EE 613 Estimation Theory - HW 4

August 22, 2008

1. (4.1) We wish to estimate the amplitudes of exponentials in noise. The observed data are

$$x[n] = \sum_{i=1}^{p} A_i r_i^n + \omega[n] \quad n = 0, 1, \ldots, N-1$$

where $\omega[n]$ is WGN with variance σ^2. Find the MVU estimator of the amplitudes and also their covariance. Evaluate your results for the case when $p = 2$, $r_1 = 1$, $r_2 = -1$ and N is even.

2. (4.3) Consider the observation matrix

$$\boldsymbol{H} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1+\epsilon \end{bmatrix}$$

where ϵ is small. Compute $(\mathbf{H}^T\mathbf{H})^{-1}$ and examine what happens as $\epsilon \to 0$. If $\mathbf{x} = [2\ 2\ 2]^T$, find the MVU estimator and describe what happens as $\epsilon \to 0$.

3. (4.6) Suppose that we have a single sinusoidal component at $f_k = k/N$ in WGN. The model is

$$x[n] = a_k \cos(2\pi f_k n) + b_k \sin(2\pi f_k n) + \omega[n] \quad n = 0, 1, \ldots, N-1.$$

Using the identity $A \cos\omega + B \sin\omega = \sqrt{A^2 + B^2} \cos(\omega - \phi)$, where $\phi = \arctan(B/A)$, we can rewrite the model as

$$x[n] = \sqrt{a_k^2 + b_k^2} \cos(2\pi f_k n - \phi) + \omega[n].$$

An MVU estimator is used for a_k, b_k, so that the estimated power of the sinusoid is

$$\hat{P} = \frac{\hat{a}_k^2 + \hat{b}_k^2}{2}.$$

A measure of detectability is $E^2(\hat{P})/\text{var}(\hat{P})$. Compare the measure when a sinusoid is present to the case when only noise is present or $a_k = b_k = 0$. Could you use the estimated power to decide if a signal is present?

4. (4.10) Consider the following special case of DC level estimation in colored Gaussian noise where

$$\mathbf{C} = \text{diag}(\sigma_0^2, \sigma_1^2, \ldots, \sigma_{N-1}^2).$$

Find the expression for the MVUE and interpret the results. What would happen to $\hat{A}$ if a single σ^2 were equal to zero?

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