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# Wireless Channel Modeling

Clarke's, Jakes' and modified Jakes' models

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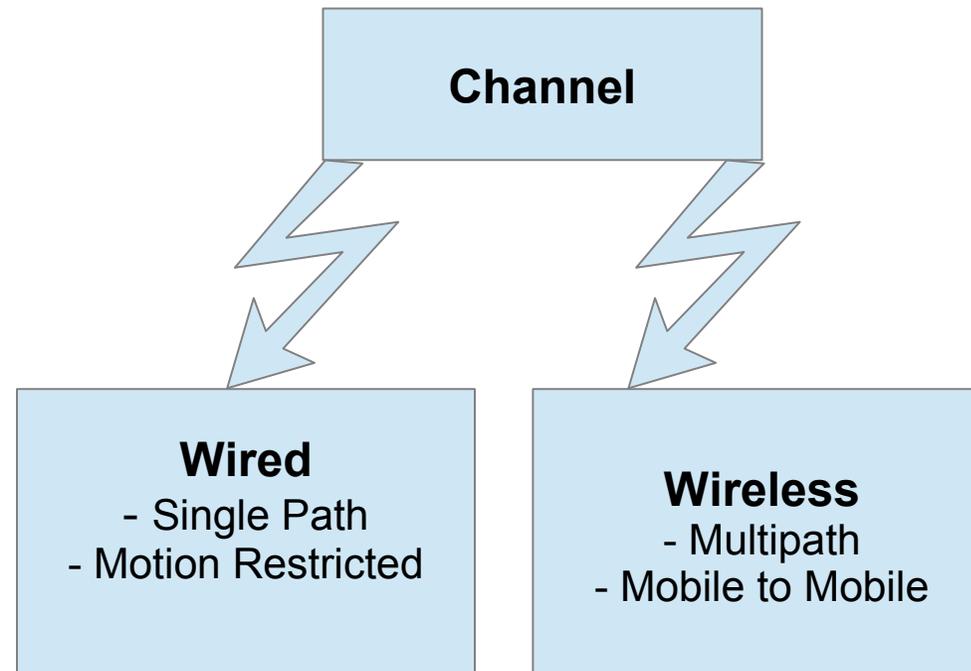
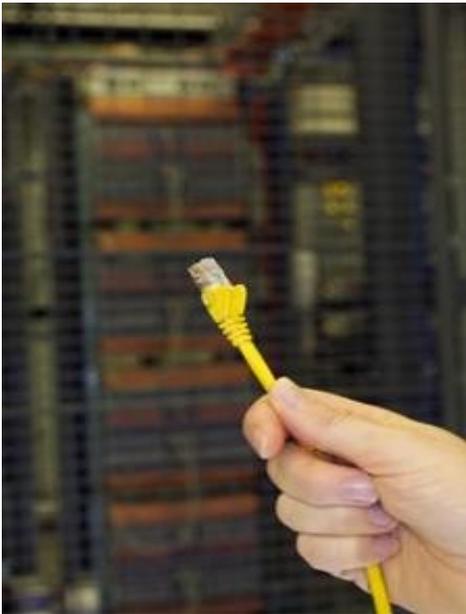
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# Outline

- What is a Channel?
- What is channel modeling?
- Wireless channel modeling
- Modeling Fading – Rayleigh
- Clarke's, Jakes' and modified Jakes' models

# What is a Channel?

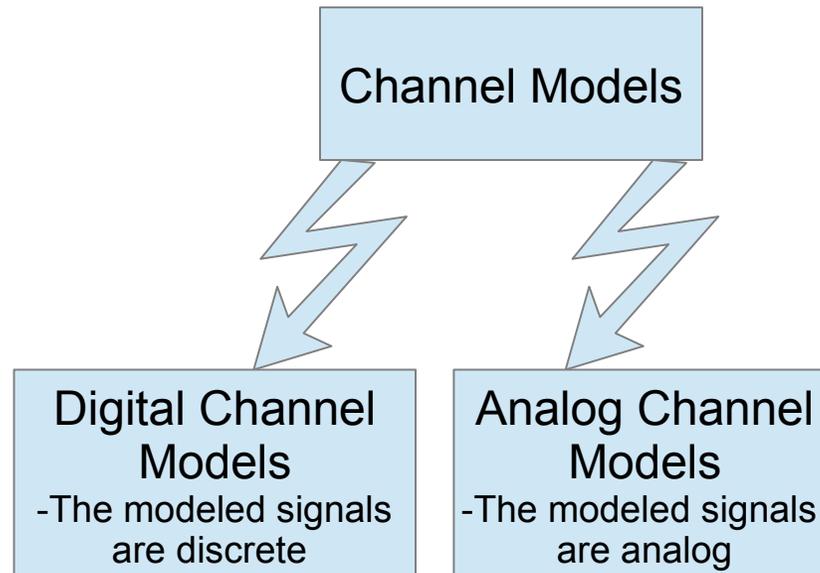
- A Communication Channel is the medium used to transmit information from one point to the other.



- Though wired channels are fast, cost-effective and secure, they are short ranged, motion restricted.

# Channel Modeling

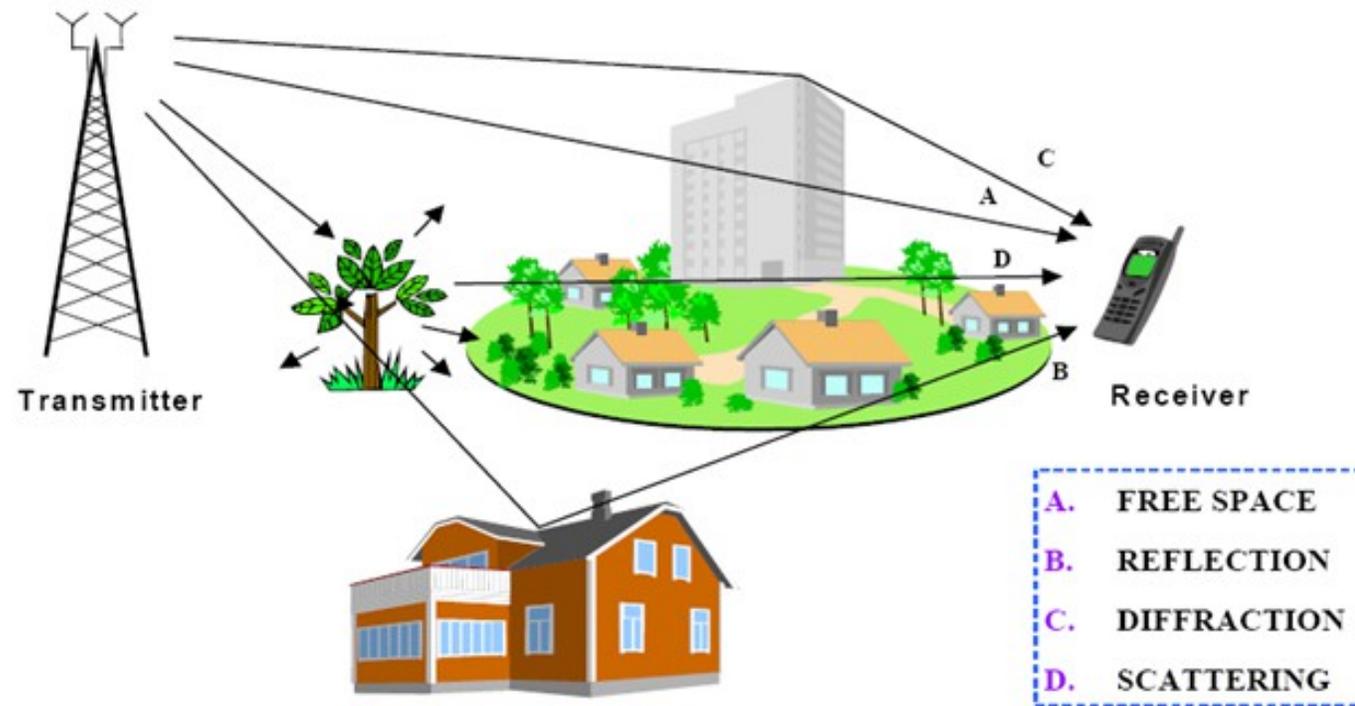
- Ideally, modeling a channel is calculating all the physical processing effecting a signal from the transmitter to the receiver.
- Why do we need to model?



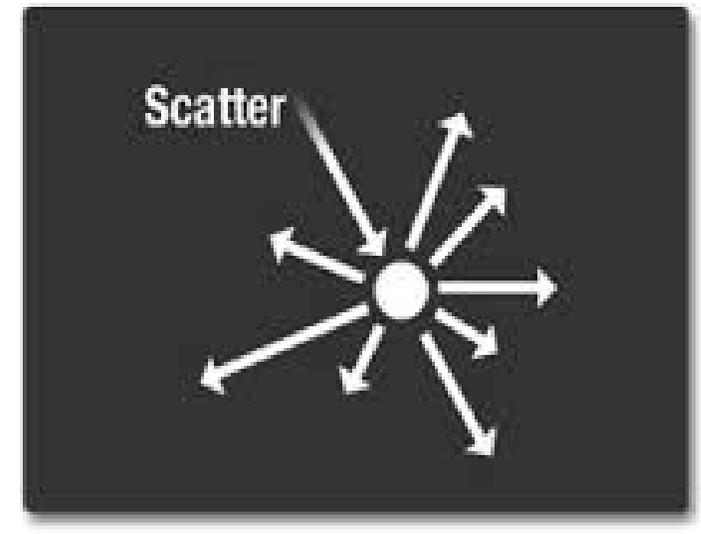
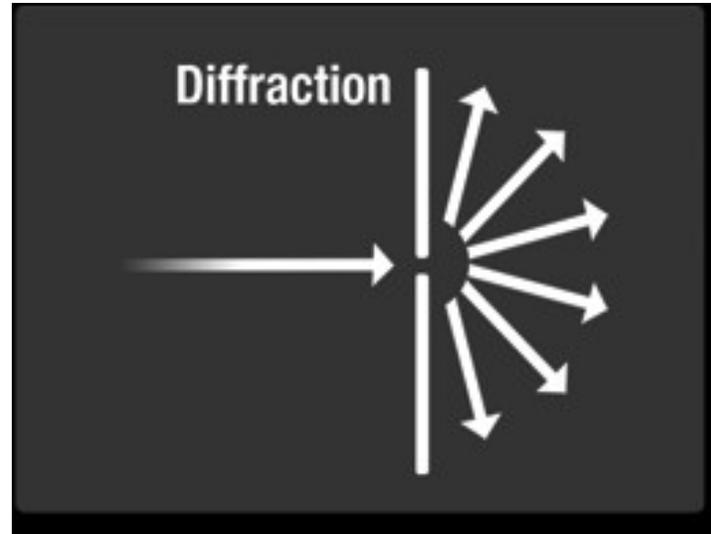
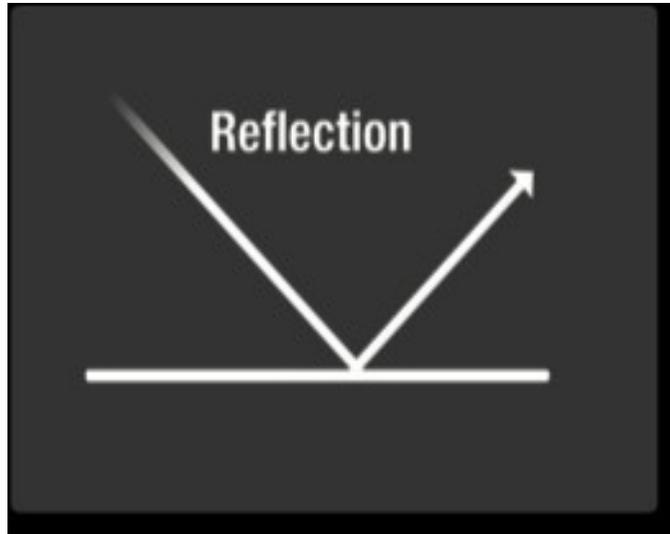
# Wireless Channel Modeling

- Three main mechanisms of electromagnetic wave propagation are:

- 1) Reflection
- 2) Diffraction
- 3) Scattering

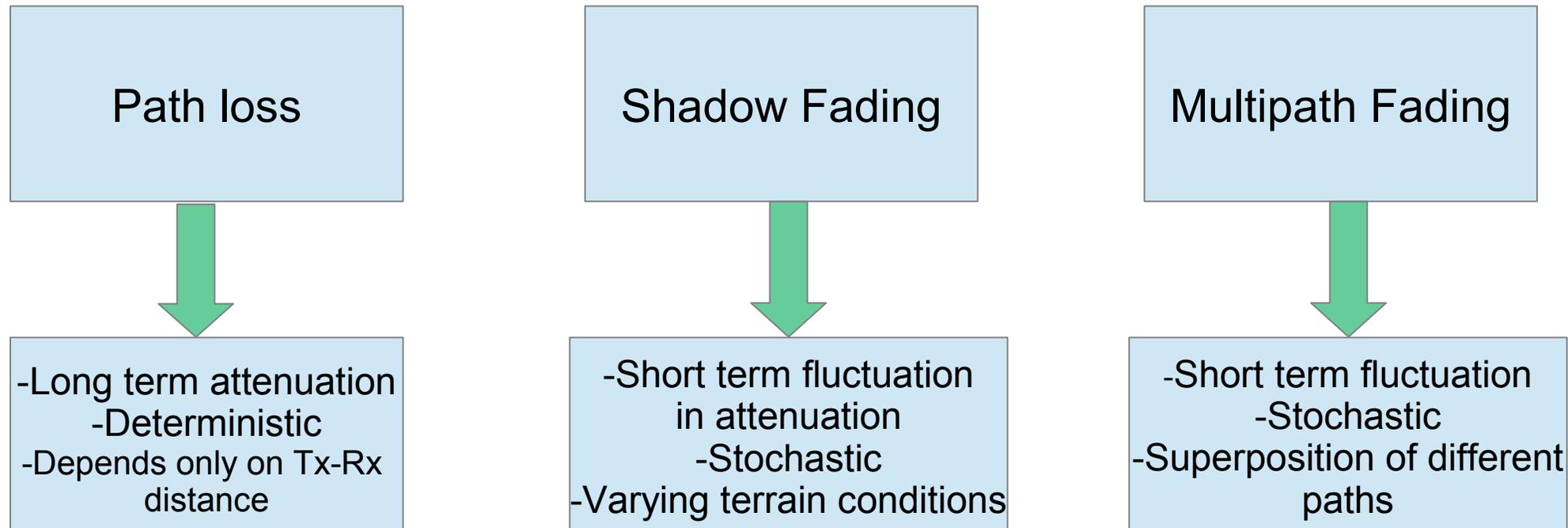


# Reflection, Diffraction & Scattering



# Wireless Channel Modeling...

- Due to these EM wave propagation mechanisms, radio propagation can be roughly described by three nearly independent phenomenon :



# Wireless Channel Modeling...

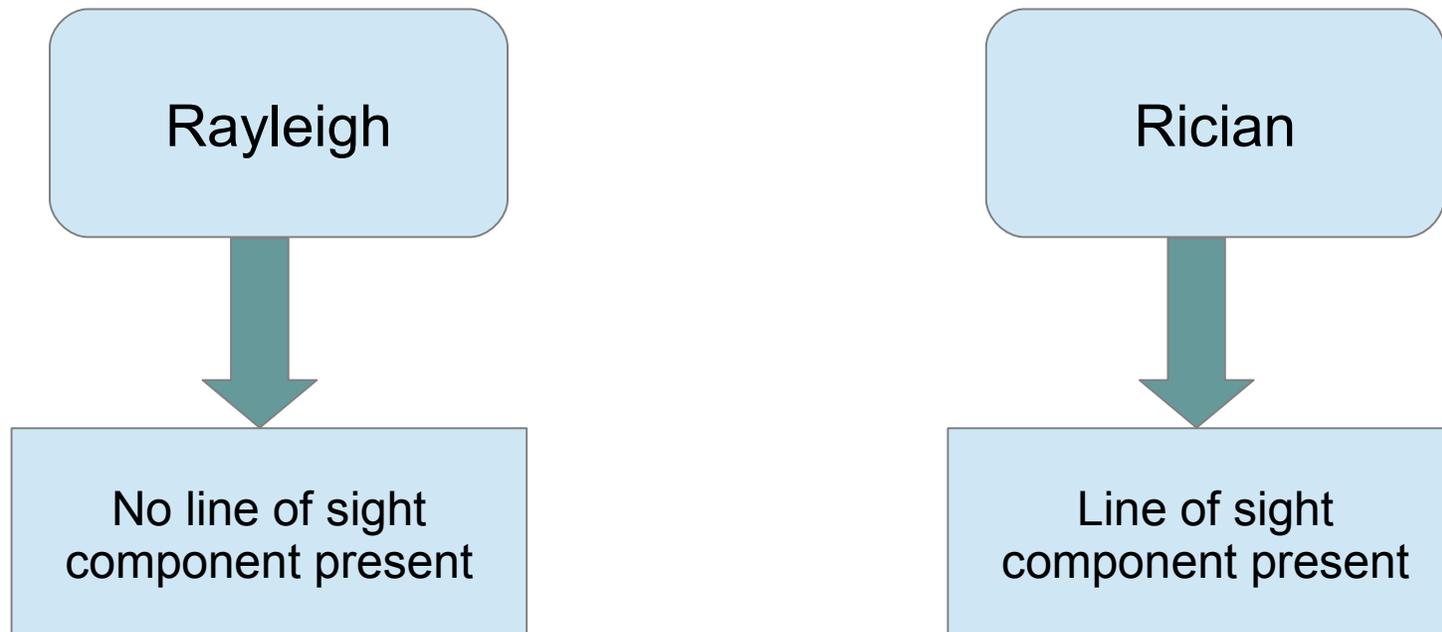
- The total attenuation can be decomposed into path loss, shadowing and multipath fading as follows:

$$a(t) = a_{PL}(t) \cdot a_{SH}(t) \cdot a_{FA}(t)$$

- In this presentation, we shall focus more on the modeling and simulation of multipath fading.
- Multipath fading in wireless communication systems is commonly modeled by Rayleigh and Rician distributions.

# Rayleigh and Rician Distribution

- A close approximation of attenuation due to multipath fading in wireless channels can be done by Rayleigh fading (for the case where no line of sight component present) and Rician fading (for the case where line of sight component present).



# Rayleigh Distribution

- A Rayleigh Random Variable  $R$  has the probability distribution:

$$PR(r) = \frac{2r}{\Omega} * \exp\left(\frac{-r^2}{\Omega}\right)$$

$$\text{where } \Omega = E(R^2)$$

- Rayleigh distribution can also be got by taking two independent and identically distributed zero mean gaussian random variables as real and imaginary parts of a complex number and then taking its magnitude.

# Rayleigh Fading Model

- For a wireless channel, the envelope of the channel response is modeled to have a Rayleigh distribution.
- Rayleigh Fading is a reasonable model when there are many objects in the environment that scatter the radio signal before it reaches the receiver.

# Delay spread & Doppler Spread

- N paths
- Different delays for each path – Delay Spread
- Different attenuation for each path
- Constant channel vs effect of motion
- Change in the carrier frequency – Doppler Spread

# Clarke's Model

- Rayleigh fading model
- Assumes isotropic scattering
- Assuming linear relationship between input and output
- Generating Rayleigh Channel model using two different Gaussian distributions.
- Gans developed a spectrum analysis for Clarke's model to include the doppler effect.

# Clarke's Model...

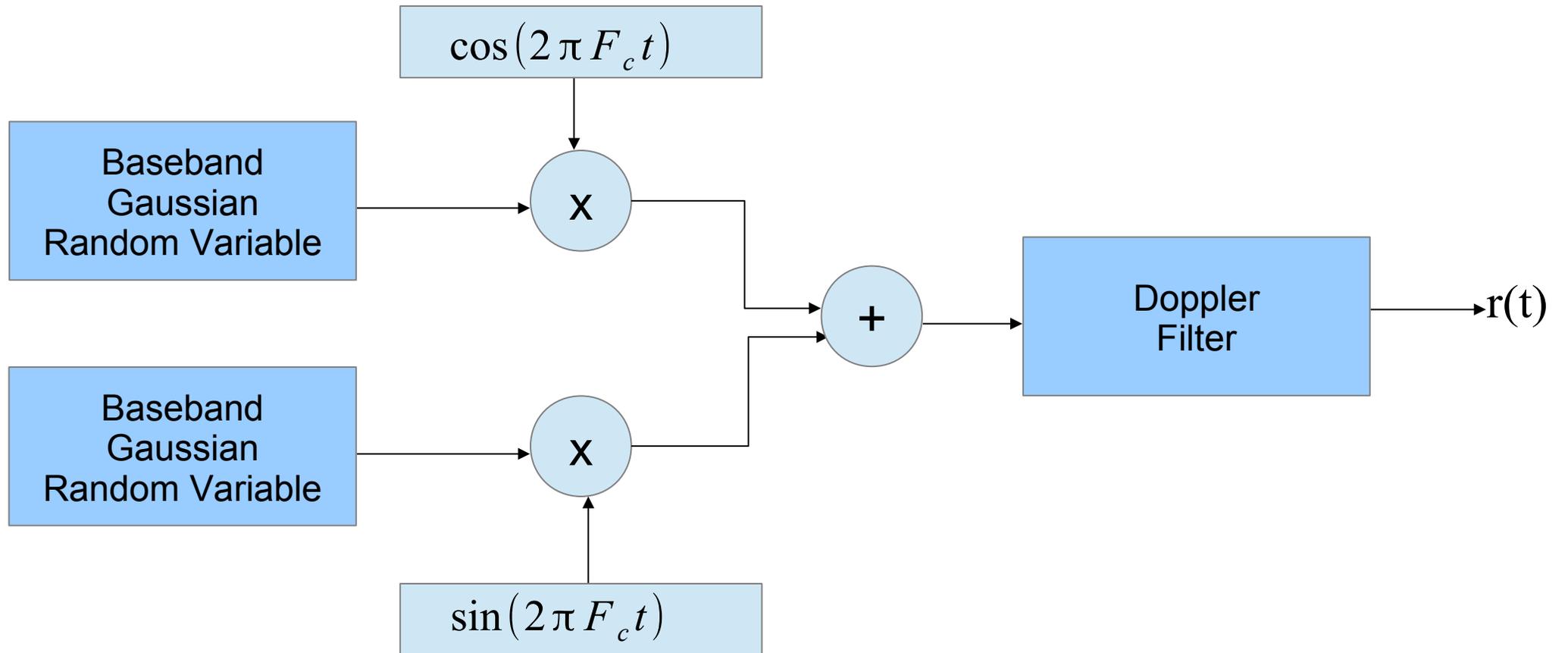
- Defining  $T_c(t)$  and  $T_s(t)$  be Gaussian random processes.
- At any time,  $T_c$  and  $T_s$  are uncorrelated zero mean Gaussian random variables.

$$r(t) = \sqrt{T_c(t)^2 + T_s(t)^2}$$

- The channel response envelope,  $r(t)$ , has a Rayleigh pdf.
- As derived by Gans, Doppler shift can be included into this channel model by passing  $r(t)$  through the filter  $s(t)$  given by:

$$S(f) = 1.5 / (\pi * Fd * \sqrt{1 - (f - Fc / Fd)^2})$$

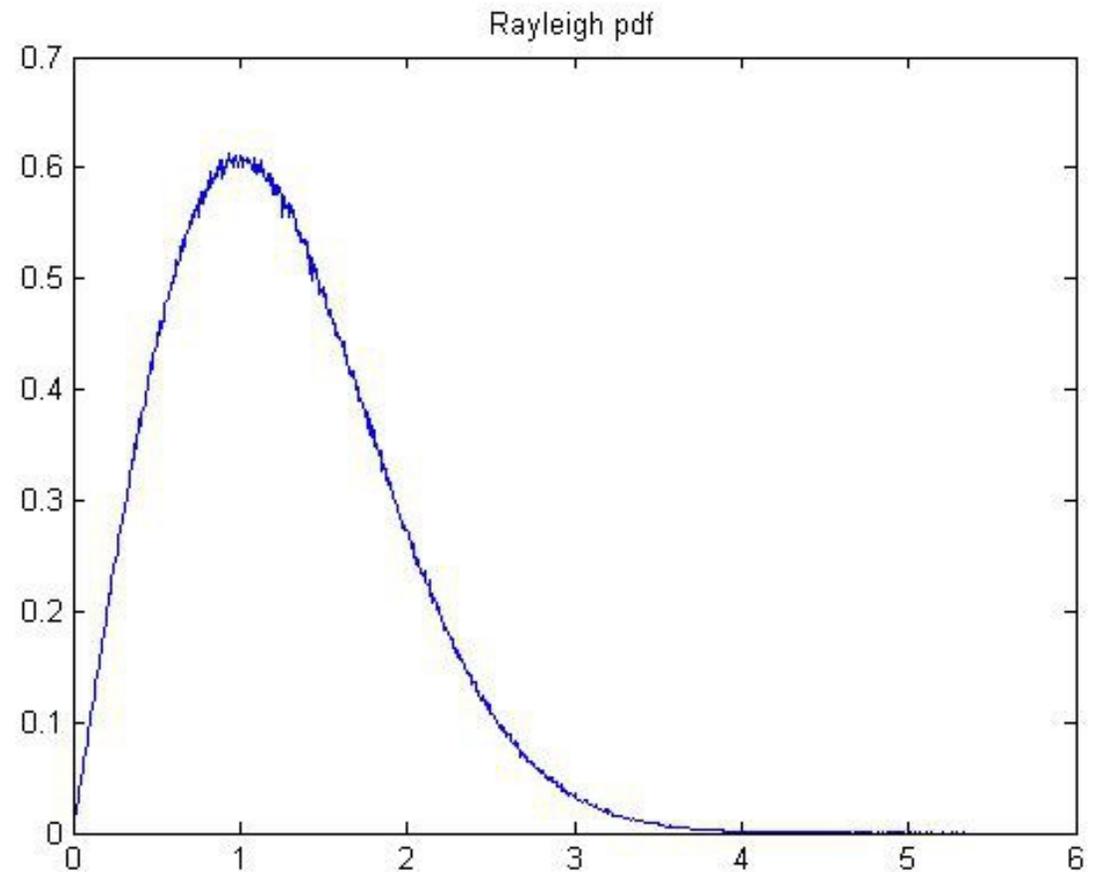
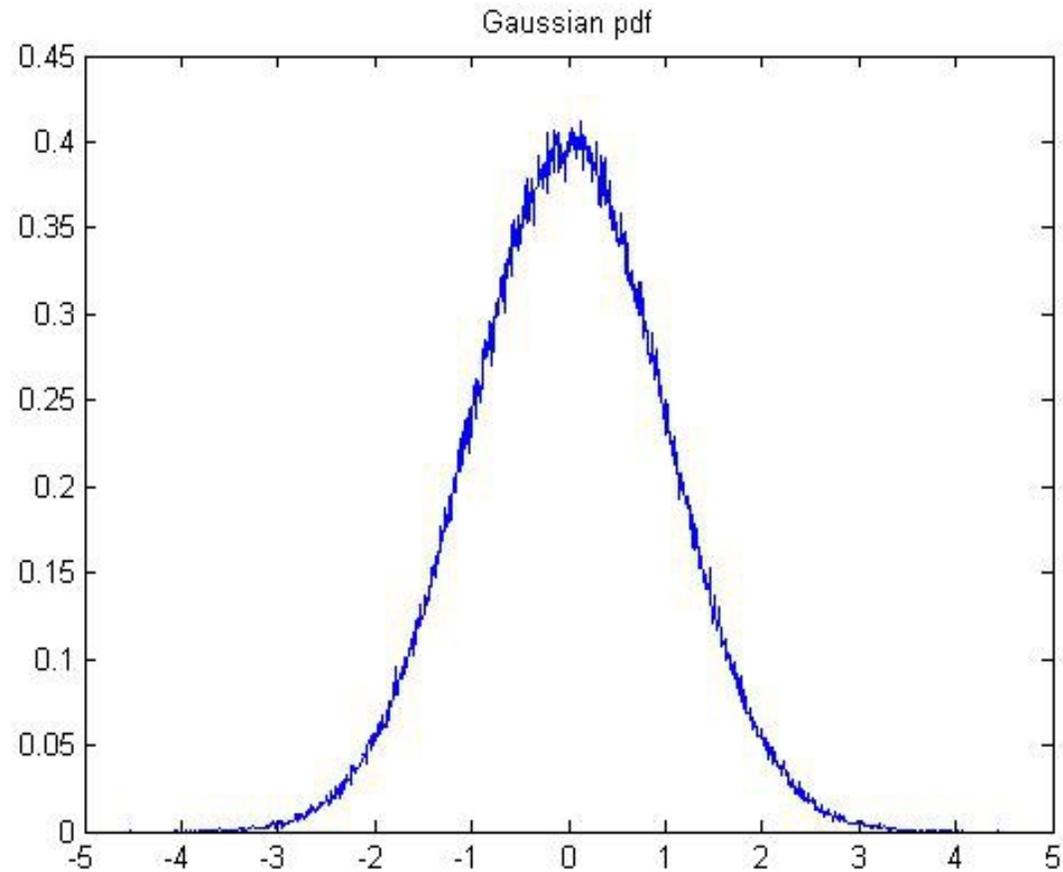
# Clarke's Model...



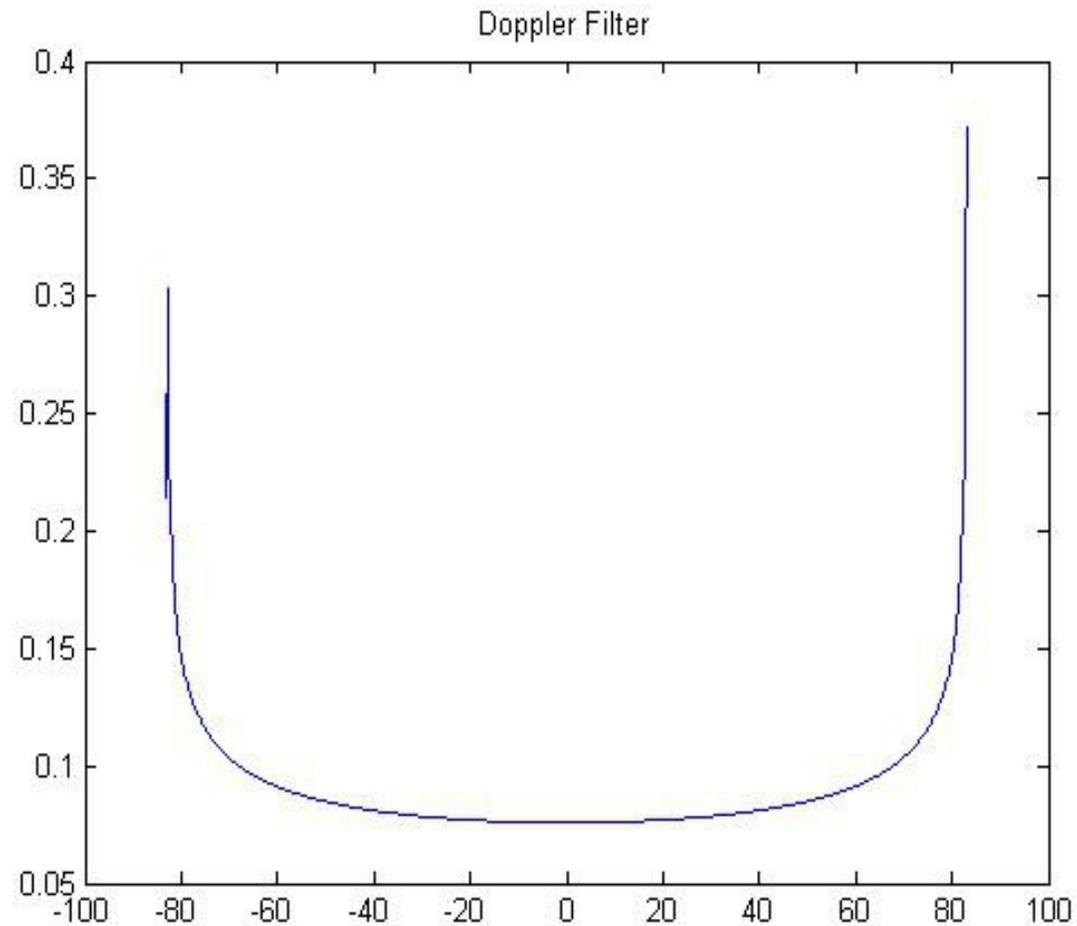
# Clarke's model simulation algorithm (Rappaport)

- Refer the section 'Clarke's Model for flat fading' in 'Wireless Communications – Principles and Practice' by Rappaport.
- The steps are clearly given in the section 'Simulating Clarke's and Gans fading model' with a block diagram showing the frequency domain implementation of a Rayleigh fading simulator at baseband.
- Repeat the above process to generate 3 such waveforms  $r(t)$  and find the cross-correlation values among them. Tabulate it.

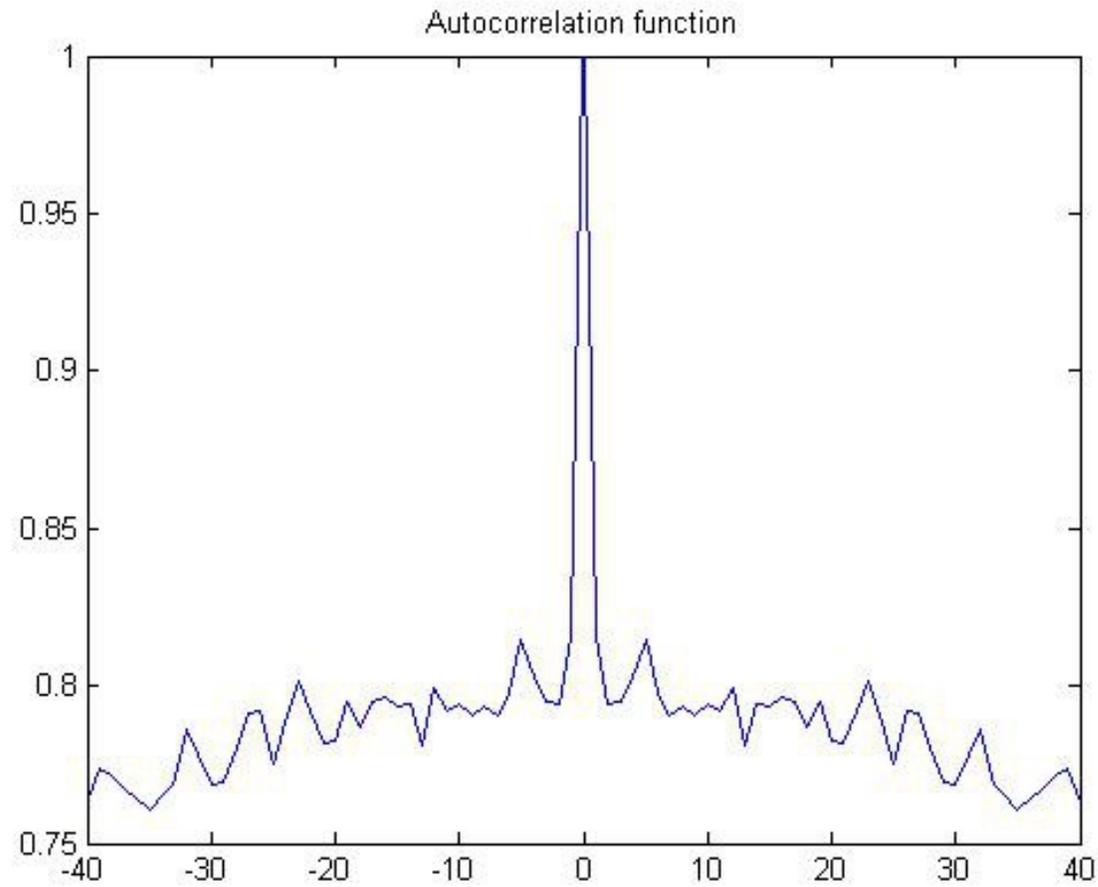
# Simulating Clarke's Model



# Simulating Clarke's Model...



# Simulating Clarke's Model...

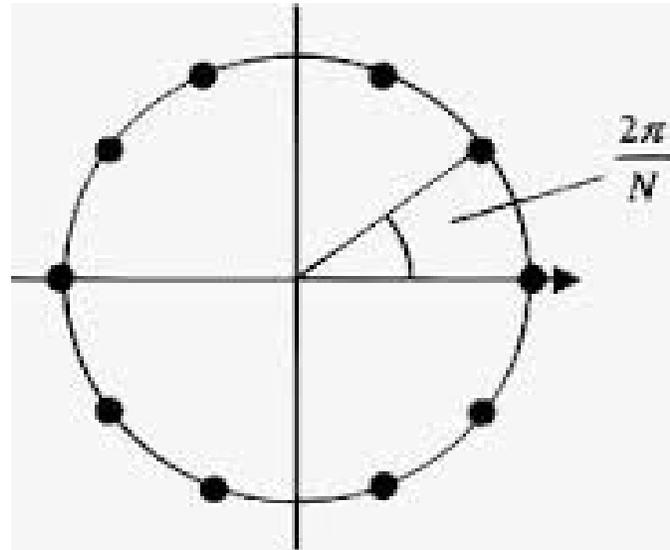


# Simulating Clarke's Model...

```
-----Correlation Coefficients (as in Table 2)-----  
Wave 1 and Wave 2  
    0.9984  
  
Wave 1 and Wave 3  
    0.9964  
  
Wave 1 and Wave 4  
    0.9989  
  
Wave 2 and Wave 3  
    0.9946  
  
Wave 2 and Wave 4  
    0.9985  
  
Wave 3 and Wave 4  
    0.9952
```

# Jakes' Channel Model

- A sum of sinusoids model.
- Assuming isotropic scattering i.e. receiver gets rays from all directions but they have a spacing of  $2\pi/N$



# Jakes' Channel Model...

- So Jakes' gets rid of the possibilities that all the rays might be coming in a small sector.
- Jakes' gives pdf of scaling as a function of angle of arrival for each path.

$$Cn^2 = f(\alpha) d\alpha$$

$$f(\alpha) = 1/2\pi \quad ; \quad d\alpha = 2\pi/N$$

$$Cn = 1/\sqrt{N}$$

- The equation for  $r(t)$  now becomes:

# Jakes' Channel Model...

$$r_I = \frac{1}{\sqrt{N}} \sum_{m=1}^N \cos(2\pi F_d \cos \alpha_m t + a_m) \quad ; \quad r_Q = \frac{1}{\sqrt{N}} \sum_{m=1}^N \sin(2\pi F_d \cos \alpha_m t + b_m)$$

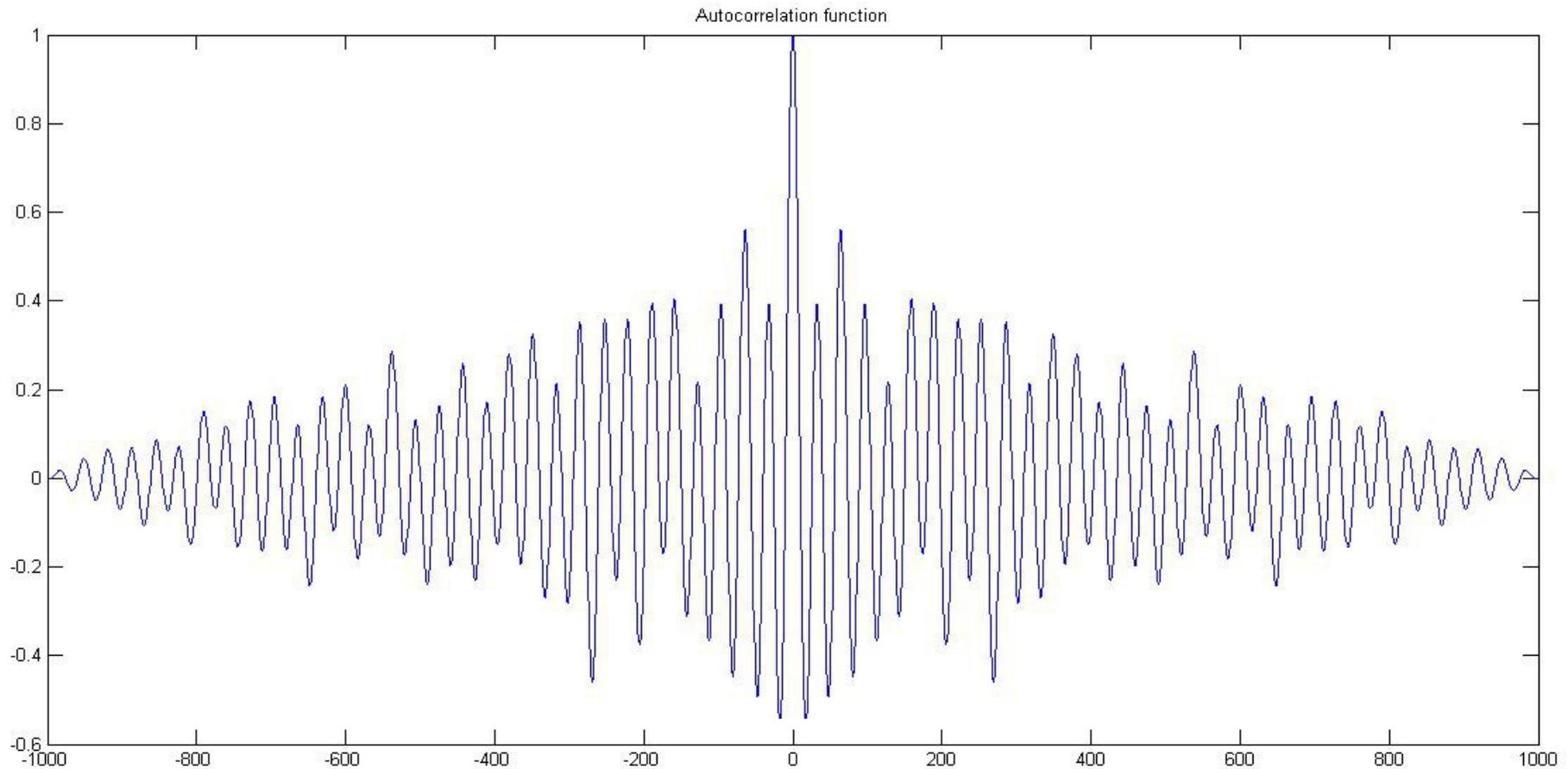
$$r(t) = r_I(t) + jr_Q(t)$$

- Where  $\alpha_m$  is the angle of arrival,  $a_m$  and  $b_m$  are random phases uniformly distributed over  $(0, 2\pi)$

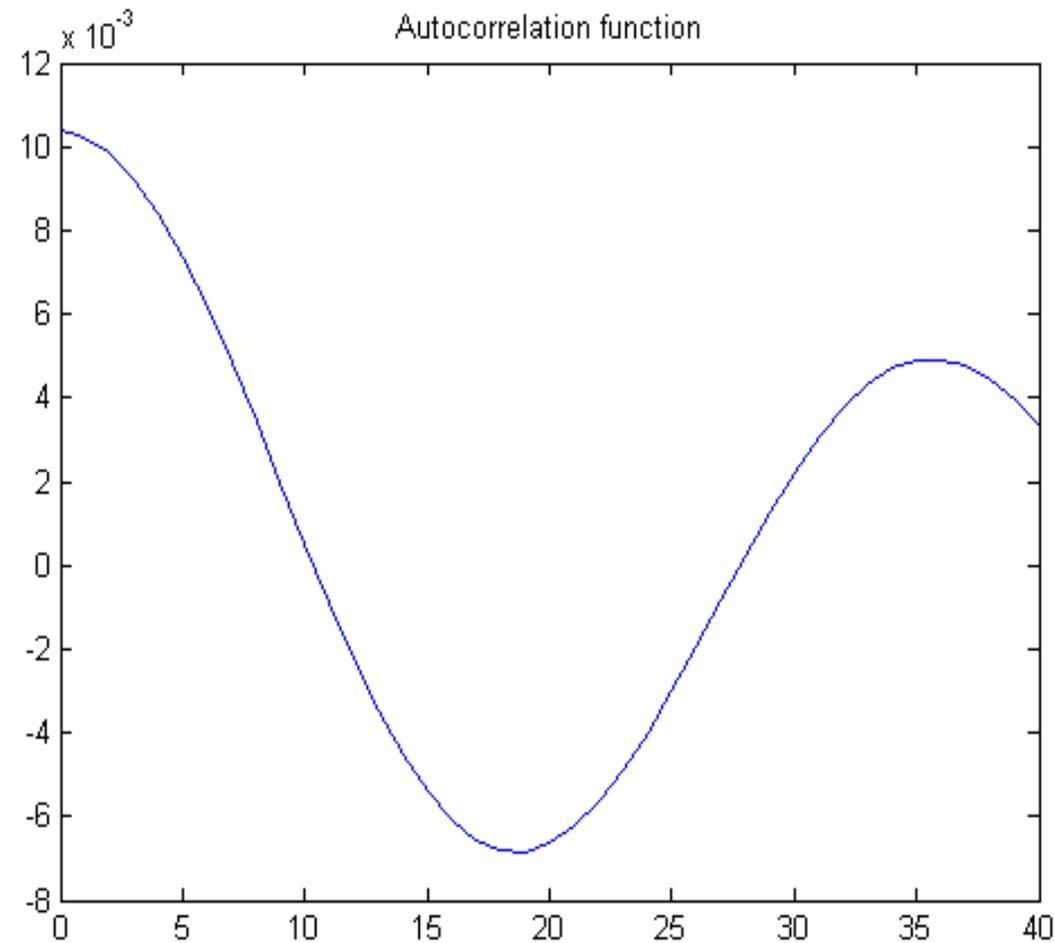
# Jakes' Model simulation algorithm

- Generate uniform random values for  $a_m, b_m, \alpha_m$  lying between 0 and  $2\pi$
- Generate a large number of samples of  $r_I(t)$  and  $r_Q(t)$
- Using them, generate the vector  $r(t) = r_I(t) + jr_Q(t)$
- Find the autocorrelation of  $r(t)$
- Repeat the above process to generate 3 different waveforms  $r(t)$  and find the cross-correlation coefficient values among them. Tabulate it.

# Simulating Jakes' Channel Model...



# Simulating Jakes' Channel Model...



# Simulating Jakes' Channel Model...

```
-----Correlation Coefficients (absolute values) (as in Table 2)-----  
Wave 1 and Wave 2  
    0.1288  
  
Wave 1 and Wave 3  
    0.2429  
  
Wave 1 and Wave 4  
    0.0645  
  
Wave 2 and Wave 3  
    0.0868  
  
Wave 2 and Wave 4  
    0.1549  
  
Wave 3 and Wave 4  
    0.1582
```

# Modified Jakes' Channel Model

- Ideally, each waveform generated must be uncorrelated to the others but there was a very high cross-correlation in Clarke's model.
- In Jakes' model, though the cross-correlation was lower than Clarke's model, it can be still be reduced
- Modified Jakes' model :
- Aim: To have negligible cross-correlation between different waveforms
- This is achieved by multiplying each waveform with a Hadamard Code.

# Hadamard Matrix

- Hadamard matrix is a square matrix whose entries are either 1s or -1s and whose rows are mutually orthogonal.
- Generating Hadamard matrix:

$$H_1 = [1]$$

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

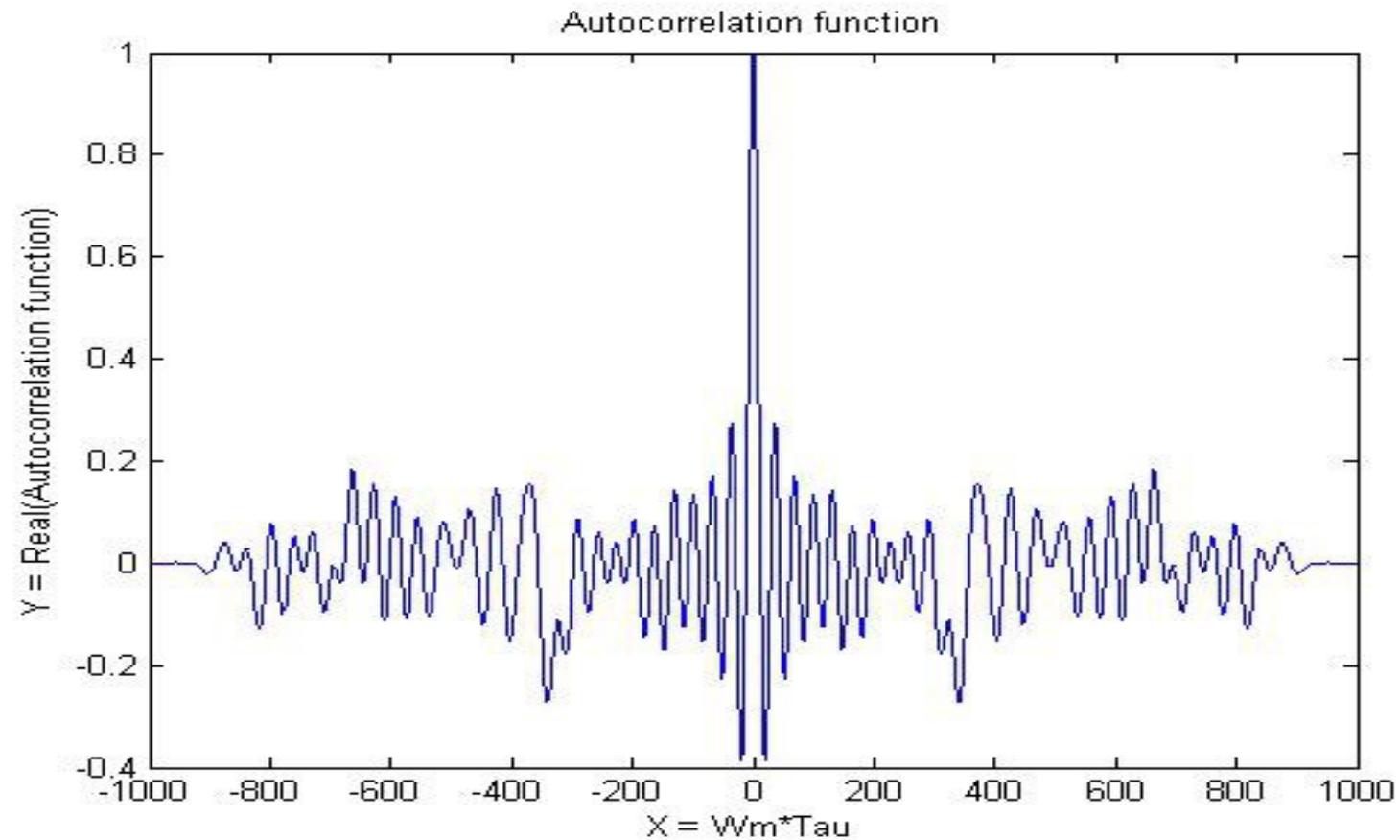
$$H_2^k = \begin{bmatrix} H_2^{k-1} & H_2^{k-1} \\ H_2^{k-1} & -H_2^{k-1} \end{bmatrix}$$

- Each channel response realization is multiplied with one row from the matrix.

# Modified Jakes' model simulation algorithm

- After generating three waveforms  $r(t)$  as given in Jakes' model:
- Make sure that each  $r(t)$  generated has  $2^k$  samples (for any integer  $k$ , say  $k=10$ ).
- Generate first 3 rows of a  $2^k \times 2^k$  Hadamard matrix or you can use the inbuilt matlab function 'hadamard' to generate the whole matrix and then use the first 3 rows.
- Multiply the 3 waveforms  $r(t)$  with a unique row of Hadamard matrix each.
- Find the cross-correlation values among the 3 waveforms generated after multiplying with the rows of Hadamard matrix. Tabulate it.

# Simulating Modified Jakes' Channel Model...



# Modified Jakes' Channel Model

-----Correlation Coefficients (absolute values) (as in Table 2)-----

Wave 1 and Wave 2

9.3495e-05

Wave 1 and Wave 3

3.2383e-04

Wave 1 and Wave 4

1.2964e-04

Wave 2 and Wave 3

3.3432e-04

Wave 2 and Wave 4

3.1335e-04

Wave 3 and Wave 4

3.1929e-04

# Conclusion

- Two major attenuations in a wireless signal: Path loss and Fading.
- Rayleigh fading channel models are fairly good approximations to model multipath fading in real life.
- Clarke's model, one of the first Rayleigh implementations, has a huge cross correlation.
- Sum of sinusoids (Jakes') Rayleigh implementations give a better model with much less cross correlation.
- Hadamard codes can be used to reduce the cross correlation even more.

# Future Work

- Better implementations of Rayleigh pdf than the sum of sinusoids method.
- Coming up with models more complex than Rayleigh which can model real life wireless channels even better.
- Using in the DRM+ radio receiver implementation.

# References

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Thank You