Overview on OFDM Design

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November 9, 2001

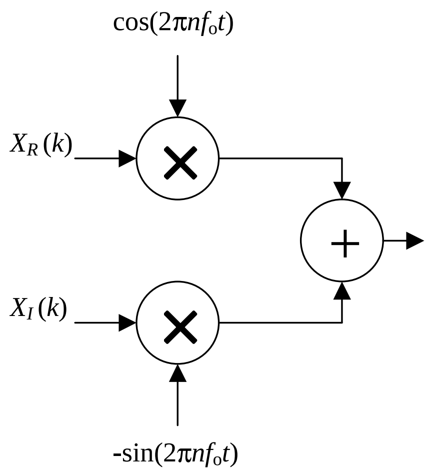
Topics

- OFDM Transmitter Design Considerations
- Design Considerations in Transmission through Multipath Channel
- Additional OFDM Receiver Design Considerations

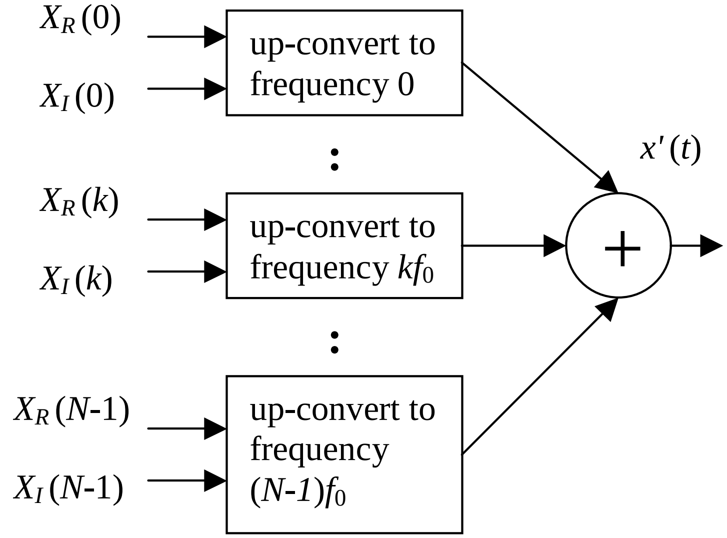
OFDM Transmitter Design Considerations

- OFDM modulation in IF vs OFDM modulation using IFFT
- OFDM symbol windowing
- OFDM spectrum

OFDM modulation using IF circuitry



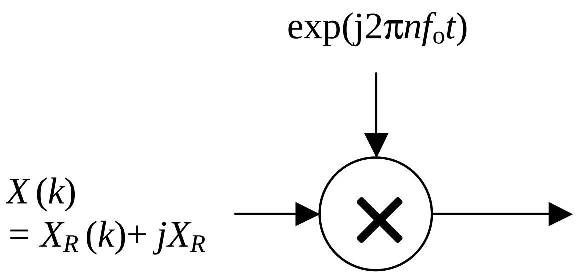
(a) One sub-carrier at IF.



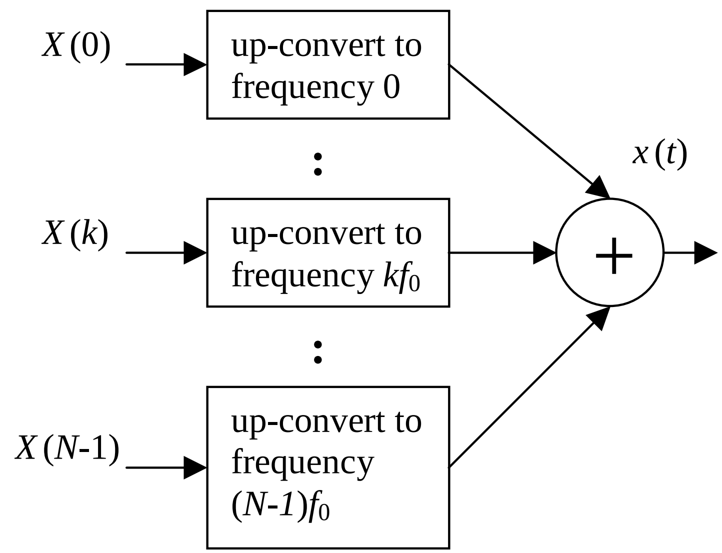
(b) OFDM modulation implemented at IF.

$$x'(t) = \sum_{k=0}^{N-1} x_R(k)\cos(2\pi k f_o t) - x_I(t)\sin(2\pi k f_o t) \quad (1)$$

OFDM modulation at baseband



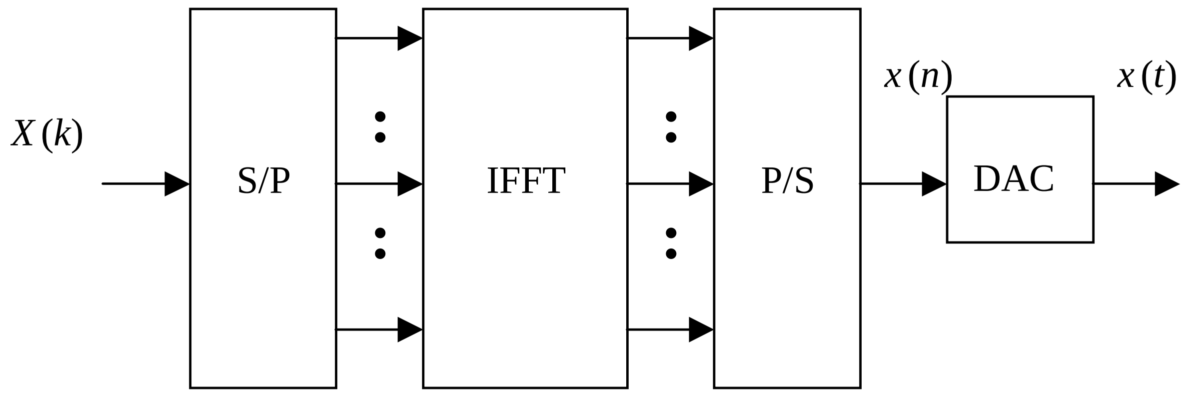
(a) One sub-carrier at baseband.



(b) OFDM modulation implemented at baseband.

$$x(t) = \sum_{k=0}^{N-1} X(k) \exp(j2\pi k f_o t) \tag{2}$$

OFDM modulation at baseband using IFFT

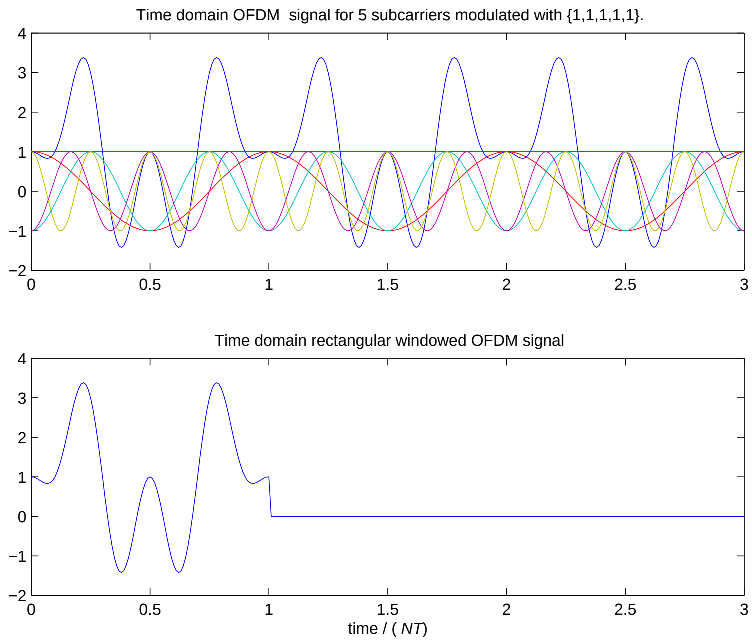


- obtain discrete values at $t = nT$, select $f_o = \frac{1}{NT}$

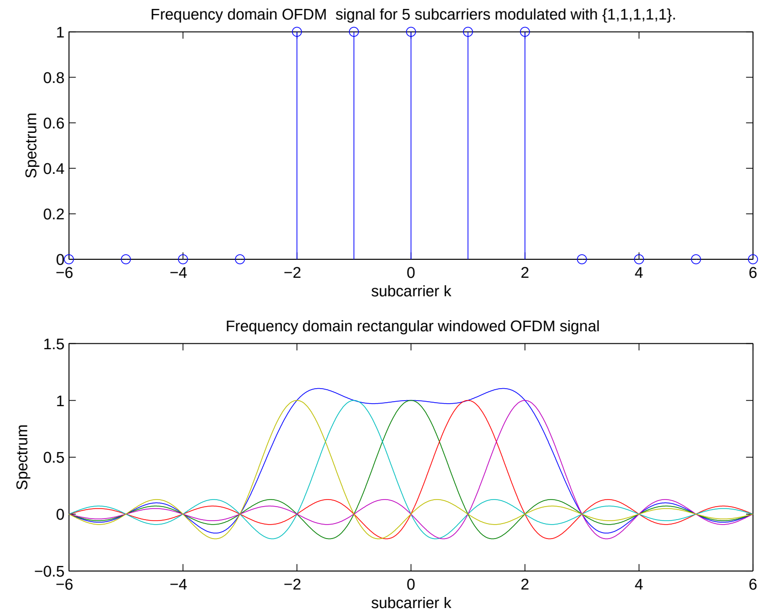
$$x(n) = x(t = nT) = \sum_{k=0}^{N-1} X(k) \exp(j2\pi nk/N) \tag{3}$$

- becomes IDFT of $\{X(k)\}_{k \text{ mod } N}$
- implemented using IFFT, then use DAC

OFDM symbol windowing - allows other OFDM symbols transmission



(a) Time domain before/after windowing.



(b) Freq. domain before/after windowing.

- before windowing: $X(f) = X(k)\delta(f - kf_o)$
- after windowing:

$$X(f)W(f) = X(k)\delta(f - kf_o) \otimes \text{sinc}(f/f_o) = X(k)\text{sinc}(f/f_o - k)$$

Note: X(f) from here on will be assumed to be windowed with rectangular window.

Spectrum efficiency η for single carrier vs OFDM

assume R is data rate of each (sub)carrier, bandwidth defined from null-to-null, N subcarriers

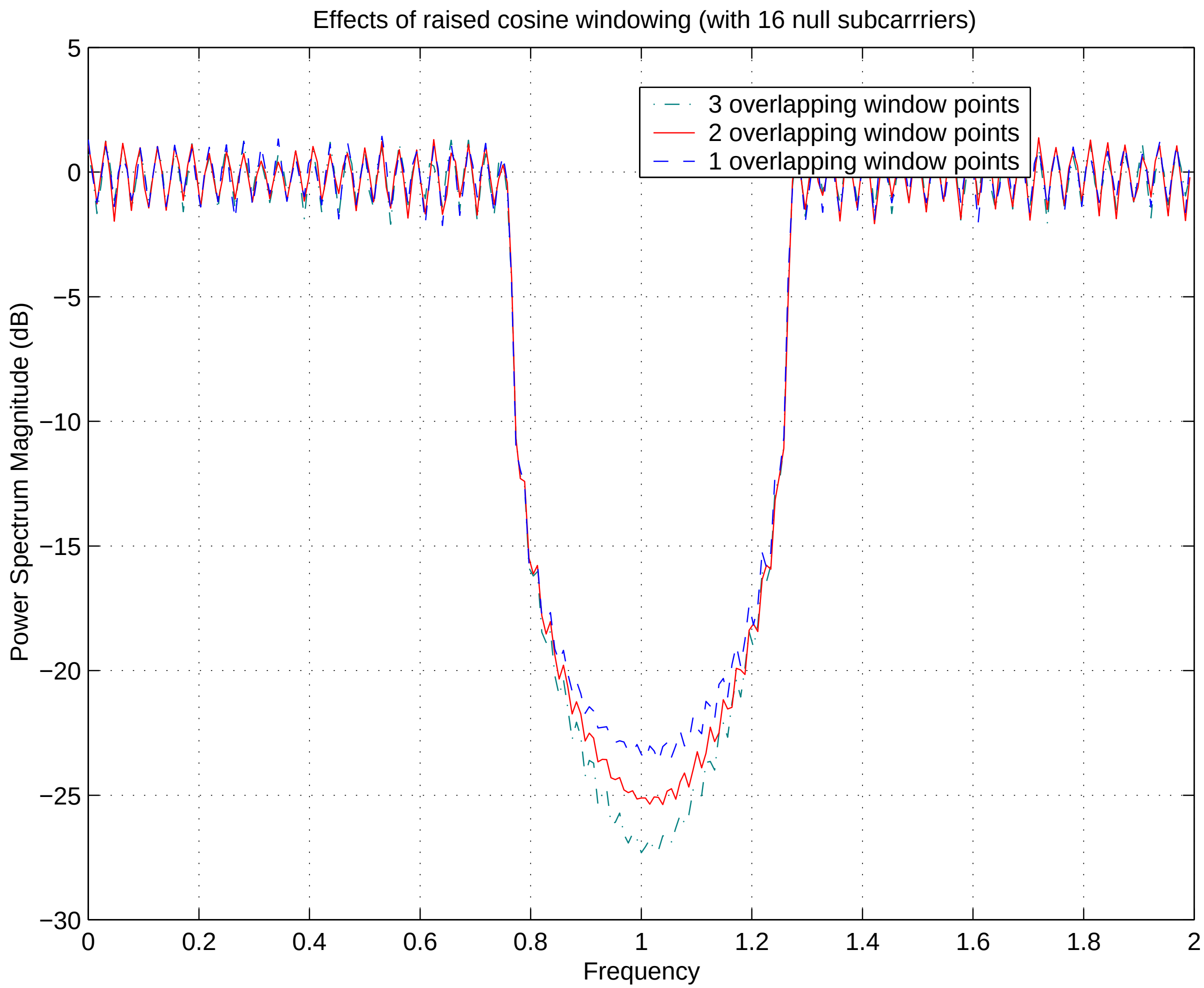
- for single carrier : $\eta = \frac{R}{2f_o}$
- for OFDM : $\eta = \frac{NR}{(N+1)f_o}$
- for OFDM : as $N \to \infty, \eta \to \frac{R}{f_o}$
- but - windowing possible for single carrier to improve η
- windowing not possible on a subcarrier to exploit implementation simplicity of IFFT

Digital to Analogue Converter (DAC)

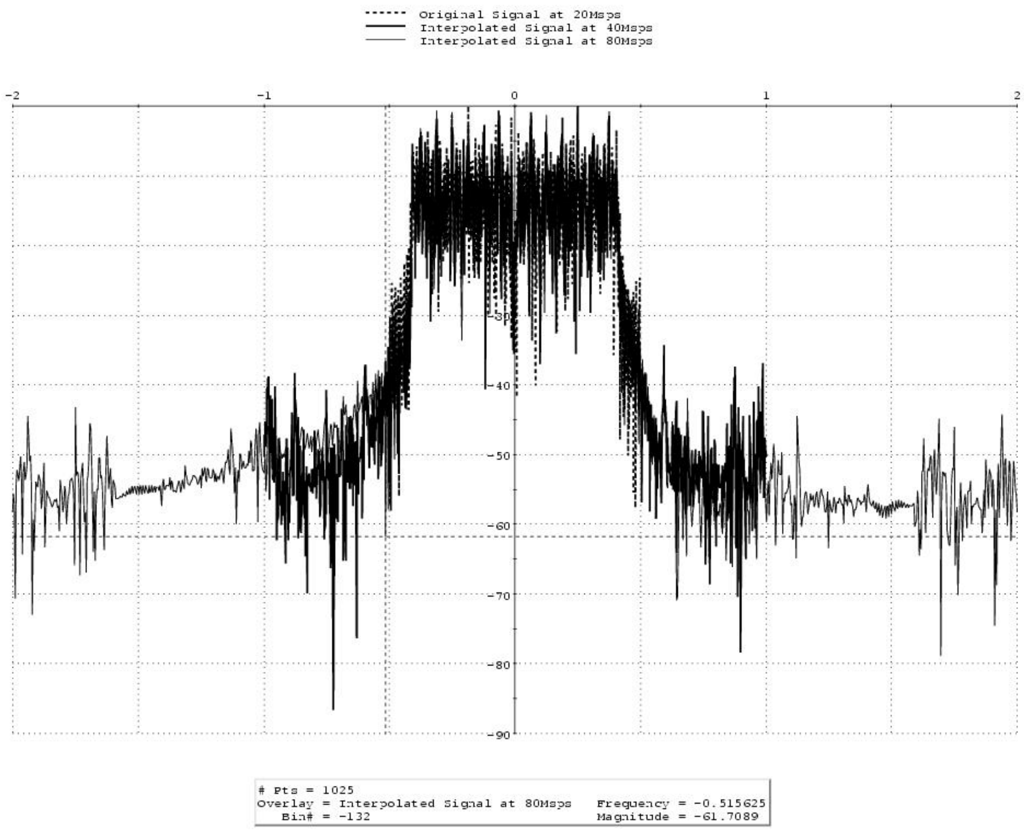
Need to perform upsampling for interpolation.

Solutions:

- null subcarriers to ease filter design
- windowing - spectrum decays at $1/|f|$ for rectangular window
- digital filtering - usually FIR



Effects of windowing and null subcarriers.



Spectrum of OFDM symbol before DAC in KRONDOR.

High peak-to-average power ratio (PAPR)

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- $x(n)$ is Gaussian distributed by law of large number - leads to high PAPR
- effect is more prominent as N increases
- requires expensive and high power amplifier with high dynamic range, or else
  - distorts time domain signal
  - non-linearity or clipping introduces out-of-band spectral regrowth
- solutions

Design Considerations in Transmission through Multipath Channel

- multi-dimension interference (MDI)
  - inter-[OFDM] symbol interference (ISI)
  - inter-[sub]carrier interference (ICI)
- use of guard interval to remove ISI
- use of cyclic prefix to remove ICI

Time selective channel

- usually valid to treat channel as time invariant over each OFDM symbol
- channel changes after every OFDM symbol

Frequency selective channel

- use discrete time baseband representation
- $y(n) = x(n) \otimes h(n) + v(n)$ where $v(n)$ is AWGN, $\otimes$ is linear convolution
- ISI occurs which spills to other OFDM symbols

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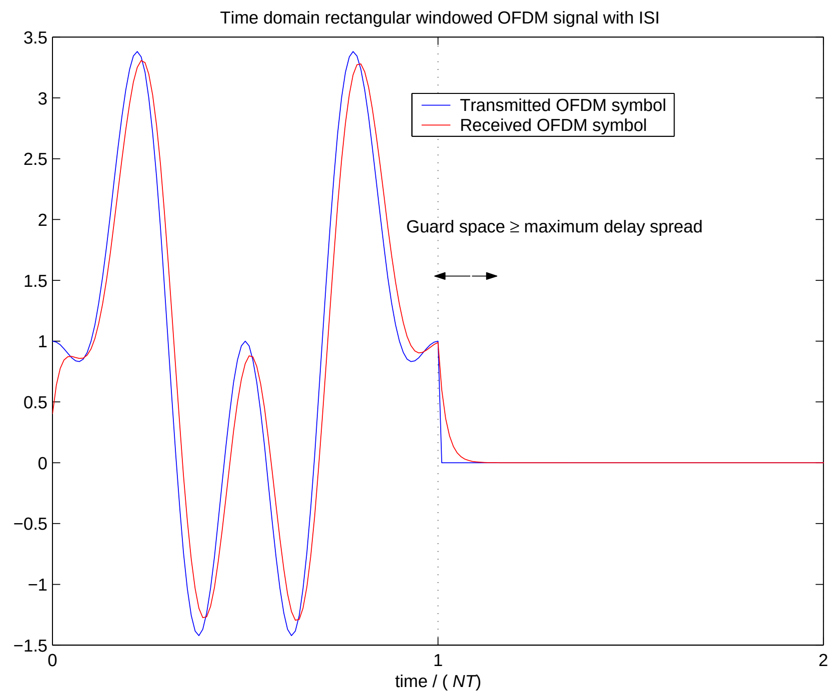
Mitigation of ISI



- append guard interval (GI) at transmitter
- GI longer than maximum delay spread
- remove GI at receiver

Introduce cyclic prefix (CP)

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- how to remove ISI within OFDM symbol? we don't
- fill appended GI with cyclic data to form CP
- define ⓒ to be circular convolution modulo N,

$$\begin{aligned} y(n) &= x(n) \otimes h(n) \\ \rightarrow \quad y_{cp}(n) &= x_{cp}(n) \otimes h(n) \\ \rightarrow \quad y(n) &= x(n) ⓒ h(n) \end{aligned} \tag{4}$$

Note: All discrete index from now on will be assumed to be modulo N.

ICI removed through cyclic prefix (CP)

- removing cyclic prefix, received time domain signal is

$$y(n) = \sum_{k=0}^{N-1} H(k)X(k) \exp(j2\pi nk/N), \tag{5}$$

where $H(k) = \sum_{n=0}^{N-1} h(n) \exp(-j2\pi nk/N)$.

- demodulation using FFT,

$$\begin{aligned} Y(k) &= \sum_{n=0}^{N-1} y(n) \exp(-j2\pi nk/N) \\ &= X(k)H(k) + V(k) \end{aligned} \tag{6}$$

when orthogonality is satisfied, i.e.
$\sum_{n=0}^{N-1} \exp(-j2\pi nk/N) \exp(j2\pi nk'/N) = \delta(k-k')$

Additional OFDM Receiver Design Considerations

- Time/frequency duality
- Timing Synchronization
- Frequency Synchronization
- Channel Estimation

Time/frequency duality

- single carrier in time selective channel vs OFDM in frequency selective channel

$$y(n) = x(n)h(n) + v(n) \rightsquigarrow Y(k) = X(k)H(k) + V(k)$$

- single carrier in frequency selective channel vs OFDM with time selective channel

$$y(n) = Ax(n)h(n) + \mathsf{ISI} + v(n) \rightsquigarrow Y(k) = BX(k)H(k) + \mathsf{ICI} + V(k)$$

  where A, B are complex attenuation factors

- ISI occurs for single carrier satisfying Nyquist pulse-shaping criterion when sampling offset occurs

- ICI occurs for OFDM when frequency offset occurs

Synchronization in general

- timing synchronization errors- correctable in a pilot based system

$$y(n + \Delta_t) \leftrightharpoons Y(k)\exp(j2\pi k\Delta_t/N) \tag{7}$$

- frequency synchronization errors when
  - carrier frequency offset
  - phase jitter
  - Doppler spread
- frequency synchronization errors- introduces ICI

$$y(n)\exp(j2\pi n\Delta/N) \leftrightharpoons Y(k + \Delta) \tag{8}$$

Windowing at receiver

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- consider windowing after removing cyclic prefix, before FFT

$$\begin{aligned} y_w(n) &= y(n)w(n) \\ &= [x(n) \circledcirc h(n)]w(n) \end{aligned} \tag{9}$$

- the DFT is

$$\begin{aligned} Y_w(k) &= [X(k)H(k)] \circledcirc W(k) \\ &= \sum_{l=0}^{N-1} X(l)H(l)W(k-l) \end{aligned} \tag{10}$$

Effects of Frequency Offset

- presence of normalized frequency offset Δ between tx and rx, (5) becomes

$$y(n) = \sum_{k=0}^{N-1} H(k)X(k)\exp(j2\pi nk/N)\exp(j2\pi n\Delta/N), \qquad (11)$$

- define $w(n;\Delta) = \exp(j2\pi n\Delta/N)$ and thus $W(k;\Delta)$ is the dirichlet kernel

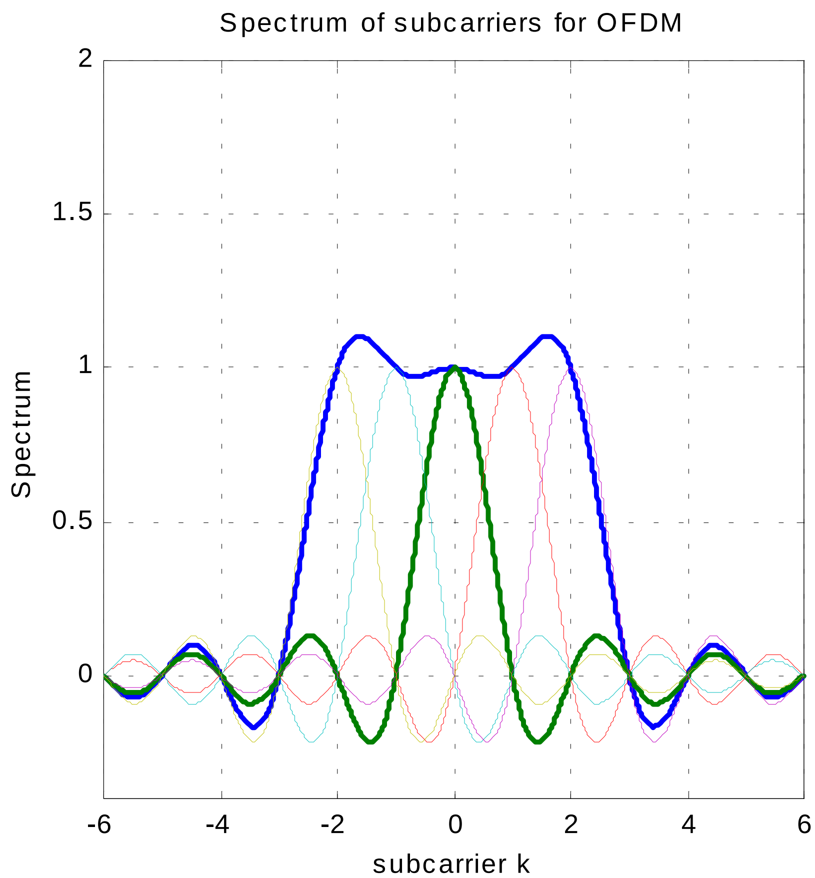
- using (9) and (10), we can re-write the DFT of (11) as

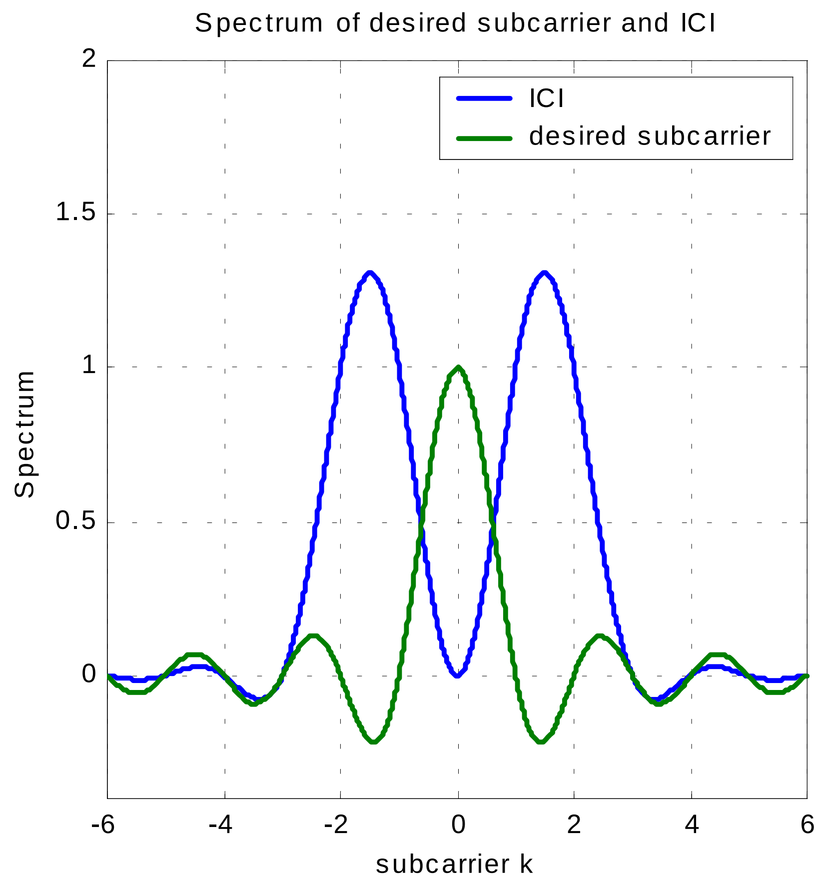
$$Y(k) = H(k)X(k)W(0;\Delta) + I(k) \qquad (12)$$

where the ICI is

$$I(k) = \sum_{l=0,l\neq k}^{N-1} H(l)X(l)W(k-l;\Delta) \qquad (13)$$

ICI due to frequency offset illustrated





ICI occurs when sampling at frequency $k + \Delta$ *instead of* k .

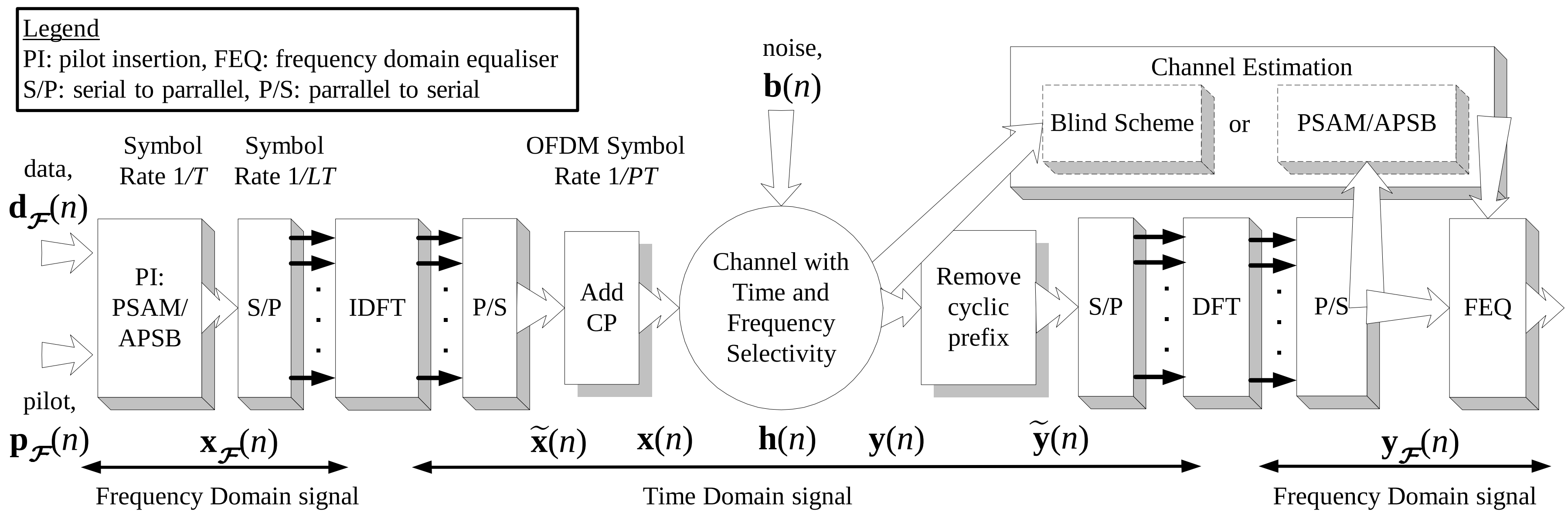
Remove ICI caused by frequency offset

- time domain methods:
  - estimate Δ using averaging of phase difference of repeated samples, multiply complex term to de-rotate signal with increasing phase
  - use windowing to reduce ICI
- frequency domain method:
  - estimate Δ, then
  - express as matrix form

$$
\begin{aligned}
\mathbf{y} &= \mathbf{W}(\Delta)\text{diag}\{\mathbf{x}\}\mathbf{h} + \mathbf{v} \qquad (14) \\
&= \mathbf{W}(\Delta)\text{diag}\{\mathbf{h}\}\mathbf{x} + \mathbf{v} \qquad (15)
\end{aligned}
$$

  - solve for $\mathbf{h}$ or $\mathbf{x}$ in the frequency domain using least squares by minimizing 2-norm of $\mathbf{v}$

OFDM model with respect channel estimation methodology



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Note: Some notations here are not inconsistent with this presentation.

Channel estimation in OFDM systems

- (semi-)blind methods based on cyclic prefix or cyclostationarity properties
- decision feedback - not popular, probably due to large processing delay in FFT
- training - substantial initialization, usually for fixed line, not for packet based system
- pilots - most common method for wireless applications

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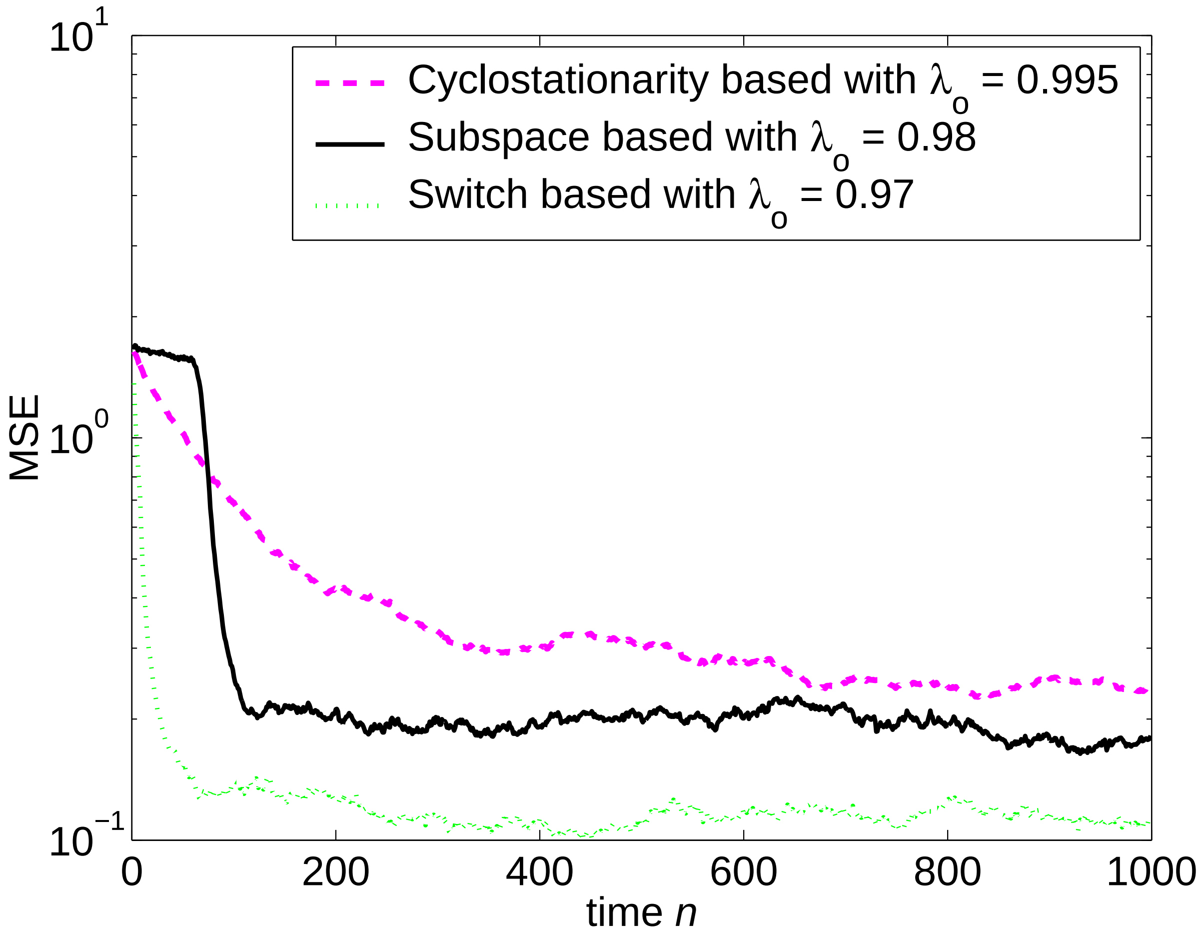

Slow convergence found in some blind schemes.

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Pilot based channel estimation

- useful to think of OFDM symbols in a 2-dimensional (2D) time/frequency grid

- usually time correlation is higher than frequency correlation

- pilots used to sample 2D channel - need to satisfy sampling theorem (2 times of coherence bandwidth/time)

- 2D filters used for interpolation