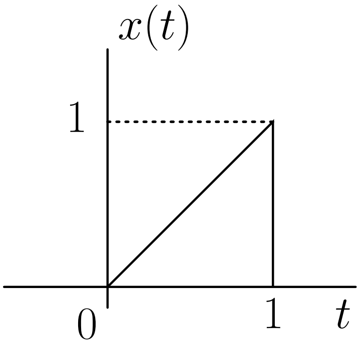
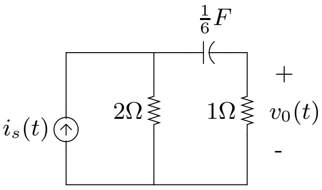
EC2102 Networks and Systems – HW 6

October 3, 2012

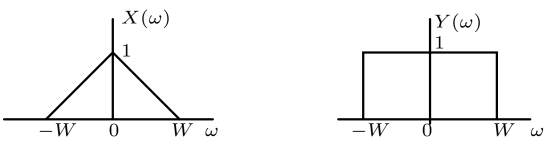
1. Show that the Fourier tranform of the triangular pulse $x(t)$ shown below is $X(\omega) = \dfrac{1}{\omega^2}(e^{-j\omega} + j\omega e^{-j\omega} - 1)$.

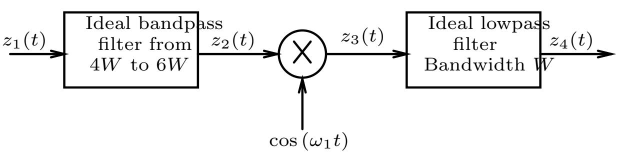


2. Consider the circuit shown below to be an LTI system with input $i_s(t)$ and output $v_0(t)$. Assume that the capacitor is initially uncharged. Using Fourier transforms, find the response to the input $i_s(t) = u(t)$ and sketch it.

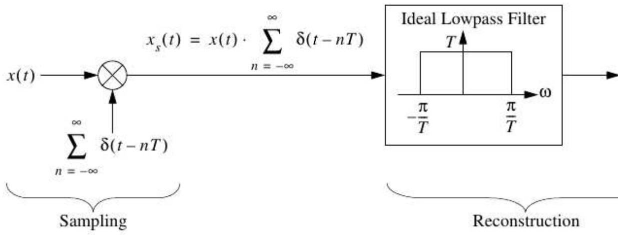


3. $x(t)$ and $y(t)$ have Fourier transforms as shown below. Sketch the Fourier transform of the various signals $z_i(t)$ for $i = 1, 2, 3, 4$ in the system shown below given that $z_1(t) = x(t)\cos(\omega_1 t) + y(t)\cos(\omega_2 t)$. Determine $z_4(t)$ in terms of $x(t)$ and $y(t)$? Assume that $\omega_1 = \omega_2 - 2W = 5W$.





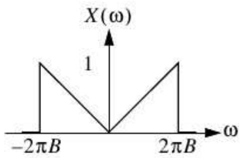
4. *Sampling* is the process whereby continuous-time signals are converted to discrete-time signals, while reconstruction is the inverse process. We will study sampling and reconstruction in depth at a later course.



In the sampling process, $x(t)$, the signal to be sampled, is multiplied by the impulse train, yielding the sampled signal $x_s(t)$. In order to perform reconstruction, we pass $x_s(t)$ through the ideal lowpass filter, yielding the reconstructed signal $x_{sr}(t)$.

(a) Write $X_s(\omega)$, the Fourier transform of $x_s(t)$, in terms of $X(\omega)$, the Fourier transform of $x(t)$. Your answer should be an explicit, closed-form expression, not one involving a convolution integral.

(b) Let $x(t)$ have the Fourier transform shown.



Sketch $X_s(\omega)$ for the cases $T = 1/(4B),\ 1/(2B),\ 1/B$.

(c) For the three cases considered in part (b), sketch $X_{sr}(\omega)$, the Fourier transform of $x_{sr}(t)$.

(d) For an arbitrary signal $x(t)$ that occupies a bandwidth of $2\pi B$ rad/s, what is the largest T such that $X_{sr}(\omega) = X(\omega)$, i.e., $x_{sr}(t) = x(t)$? Under these conditions, the original signal is reconstructed perfectly. This result is referred to as the *Nyquist Sampling Theorem.*

5. Suppose that $x(t) \longleftrightarrow X(\omega)$. Suppose that $x(t)$ is periodic with period $T_0 = 2\pi/\omega_0$.

Show that $X(\omega)$ is of the form:

$$X(\omega) = 2\pi \sum_{n=-\infty}^{\infty} X_n \delta(\omega - n\omega_0)$$

Relate the coefficients $X_n,\ -\infty < n < \infty$ to the exponential Fourier series coefficients of $x(t)$ and give a formula to find $X_n,\ -\infty < n < \infty$ in terms of $x(t)$.

6. Suppose that $x(t) \longleftrightarrow X(\omega)$. Suppose that $X(\omega)$ is periodic with period $_0 = 2\pi/T_0$, i.e.,

$$X(\omega + k\omega_0) = X(\omega),\ \forall\omega$$

   (a) Express $X(\omega)$ as a Fourier series:

$$X(\omega) = \sum_{n=-\infty}^{\infty} x_n e^{-jn\omega T_0} \qquad (1)$$

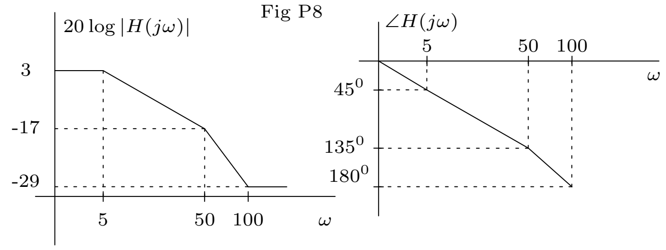
   The negative sign in the exponent is appropriate because this is a Fourier series in the frequency domain. Find an expression for the coefficients $x_n,\ -\infty < n < \infty$ in terms of $X(\omega)$. *Hint*: proceed as we did when we derived the time-domain Fourier series in class, i.e., multiply both sides of (1) by $e^{jm\omega T_0}$ and integrate over one period.

   (b) Show that $x(t)$ is of the form:

$$x(t) = \sum_{n=-\infty}^{\infty} x_n \delta(t - nT_0)$$

7. For a causal LTI system with real impulse response the Bode plot shown below, if the input is $x(t) = 0.5\cos(4t + 32^0) + 1.5\cos(30t + 48^0) + 2\sin(75t + 5^0)$, find the output $y(t)$ of the system.

Fig P8



8. For each of the four signals $x(t)$ given below: (1) calculate the unilateral Laplace transform $X(s)$ and its ROC, (2) determine if $\{s|s = j\omega, \omega \in R\} \subset ROC$ and (3) calculate the inverse Fourier transform if possible, $y(t) = F^{-1}\{X(j\omega)\}$ and explain why $x(t) = y(t)$ or $x(t) \neq y(t)$.

   (a) $x(t) = u(t-2)$

   (b) $x(t) = e^{3t}u(t)$

   (c) $x(t) = te^{t}u(t)$

   (d) $x(t) = \sin t \cdot u(t)$

9. For each of the four signals $x(t)$ given below: (1) calculate the unilateral Laplace transform $X(s)$ and its ROC and (2) if $\{s|s = j\omega,\ \omega \in R\} \subset ROC$, then calculate $X(j\omega)$, the Fourier transform of $x(t)$.

   (a) $x(t) = \sinh t \cdot u(t)$, where $\sinh t = (e^t - e^{-t})/2$

   (b) $x(t) = u(t) - u(t-3)$

   (c) $x(t) = te^{t}u(t)$

   (d) $x(t) = \sin(\omega_0 t + \phi) \cdot u(t)$