EE 613 Estimation Theory - HW 9

October 17, 2008

1. (9.2) If N IID observations $\{x[0], x[1], \ldots, x[N-1]\}$ are made from the Laplacian PDF

$$f(x;\sigma) = \frac{1}{\sqrt{2}\sigma} \exp\left(-\frac{\sqrt{2}|x|}{\sigma}\right)$$

find a method of moments estimator for σ^2.

2. (9.3) Assume that N IID samples from a bivariate Gaussian PDF are observed or $\{\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{N-1}\}$ where each $\mathbf{x}$ is a 2×1 random vector with PDF $\mathbf{x} \sim \mathcal{N}(\mathbf{0}, \mathbf{C})$. If

$$\mathbf{C} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

find a moments estimators for ρ. Also, determine a cubic equation to be solved for the MLE of ρ. Comments on the ease of implementation of the different estimators.

3. (9.4) If N IID observations $\{x[0], x[1], \ldots, x[N-1]\}$ are made from the $\mathcal{N}(\mu, \sigma^2)$ PDF, find a method of moments estimator for $\boldsymbol{\theta} = [\mu \;\; \sigma^2]^T$.

4. (9.6) For a DC level in WGN or $x[n] = A + \omega[n]$ for $n = 0, 1, \ldots, N-1$ where $\omega[n]$ is WGN with variance σ^2, the parameter A^2 is to be estimated. It is proposed to use $\widehat{A^2} = (\bar{x}^2)$. For this estimator, find the approximate mean and variance using a first-order Taylor expansion approach.

5. (10.3) The data $x[n]$ for $n = 0, 1, \ldots, N-1$ are observed, each sample having the conditional PDF

$$f(x[n]|\theta) = \begin{cases} \exp[-(x[n]-\theta)] & x[n] > \theta \\ 0 & x[n] < \theta, \end{cases}$$

and conditioned on θ the observations are independent. The prior PDF is

$$f(\theta) = \begin{cases} \exp(-\theta) & \theta > 0 \\ 0 & \theta < 0. \end{cases}$$

Find the MMSE estimator of θ.

6. (10.4) Repeat the previous problem with the conditional PDF

$$f(x[n]|\theta) = \begin{cases} \frac{1}{\theta} & 0 \leq x[n] \leq \theta \\ 0 & \text{otherwise} \end{cases}$$

and the uniform PDF $\theta \sim \mathcal{U}[0, \beta]$. What happens if β is very large so that there is little prior knowledge?

—– END —–