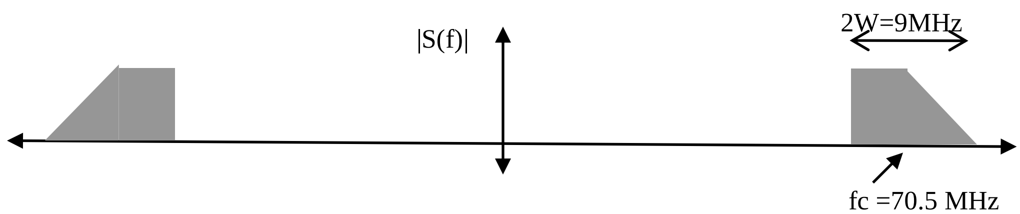
EE 5151: Communication Techniques

Sept. 2017 **Tutorial #1** KG / IITM

1. A low-pass signal of one-sided bandwidth of W=1.25MHz is sent as a DSB-SC signal. If the receiver uses an IF sampling scheme, with center frequency f_{IF} = 71MHz, determine the <u>least</u> sampling rate required.

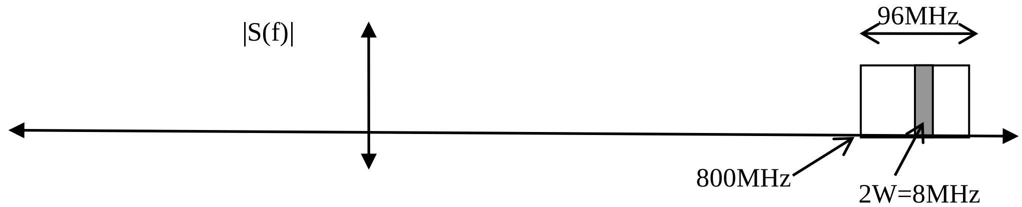
2. For the QCM signal with magnitude response as below, find the least possible band-pass sampling rate. Make a rough plot of the frequency response of the sampled sequence around 0Hz.



3. In the above problem, assume that the received signal has a phase offset of θ radians; in other words, $s(t) = m_1(t)Cos(2\pi f_c t + \theta) + m_2(t)Sin(2\pi f_c t + \theta)$. Now, what will be the time-domain representation of the sampled sequence? For the special case when $\theta = \pi/2$,what will be the samples of the received signal?

4. A QCM signal $s(t) = m_1(t)Cos(2\pi f_c t) + m_2(t)Sin(2\pi f_c t)$ has the two message signals $m_1(t)$ and $m_2(t)$ of <u>one-sided</u> bandwidth of W_1=3KHz and W_2=4KHz, respectively, and f_c=30KHz.
(a) Find the minimum band-pass sampling rate $f_s=1/T_s$ that will give un-aliased samples of the two signals.
(b) Assuming that the spectrum of $m_1(t)$ has a "triangular" shape between -3KHz to +3KHz, make a labeled, rough sketch of the spectrum of the samples $m_1(kT_s)$ between -40KHz and +40KHz.

5. A dozen DSB-SC signals of one-sided (low-pass) bandwidth W = 4MHz are present between 800MHz and 896MHz, as shown below. Describe the operations (sampling, rate-conversion, filtering) that you need to do to recover Nyquist rate samples of the 7th DSB-SC signal (i.e., the signal present between 848Mz and 856MHz).



6. A WSS random process has an auto-correlation function given by $R_X(\tau) = \frac{A^2}{2} e^{-|\tau|} Cos(2\pi f_0 \tau)$.
Assume that the random process never exceeds 6 in magnitude, and that A=6.
(a) How many uniform quantisation levels are required to provide an SQNR of at least 40dB ?
(b) If we want to increase the minimum SQNR to 60dB, how should the required number of quantisation levels change?
(c) If f_0=1MHz, what is the bit rate you will require to send the quantised samples in both of the above cases?

7. The psd of a WSS process X(t) is given by

$$S_X(f) = \begin{cases} \frac{f+5000}{5000}, -5000 \le f \le 0 \\ \frac{-f+500}{5000}, 0 < f \le 5000 \\ 0, otherwise \end{cases}$$

and the maximum amplitude of this process is 6.
(a) What is the power content of this process?
(b) If this process is sampled at fs to guarantee a guard band of 2000Hz, then what is fs?
(c) At this sampling rate, if we use a linear PCM system with 256 levels, what is the resulting SQNR in dB?
(d) What is the resulting bit rate?
(e) If we need to increase the SQNR by atleast 25dB, how many levels are required? What is the new bit rate?

8. Llyod-Max Non-uniform Quantization: Assume that a random signal x(t) with x=x(t=t_0), follows a probability density function $f_X(x)$, and the objective is to find the N-1 signal levels $\{a_1, a_2, \cdots, a_{N-1}\}$ and the corresponding N quantized values $\{\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_N\}$ such that the quantization noise variance given by

$$E_q = E[(x-x_q)^2] = \int_{-\infty}^{a_1} (x-\hat{x}_1)^2 f_X(x)\,dx + \sum_{i=2}^{N-1} \int_{a_{i-1}}^{a_i} (x-\hat{x}_i)^2 f_X(x)\,dx + \int_{a_{N-1}}^{\infty} (x-\hat{x}_N)^2 f_X(x)\,dx$$

is minimized. In other words, differentiate E_q w.r.t $\{a_1, \cdots, a_{N-1}\}$ and $\{\hat{x}_1, \cdots, \hat{x}_N\}$, equate them to zero, and show the following:

$$(i)\, a_i = \frac{\hat{x}_i + \hat{x}_{i+1}}{2} \text{ and } (ii)\, \hat{x}_i = \frac{\int_{a_{i-1}}^{a_i} x\, f_X(x)\,dx}{\int_{a_{i-1}}^{a_i} f_X(x)\,dx}$$

9. The Llyod-Max quantizer indicates that the signal level should be the mid-point of the quantization interval (see (*i*)), and that the quantized value is the "centroid" of the corresponding interval (see (*ii*)). However, a practical method to construct this non-uniform quantizer is given by the iterative method below for a *k*-bit quantizer where $N=2^k$:

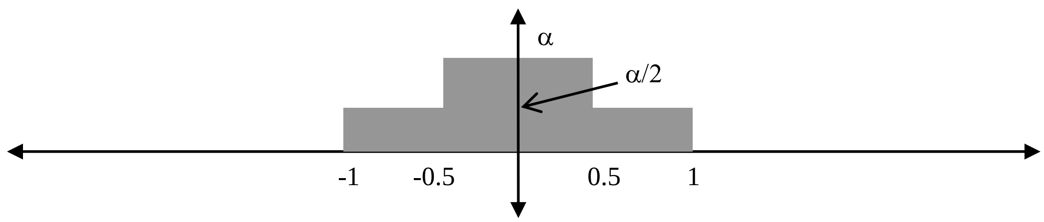
Step 1: choose *N*-1 uniform intervals $\{a_1, a_2, \cdots, a_{N-1}\}$

Step 2: find the corresponding centroids $\{\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_N\}$using (*ii*)

Step 3: re-compute $\{a_i\}$ using (*ii*) in (*i*)

Iterate between steps 2 and 3 until "convergence"

Use the above procedure for 3 iterations to define the 2-bit LM quantizer for the below source pdf. Compare the E_q obtained between the uniform quantizer (mid-tread) and the LM quantizer.



10. From "Digital Telephony" J.C.Bellamy, 3rd Ed., pp. 158-160: Problems **3.1, 3.2, 3.3, 3.4, 3.5, 3.8*, 3.14*, 3.16*, 3.17*, 3.18,** and **3.19.** The ones marked "*" are tougher ones, presumably**.**

11. In a certain digital multiplexer, 6 input streams arrive at 20MBps rate, and another 6 input streams arrive with 40MBps rate. The clock ppm in all the streams is ±1 ppm. If a 16-bit frame header and a 16-bit CRC are added to every *L*sec frame assembled by this multiplexer along with appropriate stuff-bits (and indicators), answer the following:

(a) If *L*=3secs, make a rough sketch of the assembled frame, indicating the various important fields. What is the output bit-rate in MBps?

(b) In the interest of error-detection, it is advisable to reduce the frame length (duration). For what least choice of *L* (you can express in secs or msecs) will the output bit-rate not exceed 360MBps?

12. Given a population of *N*=20,000 users, each offering E_u =0.04 Erlangs of traffic, define a 3-stage blocking switch with *k* sub-arrays in the middle-stage, each containing 250x250 cross-points such that the blocking probability $P_b = 10^{-3}$ or less. Use Lee graph approach to find this least value of *k*.

(a) Determine the number of cross-points for the above switch.

(b) For the same size of the middle-stage sub-arrays (i.e., same size of *n*) as in (a), define a non-blocking switch. How does the complexity of this switch compare to (a)?

(c) Rework value of *k* and part (a) if we require $P_b \leq 10^{-5}$.

13. A total of *N*=4096 lines have to be switched, where each line offers Eu =0.05 Erlangs of traffic. All the 3 stages of the switch are to be built using sub-arrays of size 64x64 (where in the input and output stages, not all lines need be utilized if *k* < 64).

(a) Define a blocking switch such that blocking probability $P_b = 10^{-3}$ or less. What is the complexity (including any un-utilized cross-points) ?

(b) Is it possible to build a non-blocking 3-stage switch in this case? Specify.

14. The first 400 inlets carry users with Eu=0.05 Erlangs while the next 600 inlets carry users with Eu =0.01 Erlangs. Given that the users are grouped into blocks of n=50 each, define a 3-stage block switch with overall $P_b = 10^{-2}$ or less. What is the total number of cross-points in this switch? *Hint*: The overall blocking probability is computed by considering the 4 cases, namely user from set1 calls another user in set1, or user from set1 calls user from set2, etc.

15. For the switch considered in Problem 13 (a) part, use the blocking probability expression following the work of Jacobaeus (which does **not** assume that the paths from input-to-middle stage and paths from output-to-middle stage are independent) given in eqn. (5.10) in page 239 of the book. What will be the new value of k for this case? How does this compare with your answer in 13(a)? Comment.

16. Consider a population of N=4000 users, each of E_u=0.01 Erlangs. Design a 3-stage blocking switch of least complexity such that the blocking probability $P_b=10^{-4}$ or less. What is k, and the total number of cross-points for this switch? *Hint:* To minimize the total number of cross-points, choose the input sub-array dimension n "appropriately" where $N/m=n$.

17. Consider the 5-stage switch in the book, first described in page 237, Fig. 5.9. Here, blocking is introduced also in the middle stage(s). The input has N/n_1 sub-arrays, each of dimension n_1x k_1, where N is the total population to be served by this switch. The middle-stage (which is actually a blocking switch with 3-stages) has k_1 sub-arrays, each of size N/n_1x N/n_1. Each of these sub-arrays has $N/(n_1$x $n_2)$ sub-arrays, of dimension n_2xk_2 where k_2 is the number of middle stage sub-arrays (each of dimension $N/(n_1$x $n_2)$ x $N/(n_1$x $n_2)$). Assume each user offers E_u Erlangs of traffic.

(a) Prove using the Lee-Graph approach that blocking probability of the 5-stage switch is given by

$P_b = \left\{1 - q_1^2\left[1-(1-q_2^2)^{k_2}\right]\right\}^{k_1}$ where $q_1 = (1-p_1)$ with $p_1 = \dfrac{n_1 E_u}{k_1}$ and $q_2 = (1-p_2)$ with $p_2 = \dfrac{n_2 p_1}{k_2}$.

(b) For N=50,000, and n_1=50 and n_2=50, find the 5-stage switch with minimum number of cross-points so that $P_b=10^{-8}$ or less. Assume E_u = 0.01 Erlangs each.
(c) Can you find a better choice of n_1 and n_2 for this case? (i.e., a choice that will minimize the number of cross-points further?)