EE-5060 Communication Techniques

Oct. 2010 **Tutorial #2** KG / IITM

Traffic Engineering – Erlang B formula, Multistage Switching

1. Given a switching node where the average number of call arrivals λ = 10 per minute:
(a) What is the probability that 10 or more arrivals occur in a 45 second interval?
(b) What is the probability that less than 5 arrivals occur in the 45 second interval?

2. What is the amount of traffic E that can be accepted by M=2 servers if a high blocking probability P_b = 0.50 is allowed?
(a) Repeat when the allowed P_b = 0.02.
(b) Defining the output utilization factor $\gamma = (1- P_b)E / M$, what is it for the above 2 cases of P_b?

3. Repeat the steps in Pbm. 2 for the case of M=3 servers.

4. Problems from "Digital Telephony 3rd Ed." by J.C.Bellamy, Chapter 12 (pp.568-569):**12.1** thro **12.8, 12.10*,** & **12.13*.**

5. Given a population of N=20,000 users, each offering Eu=0.04 Erlangs of traffic, define a 3-stage blocking switch with k sub-arrays in the middle-stage, each containing 250x250 cross-points such that the blocking probability $P_b \le 10^{-3}$. Use the Lee graph approach to find this least value of k.
(a) Determine the number of cross-points for the above switch.
(b) Rework value of k and part (a) if we require $P_b \le 10^{-6}$.
(c) For the same size of the middle-stage sub-arrays (i.e., same size of m and n) as in (a), define a non-blocking switch. How does the complexity of this switch compare to (a)?
(d) For these N=20,000 users, what will be the least complexity of a 3-stage non-blocking switch if one had the flexibility to choose any n (and k) ? (Recall in our notation: N=nm)

6. A total of N=4096 lines have to be switched, where each line offers Eu=0.05 Erlangs of traffic. All the 3 stages of the switch are to be built using sub-arrays of size 64x64 (where in the input and output stages, not all lines need be utilized if k <64).
(a) Define a blocking switch such that blocking probability $P_b \le 10^{-3}$. What is it's complexity (including the unutilized cross-points)?
(b) Is it possible to build a non-blocking 3-stage switch in this case? If so, specify the same and it's complexity.

7. The first 400 inlets carry users with Eu=0.05 Erlangs while the next 600 inlets carry users with Eu=0.01 Erlangs. Given that the users are grouped into blocks of n=50 each, define a 3-stage blocking switch with overall $P_b \le 10^{-2}$. What is the total number of cross-points in this switch? *Hint*: The overall P_b is computed by considering the 4 cases, namely user from set1 calls another user in set1, or user from set1 calls user from set2, etc.

8. Problems from "Digital Telephony 3rd Ed." by J.C.Bellamy, Chapter 5 (pp.274):**5.2, 5.3** (Lee Graph only),**5.4*thro 5.8*.**

9. For the switch considered in Problem 5 (a) part, use the blocking probability expression following the work of Jacobaeus (which does **not** assume that the paths from input-to-middle stage and paths from output-to-middle stage are independent) given in eqn. (5.10) in page 239 of the book. What will be the new value of k for this case? How does this compare with your answer in 5(a)? Comment.

10. Consider a population of N=4000 users, each of E_u=0.01 Erlangs. Design a 3-stage blocking switch of least complexity such that the blocking probability $P_b=10^{-4}$ or less. What is k, and the total number of cross-points for this switch? *Hint:* To minimize the total number of cross-points, choose the input sub-array dimension n "appropriately" where $N/m=n$.

11. Consider the 5-stage switch in the book, first described in page 237, Fig. 5.9. Here, blocking is introduced also in the middle stage(s). The input has N/n_1 sub-arrays, each of dimension n_1x k_1, where N is the total population to be served by this switch. The middle-stage (which is actually a blocking switch with 3-stages) has k_1 sub-arrays, each of size N/n_1x N/n_1. Each of these sub-arrays has $N/(n_1$x $n_2)$ sub-arrays, of dimension n_2xk_2 where k_2 is the number of middle stage sub-arrays (each of dimension $N/(n_1$x $n_2)$ x $N/(n_1$x $n_2)$). Assume each user offers E_u Erlangs of traffic.

(a) Prove using the Lee-Graph approach that blocking probability of the 5-stage switch is given by
$P_b = \left\{1-q_1^2\left[1-(1-q_2^2)^{k_2}\right]\right\}^{k_1}$ where $q_1=(1-p_1)$ with $p_1=\dfrac{n_1E_u}{k_1}$ and $q_2=(1-p_2)$ with $p_2=\dfrac{n_2p_1}{k_2}$.

(b) For N=50,000, and n_1=50 and n_2=50, find the 5-stage switch with minimum number of cross-points so that $P_b=10^{-8}$ or less. Assume E_u = 0.01 Erlangs each.

(c) Can you find a better choice of n_1 and n_2 for this case? (i.e., a choice that will minimize the number of cross-points further?)