bar{s}_1 \longrightarrow \bar{s}_1^T \bar{s}_1 = E$

For the given signal set



&

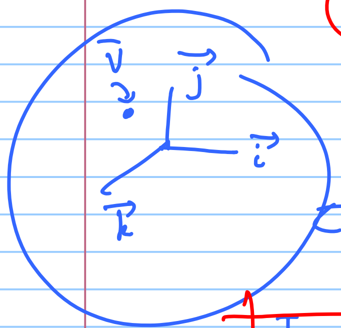


$$\int_0^T \phi_1(t)\,\phi_2(t)\,dt = 0$$

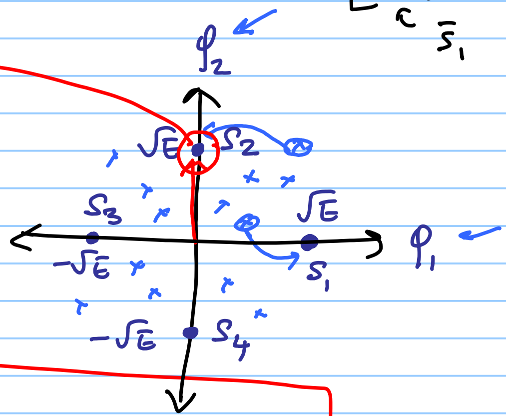
$$s_1(t) = \sqrt{E}\,\phi_1(t) + 0\,\phi_2(t) \longrightarrow \underbrace{\begin{bmatrix}\sqrt{E}\\ 0\end{bmatrix}}_{\bar{s}_1}$$

$s_1 = (\sqrt{E}, 0)$; $s_2(t) \to s_2 = (0, \sqrt{E})$;

$s_3 = (-\sqrt{E}, 0)$; $s_4 = (0, -\sqrt{E})$;



$(\sqrt{E})^2 = E$



Π ✓

compact

ortho-normal

basis

Energy $\int_0^T s_1^2(t)\,dt \longleftrightarrow \bar{s}_1^{\,T}\,\bar{s}_1$

homomorphism

$s(t) \longleftrightarrow \bar{s} \longleftrightarrow (s_a + j\,s_b)$

Cross correlation →

$$\boxed{\int_0^T s_1(t)\, s_2(t)\, dt \longleftrightarrow \bar{s}_1^{\,T} \bar{s}_2}$$

64

M ⟷ N 12

waveforms or signals in the signal set $s_i(t)$, $i = 1 \ldots M$

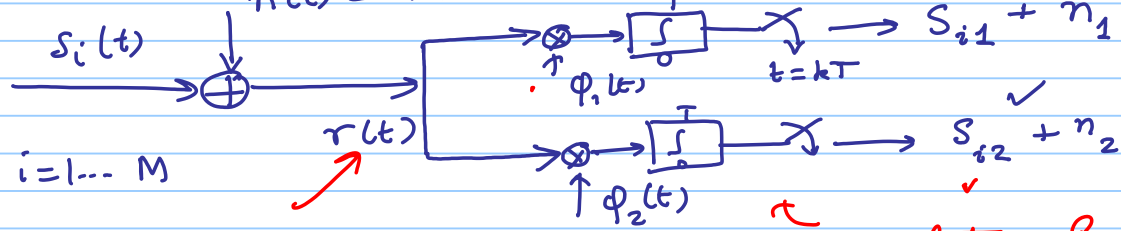
basis functions $\phi_j(t)$, $j = 1 \ldots N$

$N \ll M$

N = 2

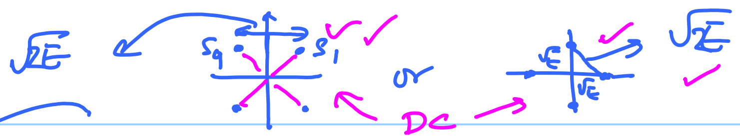
$s_i(t) \longrightarrow \bar{s}_i \leftarrow (2 \times 1)$

$n(t) \leftarrow$ AWNoise (Gaussian)

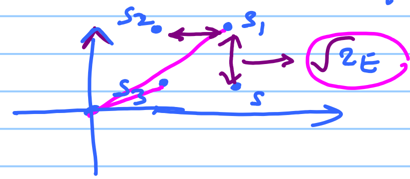


$i = 1 \ldots M$

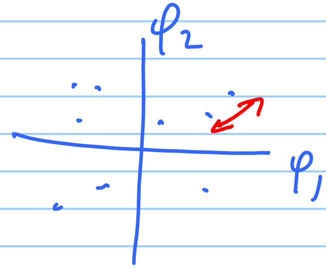
correlation R_x



Resources ⟶ Time / Frequency / Power



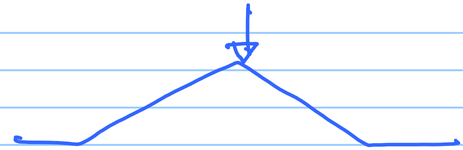
2^L

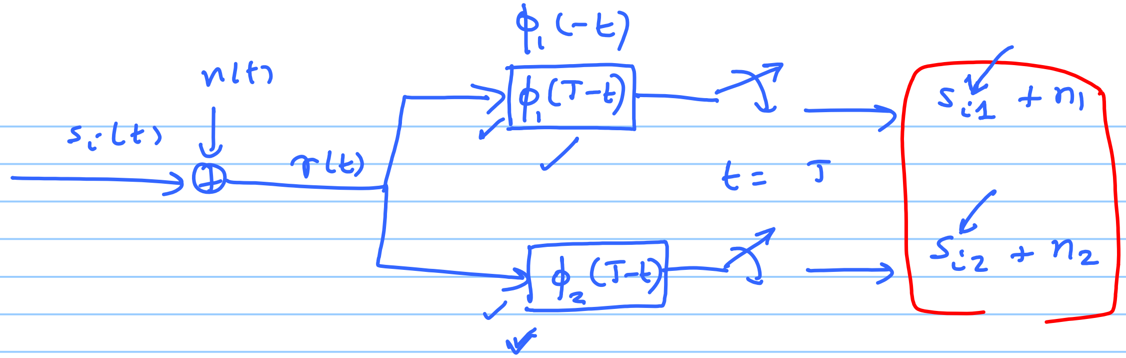


FSK

orthogonal signals

$$\boxed{s_i(t) = s_{i2}\, \varphi_1(t) + s_{i2}\, \varphi_2(t) \quad i = 1 \ldots M}$$





Matched filter Rx

← vector repn. (Discrete-time)