EE 613 Estimation Theory - HW 3

August 14, 2008

1. (3.1) If $x[n]$ for $n = 0, 1, \ldots, N-1$ are IID according to $\mathcal{U}[0, \theta]$, show that the regularity condition does not hold or that

$$E\left[\frac{\partial \ln f(\mathbf{x}; \theta)}{\partial \theta}\right] \neq 0 \quad \text{for all } \theta > 0$$

Hence, the CRLB cannot be applied to this problem.

2. (3.2) If a single sample $x[0] = A + \omega[0]$ is observed and $\omega[0]$ has the PDF $f(\omega[0])$ which can be arbitrary, show that the CRLB for A is

$$\operatorname{var}(\hat{A}) \geq \left[\int_{-\infty}^{\infty} \frac{\left(\frac{(df(u)}{du}\right)^2}{f(u)} du\right]^{-1}$$

Evaluate this for the Lapalcian PDF

$$f(\omega[0]) = \frac{1}{\sqrt{2}\sigma} \exp\left(-\frac{\sqrt{2}|\omega[0]|}{\sigma}\right)$$

and compare the result with the Gaussian case.

3. (3.9) We observe two samples of a DC level in *correlated* Gaussian noise

$$\begin{aligned} x[0] &= A + \omega[0] \\ x[1] &= A + \omega[1] \end{aligned}$$

where $\mathbf{w} = [\omega[0] \quad \omega[1]]^T$ is zero mean with covariance matrix

$$\mathbf{C} = \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

The parameter ρ is the correlation coefficient between $\omega[0]$ and $\omega[1]$. Compute the CRLB for A and compare it to the case when $\omega[n]$ is WGN or $\rho = 0$. Also, explain what happens when $\rho \to \pm 1$.

4. (3.11) For a 2×2 Fisher information matrix

$$\mathbf{I}(\boldsymbol{\theta}) = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

which is positive definite, show that

$$[\mathbf{I}^{-1}(\boldsymbol{\theta})]_{11} = \frac{c}{ac - b^2} \geq \frac{1}{a} = \frac{1}{[\mathbf{I}(\boldsymbol{\theta})]_{11}}$$

What does this say about estimating a parameter when a second parameter is either known or unknown? When does the equality hold and why?

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