HW8 solutions (draft)

Q1)

$x(t) = u(t) \quad y(t) = (4e^{-t} - 3e^{-2t})u(t)$

$(b) \quad H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{4}{s+1} - \frac{3}{s+2}}{\frac{1}{s}} = \frac{s(s+5)}{(s+1)(s+2)}$

$(a) \quad H(s) = 1 + \frac{2s-2}{(s+1)(s+2)} = 1 - \frac{4}{s+1} + \frac{6}{s+2}$

$\Rightarrow h(t) = \delta(t) - 4e^{-t}u(t) + 6e^{-2t}u(t)$

$(c) \quad Y(s) = \frac{s(s+5)}{(s+1)(s+2)(s+4)} = \frac{-4}{3(s+1)} + \frac{3}{s+2} + \frac{-2}{3(s+4)}$

$\Rightarrow y(t) = (\frac{-4}{3}e^{-t} + 3e^{-2t} - \frac{2}{3}e^{-4t})u(t)$

$(d) \quad H(j\omega) = \frac{j\omega(j\omega+5)}{(j\omega+1)(j\omega+2)}$

$|H(j\omega)| = \frac{\omega\sqrt{\omega^2+25}}{\sqrt{\omega^2+1}\sqrt{\omega^2+4}}$

$\angle H(j\omega) = 90^o + \tan^{-1}(\frac{\omega}{5}) - \tan^{-1}(\frac{\omega}{2}) - \tan^{-1}(\omega)$

$\omega = 2 \Rightarrow y(t) = |H(j2)|cos(2t + \angle H(j2))$

$y(t) = 1.7cos(2t + 3.36^o)$

Q2)

Q2a

Poles at $s = -5, -3$.
Zero at $s = 2$.
The pole zero plot is shown in Fig.1. This system is BIBO stable as all the poles are in LHP.

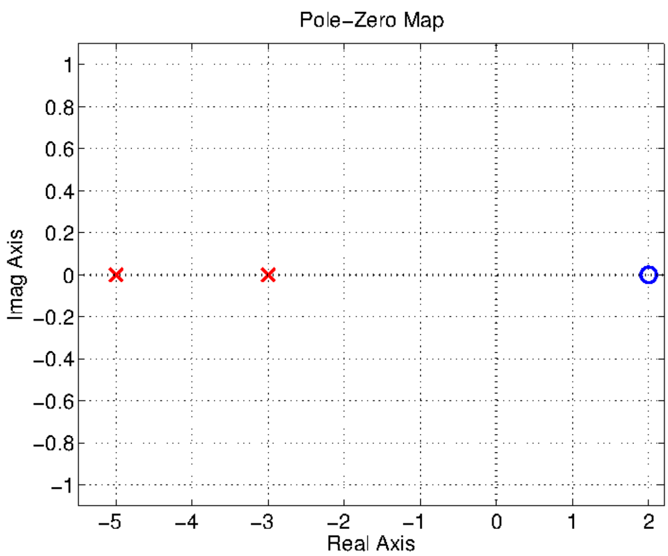


Figure 1: Pole Zero plot for Q2a

Q2b

Poles at $s = -3, \ -2, \ -2.$
Zero at $s = -1.$
The pole zero plot is shown in Fig.2. This system is BIBO stable as all the poles are in LHP.

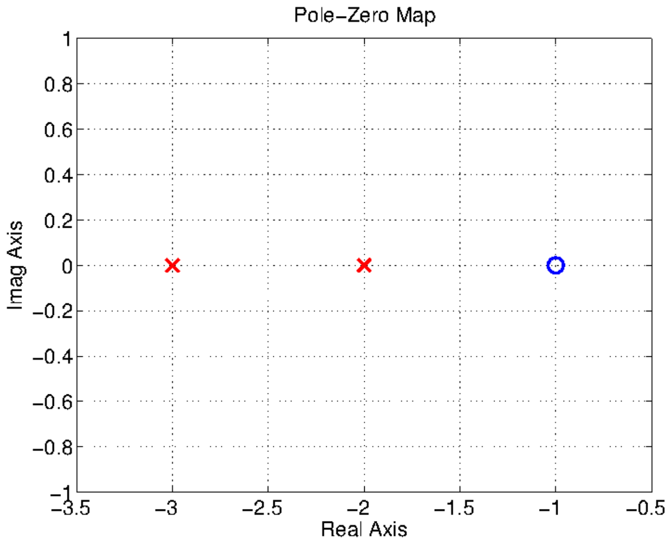


Figure 2: Pole Zero plot for Q2b

Q2c

Poles at $s = 0, \ -2.$
Zero at $s = -0.25 + 0.66j, \ -0.25 - 0.66j.$
The pole zero plot is shown in Fig.3. This system is BIBO stable as the poles are in LHP and the pole on the $j\omega$ axis is simple.

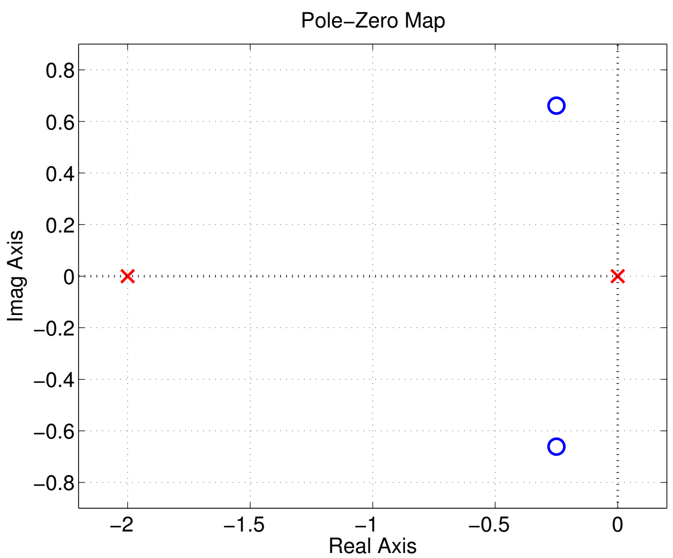


Figure 3: Pole Zero plot for Q2c

Q2d

Poles at $s = -2, +j, +j, -j, -j$.
Zero at $s = -0.5$.
The pole zero plot is shown in Fig.. This system is **NOT** BIBO stable as there are 2 poles each at $+j$ and $-j$.

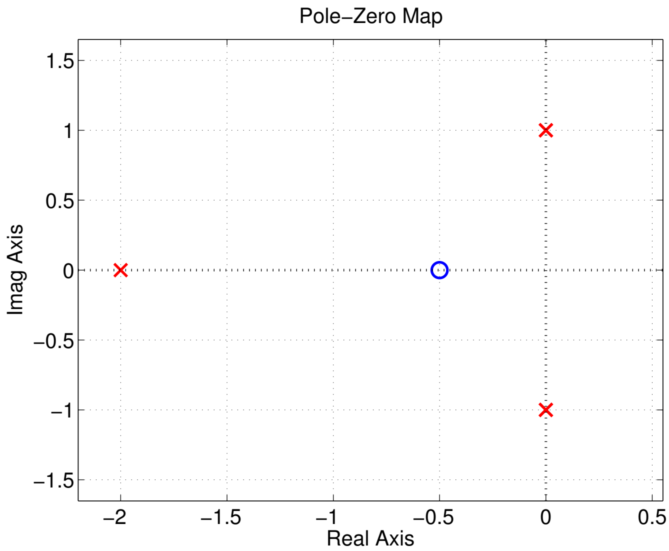


Figure 4: Pole Zero plot for Q2d

Q3)

We can write the transfer function as

$$H(s) = K\frac{(s+a)}{(s+1)^2+1}$$

where K is a constant and $-a$ is the zero location.
The step response of this system is

$$V(s) = \frac{K}{s}\frac{(s+a)}{(s+1)^2+1}$$

Using partial fractions, we can write $V(s)$ as

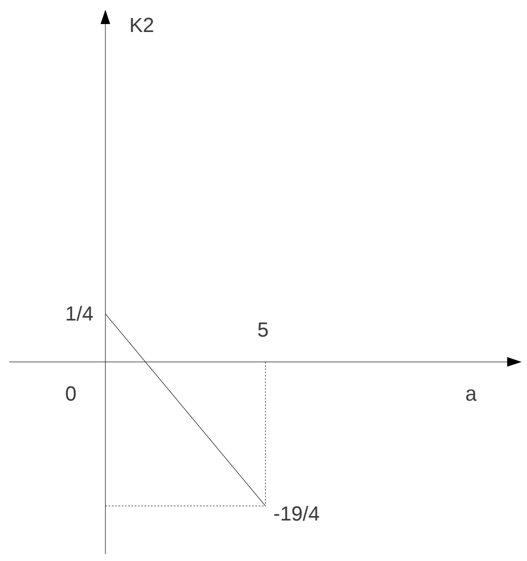
$$V(s) = \frac{K_1 s + K_2}{(s+1)^2+1} + \frac{K_3}{s}$$

From this we can find $K_1, K_2 and K_3$ as

$$K_1 = -\frac{a}{2}$$
$$K_2 = -a + \frac{1}{4}$$
$$K_3 = \frac{a}{2}$$

The output term that is of interest is

$$\frac{K_2}{(s+1)^2+1}$$



The inverse transform gives

$$(-a + \frac{1}{4}) \exp(-t) \sin(t + \phi)$$

From this, we get

$$K_2 = \frac{1}{4} - a$$

Q4

Let the transfer function be

$$H(s) = \frac{(s+a)(s+b)}{(s+1)^2+1)}$$

Using the eigenfunction property, for 6V DC input, the steady state response is

$$\begin{aligned} 6H(j0) &= [\frac{(j\omega+a)(j\omega+b)}{(j\omega+1)^2+1)}]|_{\omega=0} \\ &= 3ab \end{aligned}$$

From the given data, $3ab = 0$.

For the input $\sin(t)$, the output of the system(again using eigenfunction property) will be

$$|H(j\omega)|_{\omega=1} \sin(t + \phi)$$

$$|H(j)| = \frac{\sqrt{a^2+1}\sqrt{b^2+1}}{\sqrt{5}}$$

From the steady resonpse given for $\sin(t)$,

$$0.6\sin(t) + 0.8\cos(t) = \sin(t + \frac{4}{3})$$

This implies that $|H(j)| = 1$ and hence

$$a^2 + b^2 + (ab)^2 + 1 = 5$$

Solving for a and b, we get $a = 0$, $b = 2$. The transfer function of the system is given by

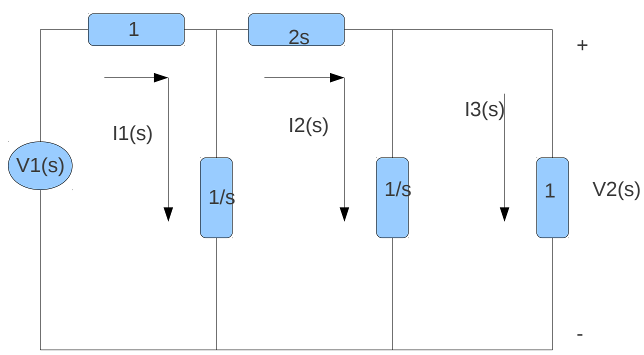
$$H(s) = \frac{s(s+2)}{(s+1)^2+1}$$

For the input $\sin(2t)$, the steady state current would be

$$|H(j2)|\sin(2t + \angle H(j2)) =$$

$$\frac{2\sqrt{2}}{5}\sin(2t + 0.322)$$

Q5(a))



Writing KVL for the three loops give

$$Loop1 : (1 + \frac{1}{s})I_1(s) - \frac{1}{s}I_2(s) = V_1(s)$$
$$Loop2 : \frac{-1}{s}I_1(s) + (2s + \frac{2}{s})I_2(s) - \frac{1}{s}I_3(s) = 0$$
$$Loop3 : \frac{-1}{s}I_2(2) + (1 + \frac{1}{s})I_3(s) = 0$$

And also $V_2(s) = I_3(s)$ and $V_2(s) = (I_2(s) - I_3(s))\frac{1}{s}$.
Solving the above equations, we get

$$I_2(s) = (s + 1)V_2(s)$$

Finding $I_1(s)$ in terms of $V_2(s)$ using $I_2(s)$ and $I_3(s)$, we get

$$I_1(s) = 2(s^2 + 1)(s + 1) - 1$$

Using $I_1(s)$ in the equation for loop 1, we get

$$\frac{V_2(s)}{V_1(s)} = \frac{1}{2(s^2 + s + 1)(s + 1)}$$

b

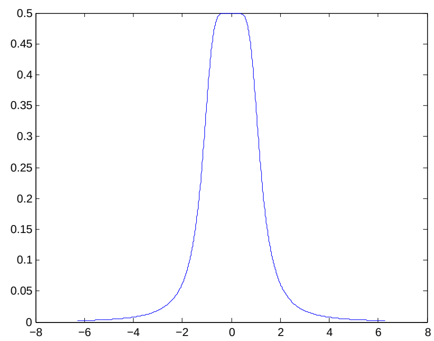
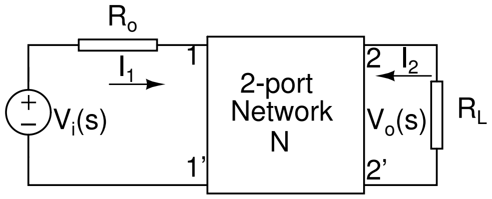


Figure 5: Magnitude spectrum of H(s)

Q6)



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = Vi - I_1 \times R_o$$
$$\implies V_i = I_1(R_o + Z_{11}) + I_2 Z_{12} \quad (1)$$
$$V_o = V_2 = I_1 Z_{21} + I_2 Z_{22} = -I_2 R_L$$
$$\implies I_1 = -I_2 \frac{(Z_{22} + R_L)}{Z_{21}} \quad (2)$$

From equations (1) **and** (2)**,**

$$V_i = I_2 \left[\frac{Z_{12}Z_{21} - (Z_{11} + R_o)(Z_{22} + R_L)}{Z_{21}} \right]$$
$$\implies \frac{V_o}{V_i} = \frac{Z_{21} R_L}{(Z_{11} + R_o)(Z_{22} + R_L) - Z_{12}Z_{21}}$$

Q7)

$$Z = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}$$

From the Z matrix, $Z_{11} = 0, Z_{12} = -r, Z_{21} = r$ **and** $Z_{22} = 0$
$V_{11'}$ **and** $V_{22'}$ **can be written as**

$$V_{11'} = I_1 Z_{11} + I2 Z_{12}$$
$$V_{22'} = I_1 Z_{21} + I2 Z_{22}$$
$$V_{11'} = -r I_2 \quad (3)$$
$$V_{22'} = r I_1 \quad (4)$$
$$V_{22'} = -I_2 Z_L(s) \quad (5)$$

From equations (4) **and** (5)**,**

$$I_2 = \frac{-I_1 r}{Z_L(s)}$$

substituting I_2 **in**(3)

$$\frac{V_{11'}}{I_1} = r^2 Z_L(s)$$

$$Z_i n(s) = \frac{V_{11'}}{I_1} = r^2 Z_L(s)$$

Given $Z_L(s) = \frac{1}{sC}$**,** $Z_i n(s) = s\, r^2 C$
$Z_i n(s)$ **is of the forn sL with inductance** $r^2 C$