EE 613 Estimation Theory - HW 6
September 11, 2008

1. (6.2) In the estimation of DC in zero-mean white noise of arbitrary PDF with variance σ^2, problem, if the noise variance is given by $\sigma^2 = n + 1$, examine what happens to the variance of the BLUE as $N \to \infty$. Repeat for $\sigma^2 = (n+1)^2$ and explain your results.

2. (6.3) Consider the estimation of DC in zero-mean Gaussian noise, and assume that the noise samples are correlated with the covariance matrix

$$\boldsymbol{C} = \sigma^2 \begin{bmatrix} 1 & \rho & 0 & 0 & \cdots & 0 & 0 \\ \rho & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \rho & \cdots & 0 & 0 \\ 0 & 0 & \rho & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & \rho \\ 0 & 0 & 0 & 0 & \cdots & \rho & 1 \end{bmatrix}$$

where $|\rho| < 1$ and N, the dimension of the matrix, is assumed to be even. $\boldsymbol{C}$ is a block-diagonal matrix and so is easily inverted. Find the BLUE and its variance and interpret your results.

3. (6.4) The observed samples $\{x[0], x[1], \ldots, x[N-1]\}$ are IID according to the following PDFs:

   a. Laplacian
   $$f(x[n]; \mu) = \frac{1}{2} \exp[-|x[n] - \mu|]$$
   b. Gaussian
   $$f(x[n]; \mu) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(x[n] - \mu)^2\right].$$

   Find the BLUE of the mean μ in both cases. What can you say about the MVU estimator for μ?

4. (6.7) Assume that $x[n] = As[n] + \omega[n]$ for $n = 0, 1, \ldots, N-1$ are observed, where $\omega[n]$ is zero-mean noise with covariance matrix $\mathbf{C}$ and $s[n]$ is a known signal. The amplitude A is to be estimated using a BLUE. Find the BLUE and discuss what happens if $\mathbf{s} = [s[0]\ s]1] \ \ldots s[N-1]]^T$ is an eigenvector of $\mathbf{C}$. Also, find the minimum variance.

5. If x and y are distributed according to a bivariate Gaussian PDF, then show that the conditional PDF $f(y|x)$ is also Gaussian with mean

$$E[y|x] = E[y] + \frac{\text{cov}(x, y)}{\text{var}(x)}(x - E[x])$$

and variance

$$\text{var}(y|x) = \text{var}(y) - \frac{\text{cov}^2(x, y)}{\text{var}(x)}.$$

—— END ——