# EC2102 Networks and Systems – HW 5

September 24, 2012

1. Sketch the following functions:

   (a) $\mathrm{rect}(t/2)$, (b)$\mathrm{rect}((t-10)/8)$, and (c) $\mathrm{sinc}(t/5)\cdot\mathrm{rect}(t/10)$. [$\mathrm{sinc}(x)=\frac{\sin(\pi x)}{\pi x}$]

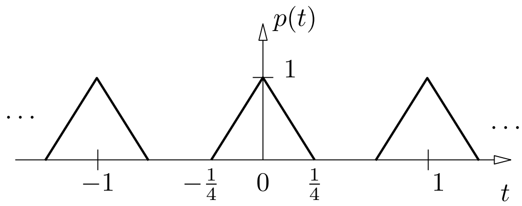
2. Show that $\displaystyle\int_{-\infty}^{\infty}\mathrm{sinc}(x)dx=\int_{-\infty}^{\infty}\mathrm{sinc}^2(x)dx=1.$

3. For each of the following signals $x(t)$, compute the Fourier transform $X(\omega)$.

   (a) $x(t)=e^{-|t|/2}$

   (b) $x(t)=\sin(2\pi t)\cdot[e^{-t}u(t)]$

4. Let $p(t)$ be periodic traingular pulse train shown below.




   (a) Calculate $P_n$, the exponential Fourier series coefficients of $p(t)$. Sketch $P_n$ vs. $n$. Calculate $P(\omega)$, the Fourier transform of $p(t)$. Sketch $P(\omega)$ vs. $\omega$.

   (b) Let $x(t)$ be an aperiodic signal having Fourier transform $X(\omega)$. Define $y(t)=p(t)\cdot x(t)$. Find an expression for $Y(\omega)$, the Fourier transform of $y(t)$.

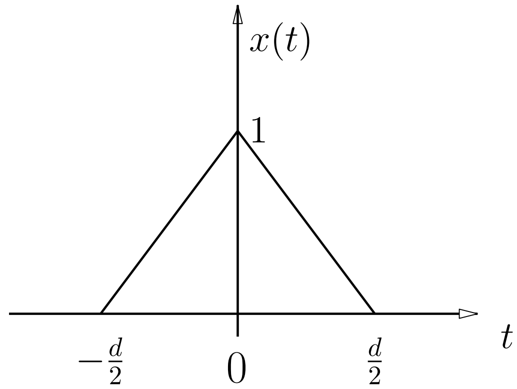
   (c) Let $x(t)=\mathrm{sinc}(t)$. Sketch $Y(\omega)$.

5. Find the Fourier transform of $x(t)$ shown below in three different ways:

   (a) directly through integration,

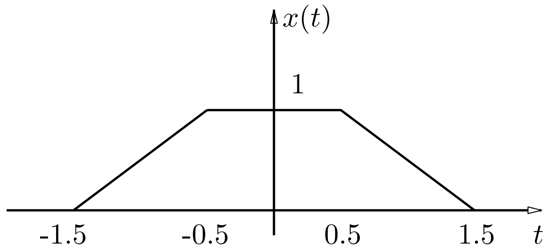
   (b) using the time-differentiation property of the Fourier transform, and

   (c) using the Fourier transform of the $\mathrm{rect}(\cdot)$ function and the convolution property of the Fourier transform.




   Sketch the Fourier transform $X(\omega)$.

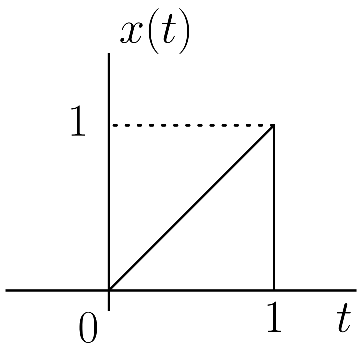
6. Find the Fourier transform $X(\omega)$ of the signal $x(t)$ shown below.



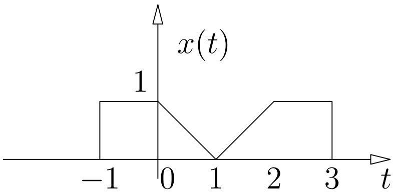

7. Find the energy of the signal $x(t)=e^{-at}u(t)$. Determine the frequency $W$ (in rad/s) so that the energy contributed by the spectral components of all the frequencies below $W$ is 95% of the signal energy $E_x$.

8. If $f_1(t)=2\cdot\mathrm{rect}\,(t/4)$ and $f_2(t)=\mathrm{rect}\,(t/2)$, find $g(t)=f_1(t)\star f_2(t)$ and $G(\omega)$. Sketch the maginitude and phase of $G(\omega)$.

9. The Fourier tranform of the triangular pulse $x(t)$ shown below is $X(\omega)=\dfrac{1}{\omega^2}(e^{-j\omega}+j\omega e^{-j\omega}-1)$. Using this, find the Fourier transforms of $x_1(t)=\mathrm{rect}(t)\star\mathrm{rect}(t)$ and $x_2(t)=\mathrm{rect}(t/2)$.



10. Let $X(\omega)$ be the Fourier transform of the signal $x(t)$ shown below. Do the following computations without explicitly evaluating $X(\omega)$.




(i) Find $X(0)$. (ii) Evaluate $\int_{-\infty}^{\infty} X(\omega) d\omega$. (iii) Evaluate $\int_{-\infty}^{\infty} X(\omega) e^{j\omega} d\omega$. (iv) Evaluate $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$. (v) Sketch the inverse Fourier transform of $\mathrm{Re}[X(\omega)]$.

11. Consider the circuit shown below to be an LTI system with input $i_s(t)$ and output $v_0(t)$. Assume that the capacitor is initially uncharged. Using Fourier transforms, find the response to the input $i_s(t) = u(t)$ and sketch it.