EE 613 Estimation Theory - HW 11

November 07, 2008

1. (12.1) Consider the quadratic estimator

$$\hat{\theta} = ax^2[0] + bx[0] + c$$

of a scalar parameter θ based on the single data sample $x[0]$. Find the coefficients a, b, c that minimize the Bayesian MSE. If $x[0] \sim \mathcal{U}[-\frac{1}{2}, \frac{1}{2}]$, find the LMMSE estimator and the quadratic MMSE estimator if $\theta = \cos 2\pi x[0]$. Also, compare the minimum MSEs.

2. (12.6) We observe the data $x[n] = s[n] + \omega[n]$ for $n = 0, 1, \ldots, N-1$, where $s[n]$ and $\omega[n]$ are zero-mean, WSS random processes which are uncorrelated with each other. The ACFs are

$$r_{ss}[k] = \sigma_s^2 \delta[k]$$
$$r_{\omega\omega}[k] = \sigma^2 \delta[k].$$

Determine the LMMSE estimator of $\mathbf{s} = [s[0],\ s[1], \ldots,\ s[N-1]]^T$ based on $\mathbf{x} = [x[0],\ x[1], \ldots,\ x[N-1]]^T$ and the corresponding minimum MSE matrix.

3. (12.14) In this problem we examine the interpolation of a data sample. We assume that the data set $\{x[n-M], \ldots, x[n-1], x[n+1], \ldots, x[n+M]\}$ is available and that we wish to estimate or *interpolate* $x[n]$. The data and $x[n]$ are assumed to be a realization of a zero mean WSS random process. Let the LMMSE estimator of $x[n]$ be

$$\hat{x}[n] = \sum_{k=-M, k\neq 0}^{M} a_k x[n-k].$$

Find the set of linear equations to be solved for the weighting coefficients by using the orthogonality principle. Next, prove that $a_{-k} = a_k$ and explain why this must be true. See also Kay[1] for a further discussion of interpolation.

4. (12.20) Consider an AR(N) process

$$x[n] = -\sum_{k=1}^{N} a[k]x[n-k] + u[n]$$

where $u[n]$ is white noise with variance σ_u^2. Prove that the optimal one-step linear predictor of $x[n]$ is

$$\hat{x}[n] = -\sum_{k=1}^{N} a[k]x[n-k].$$

Also, find the minimum MSE.

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[1] kay, S.,“Some Results in Linear Interpolation Theory”, IEEE Trans. ASSP, Vol. 31, pp.746-749, June 1983